

SEPARATORS

SEPARATORS

WITH NON-HEREDITARY PROPERTIES

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WITH NON-HEREDITARY PROPERTIES

Pinar Heggernes, Pim van't Hof, Dániel Marx, and Yngve Villanger

2-CONNECTED SEPARATORS

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The Treewidth Reduction Theorem

2-CONNECTED SEPARATORS

The Treewidth Reduction Theorem

2-CONNECTED STEINER TREES

2-CONNECTED SEPARATORS

The Treewidth Reduction Theorem

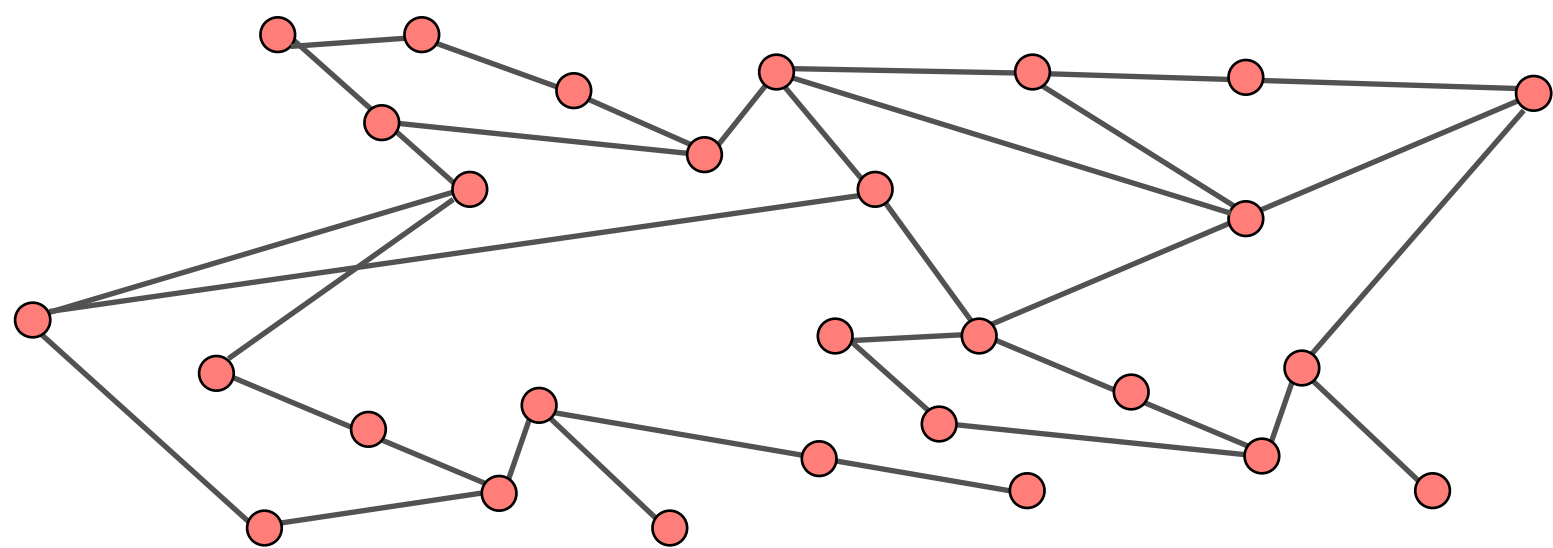
2-CONNECTED STEINER TREES

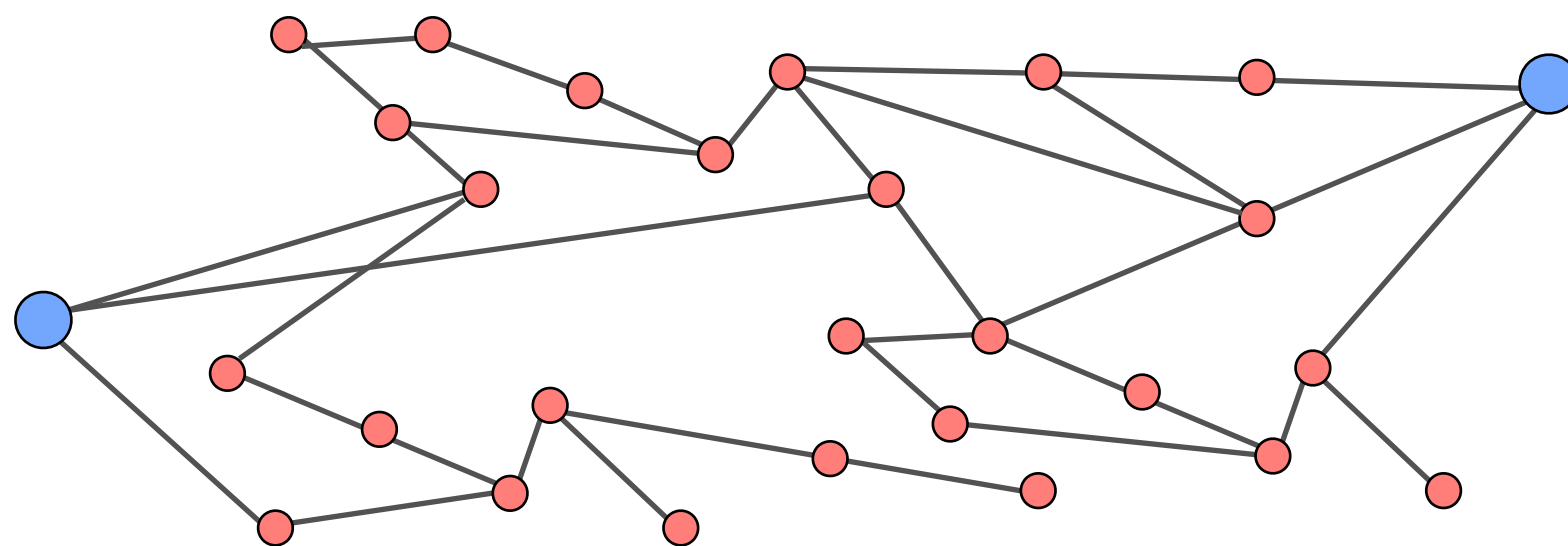
Some Structural Observations

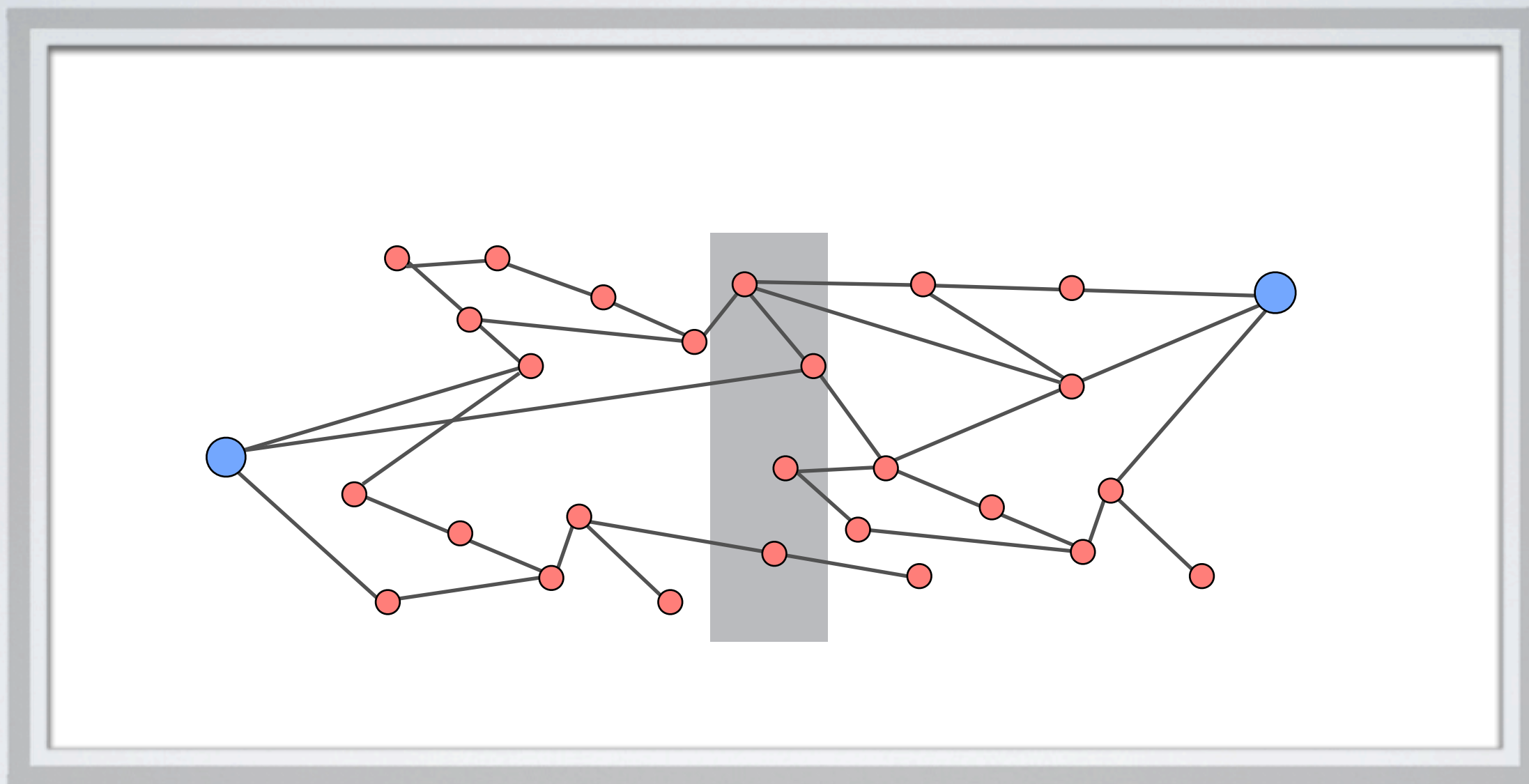


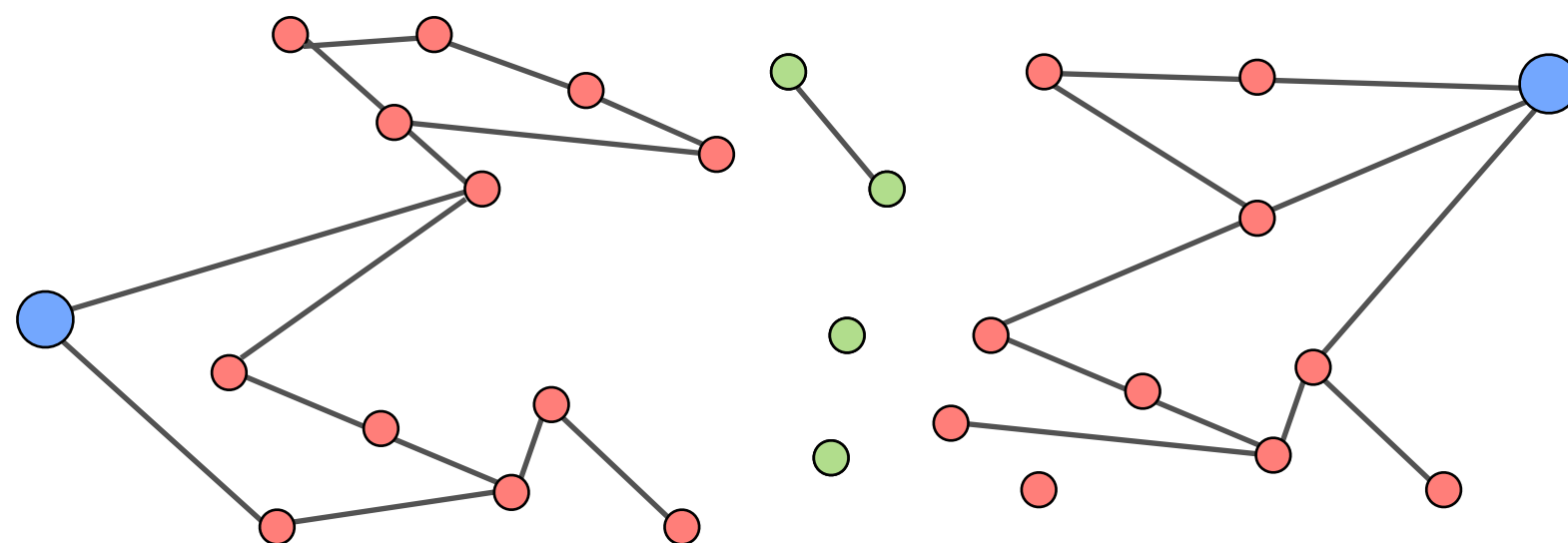


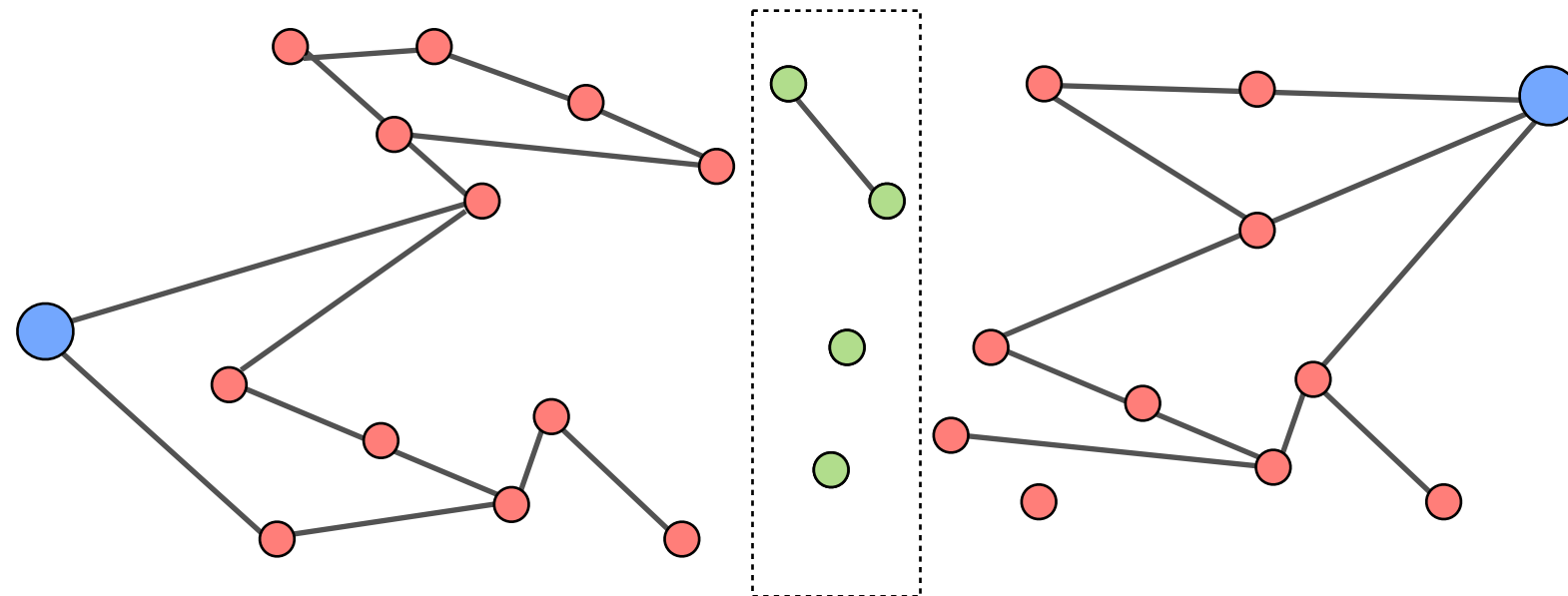
PRELIMINARIES

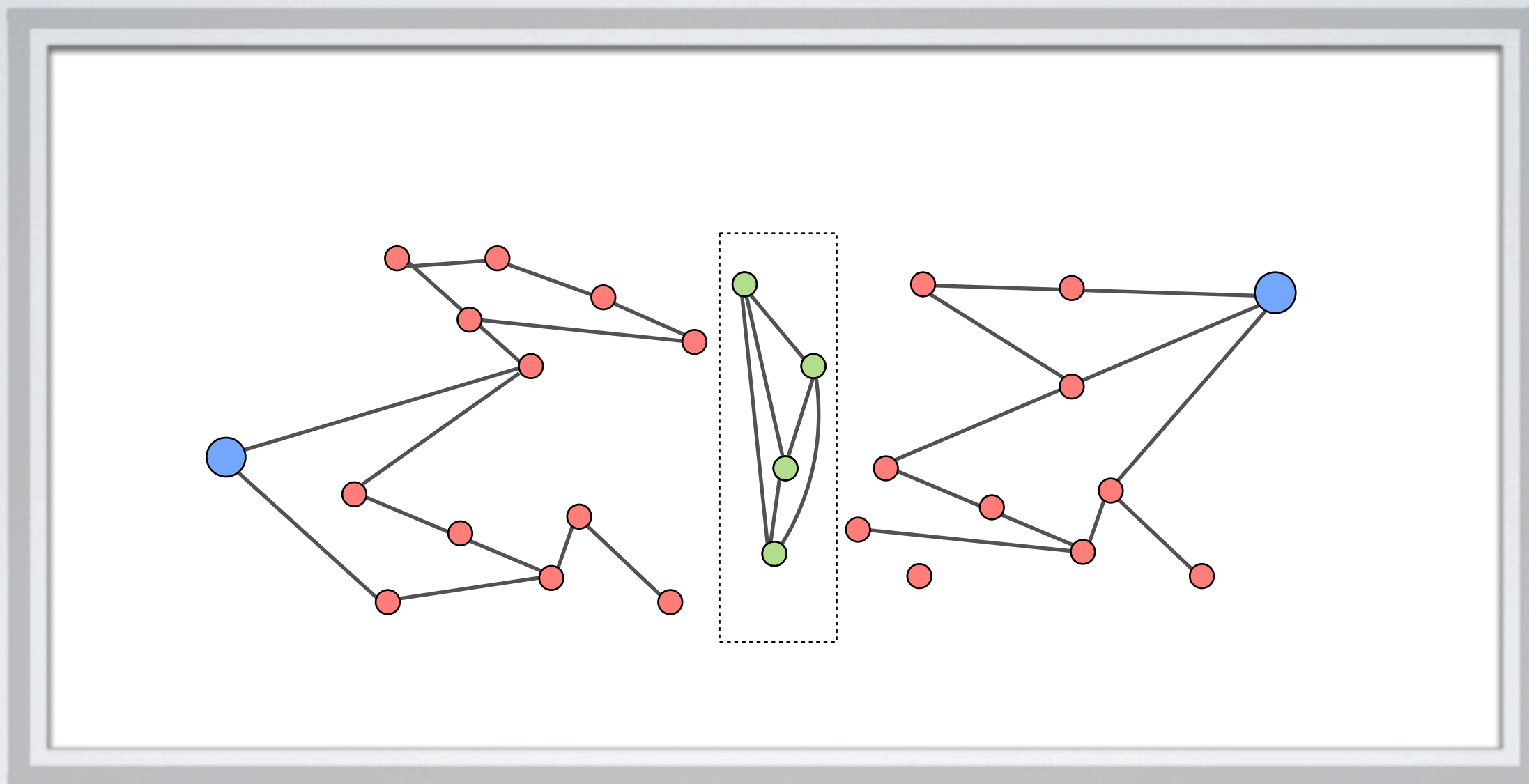






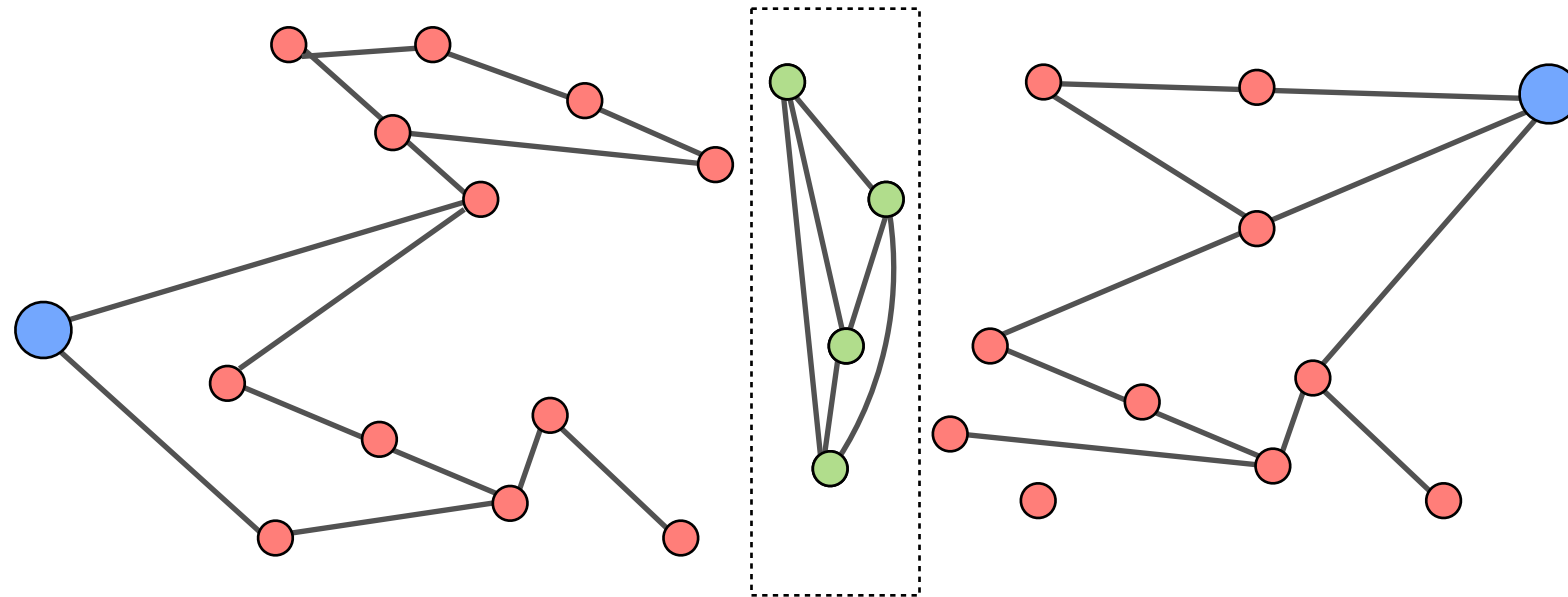




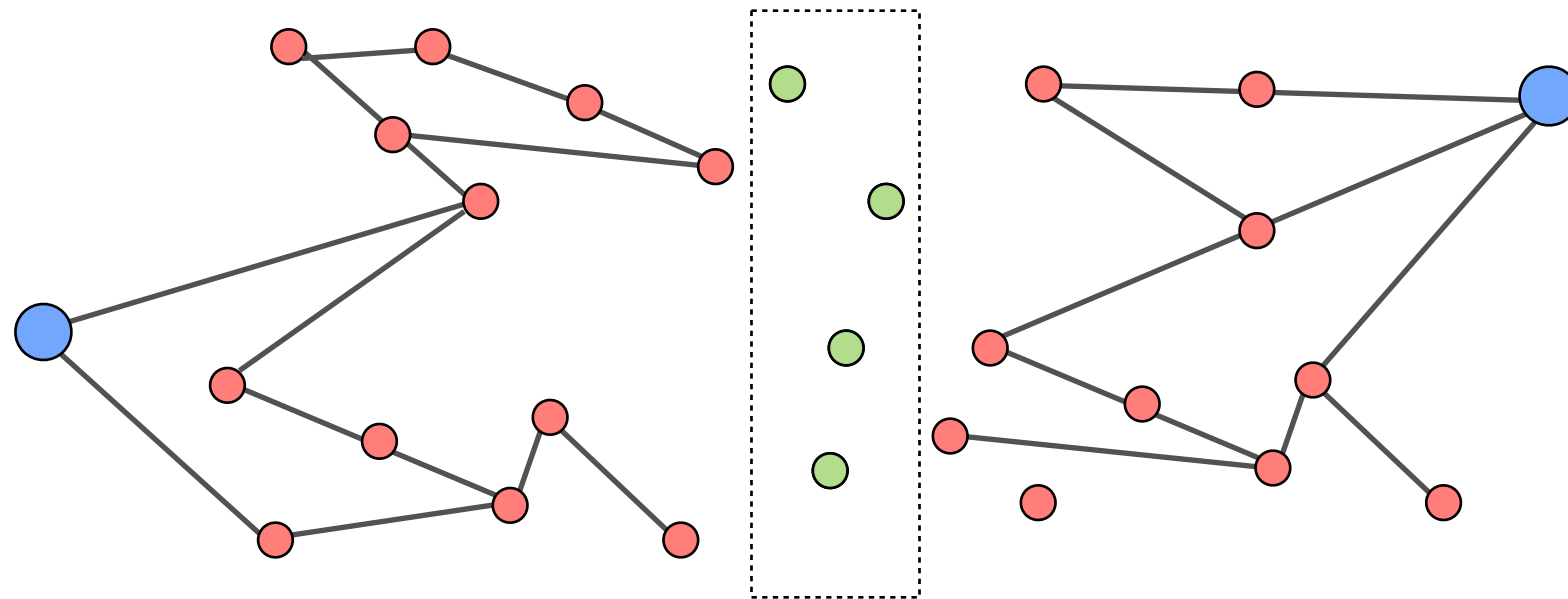


Cliques

Polynomial Time

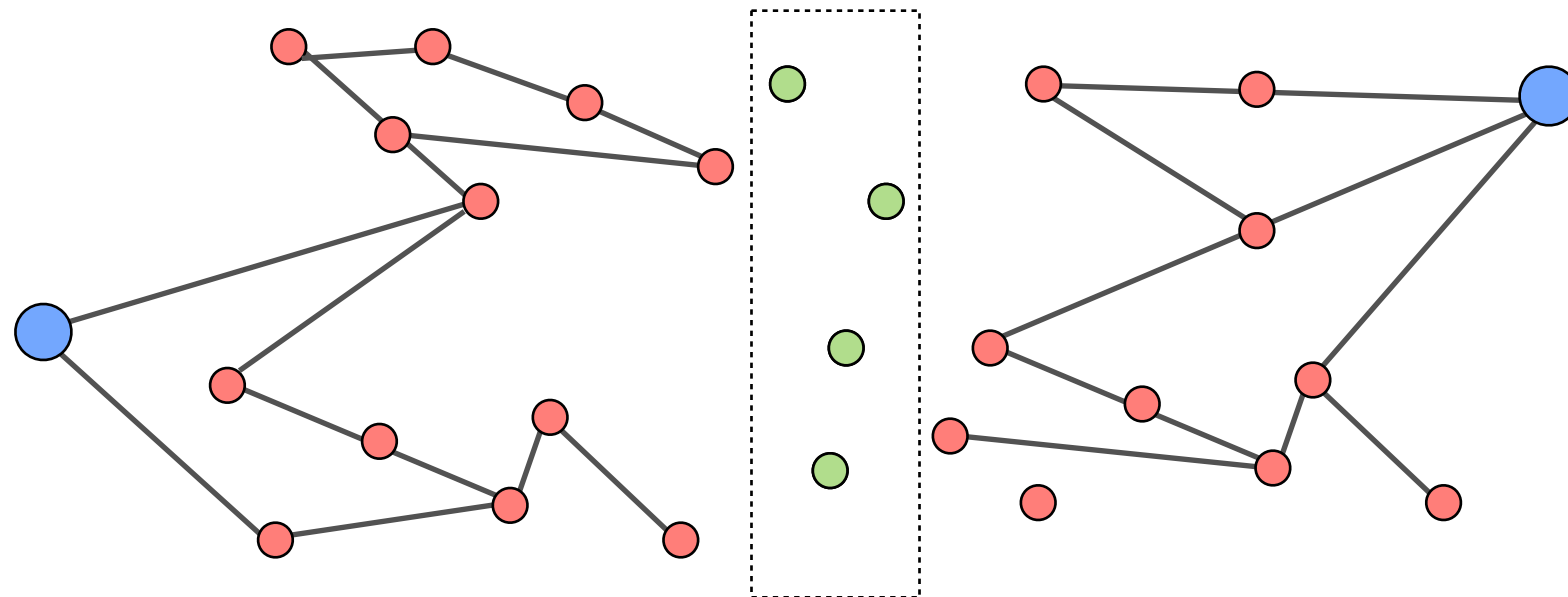


Cliques



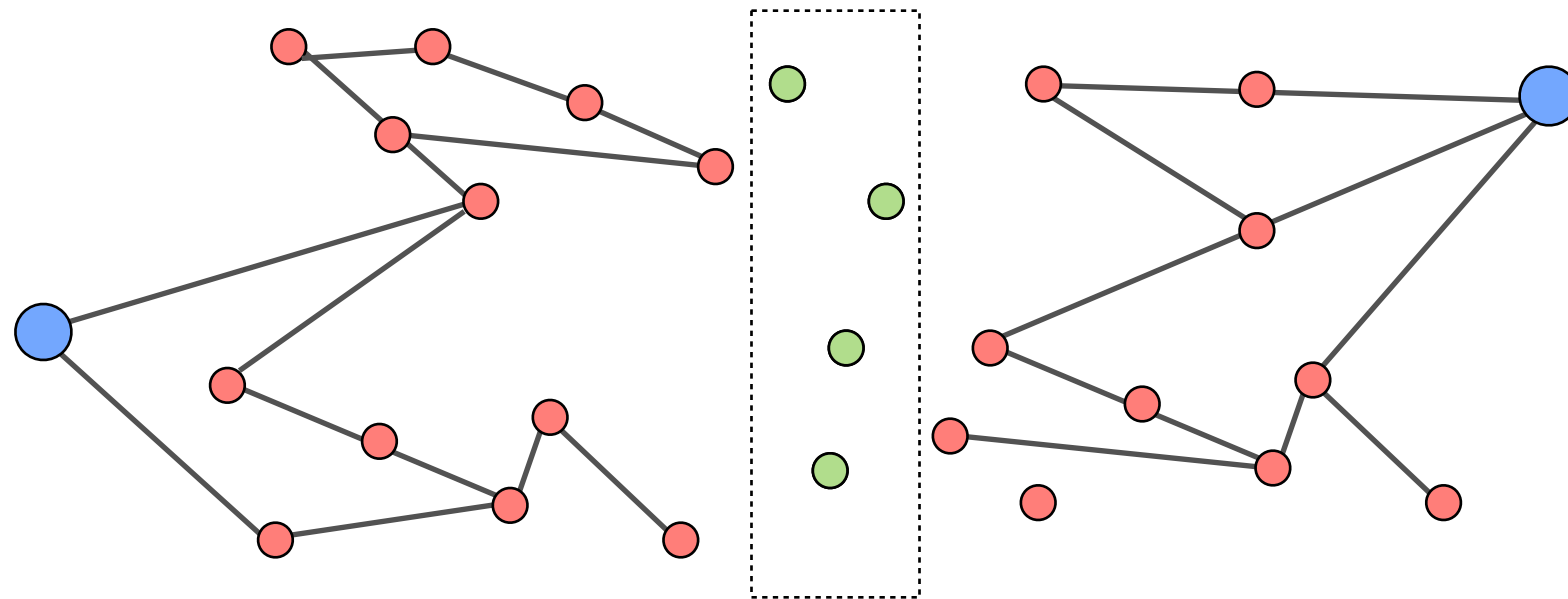
Independent Set

Fixed Parameter Tractable



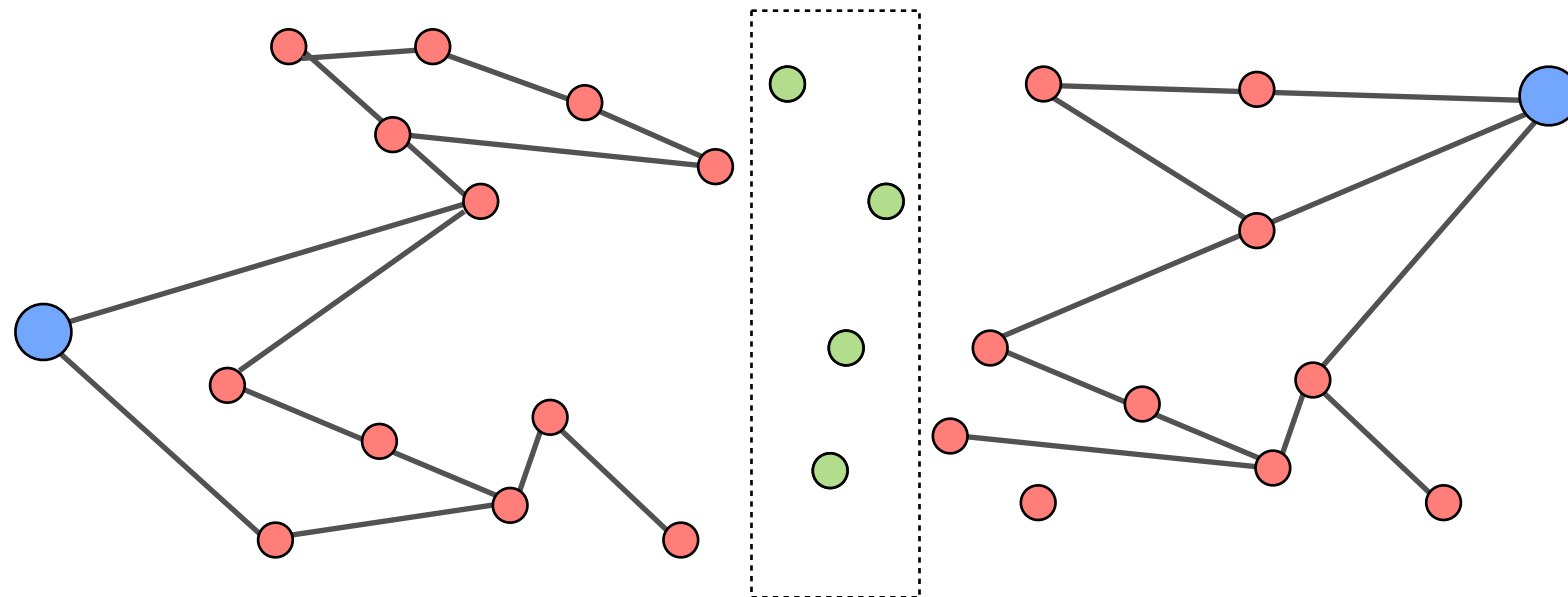
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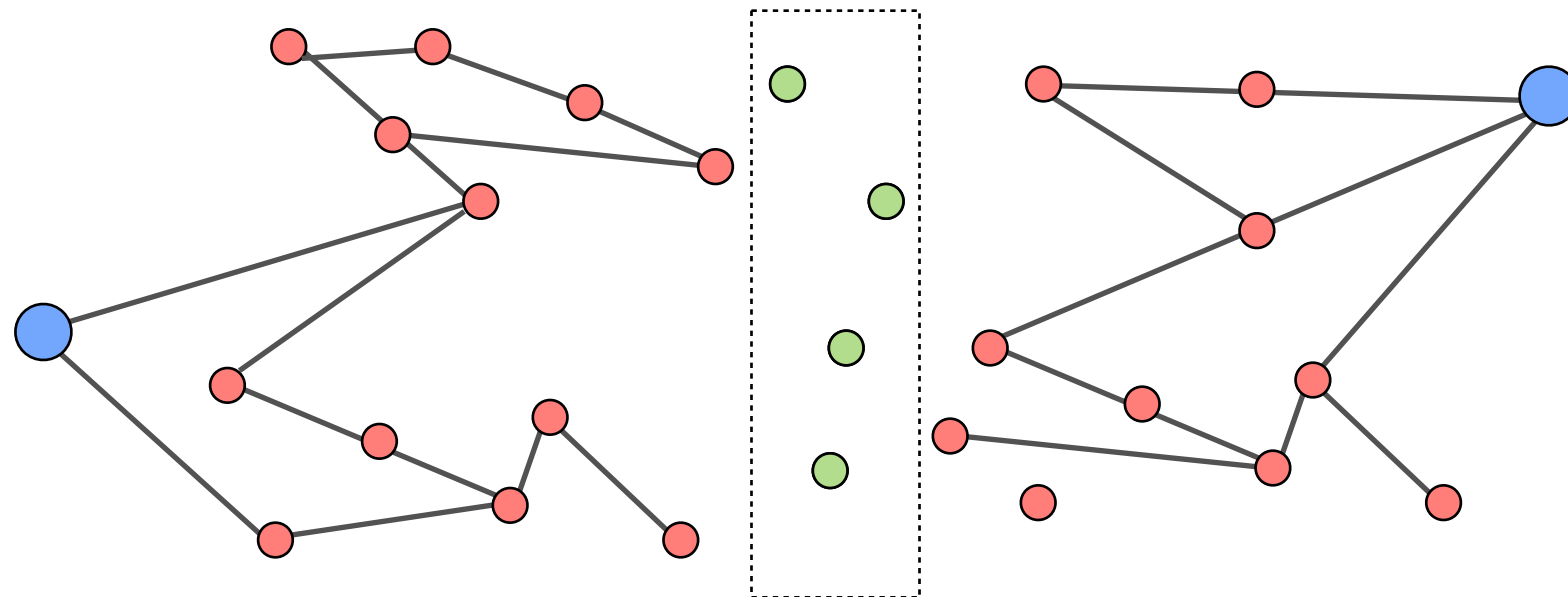
Your favorite Hereditary Property

Fixed Parameter Tractable



Tarjan; Marx, Sullivan and Razgon

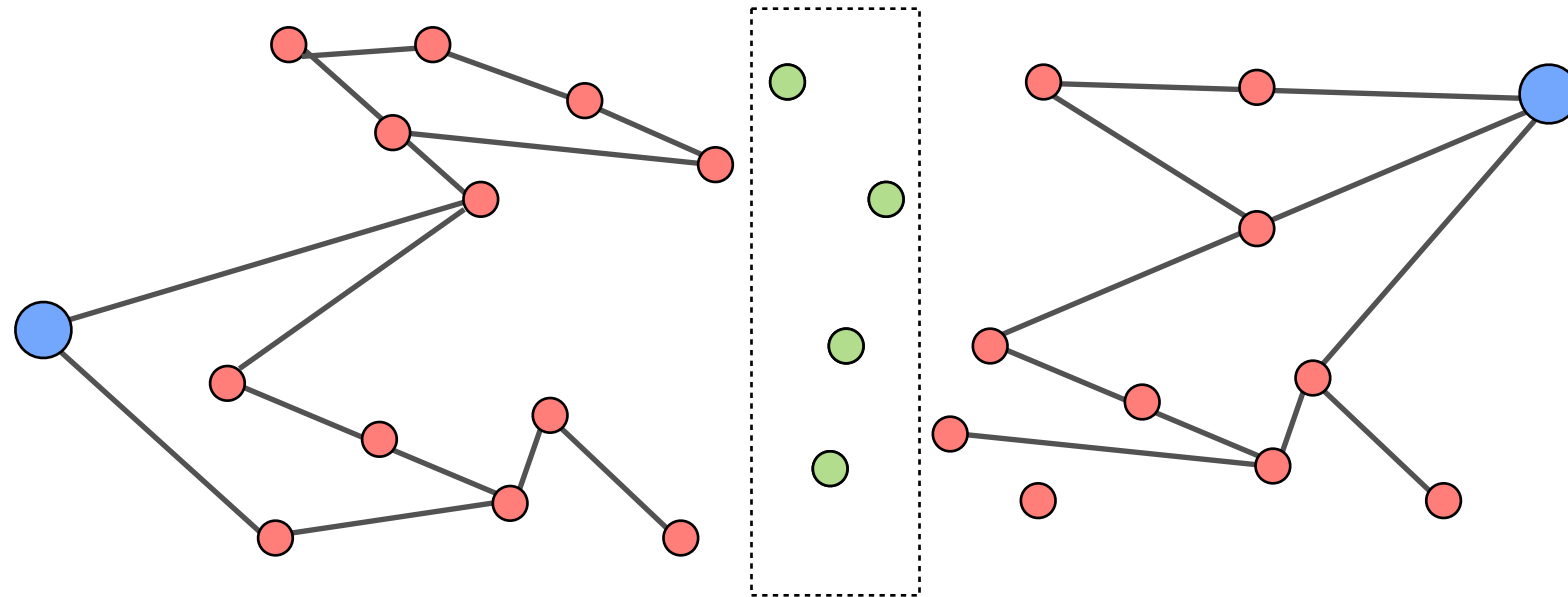
Your favorite Hereditary Property



Tarjan; Marx, Sullivan and Razgon

What about non-hereditary properties?

?



Tarjan; Marx, Sullivan and Razgon

What about non-hereditary properties?

Connected Separators

2- Connected Separators

c- Connected Separators

c- Connected Separators

O-Regular Separators

c -Connected Separators

c -Regular Separators

c -Connected Separators

c -Regular Separators

1-Diameter Separators

c -Connected Separators

c -Regular Separators

c -Diameter Separators





2-CONNECTED SEPARATORS

THE 2-CONNECTED SEPARATOR PROBLEM

THE 2-CONNECTED SEPARATOR PROBLEM

THE 2-CONNECTED SEPARATOR PROBLEM

A (s,t) separator of size at most k
that induces a 2-connected subgraph.

THE 2-CONNECTED SEPARATOR PROBLEM

"Easy" on graphs of small treewidth.

(Due to MSO expressibility.)

THE 2-CONNECTED SEPARATOR PROBLEM

"Easy" on graphs of small treewidth.
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General graphs



Equivalent instances with small
treewidth

THE 2-CONNECTED SEPARATOR PROBLEM

The Treewidth Reduction Theorem

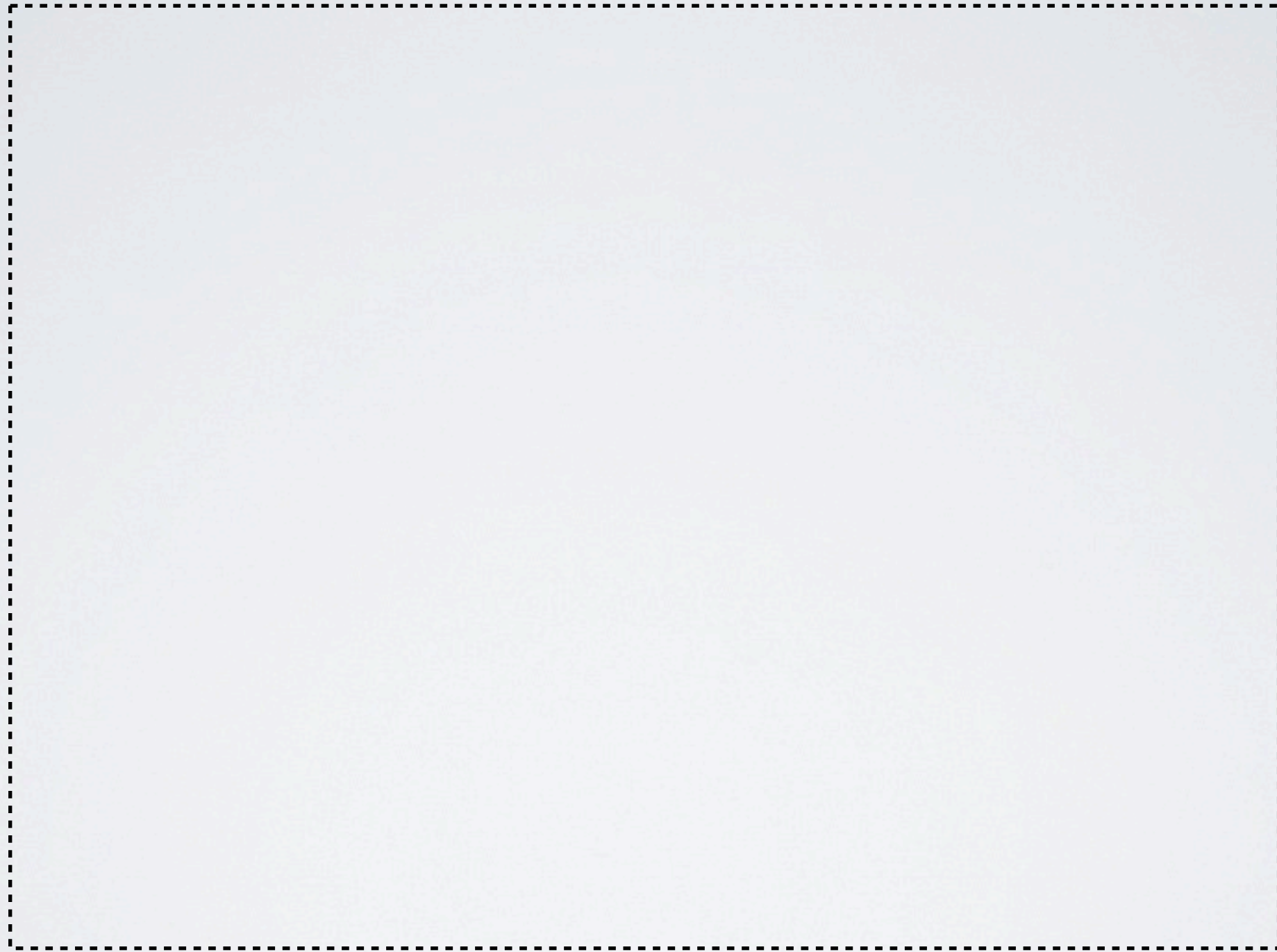
General graphs



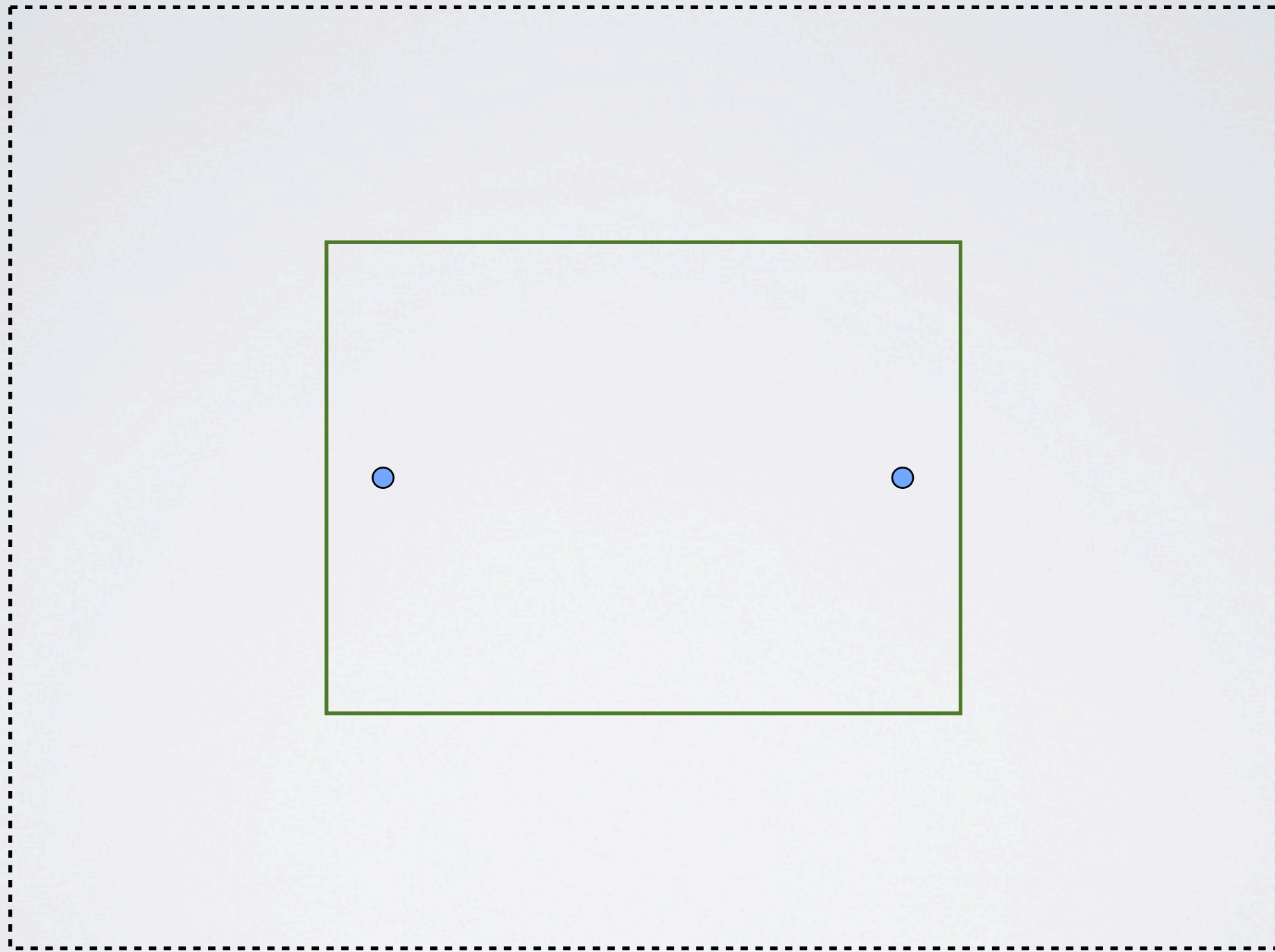
Equivalent instances with small
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THE 2-CONNECTED SEPARATOR PROBLEM

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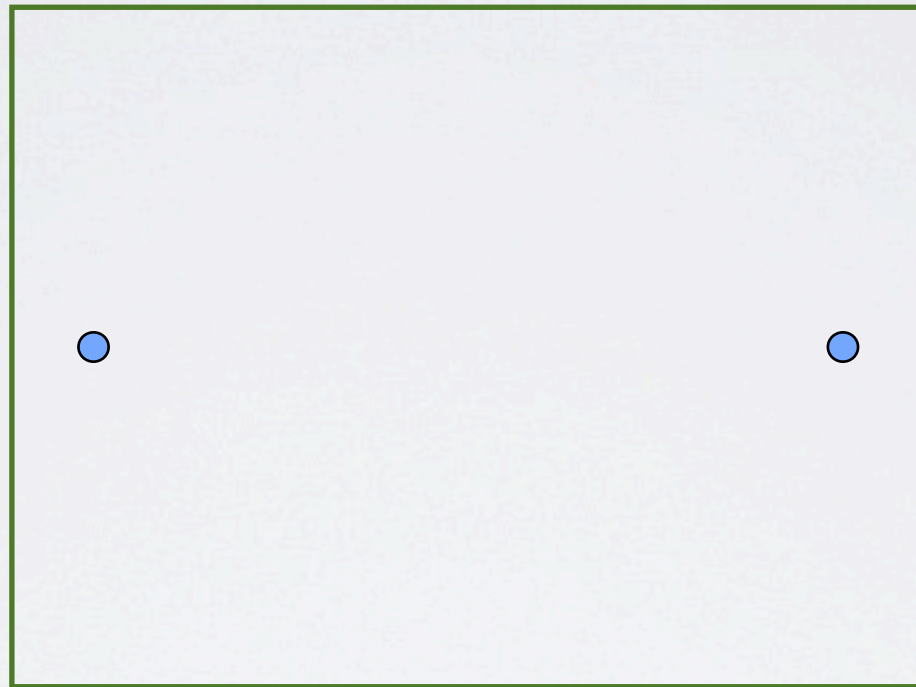


THE 2-CONNECTED SEPARATOR PROBLEM



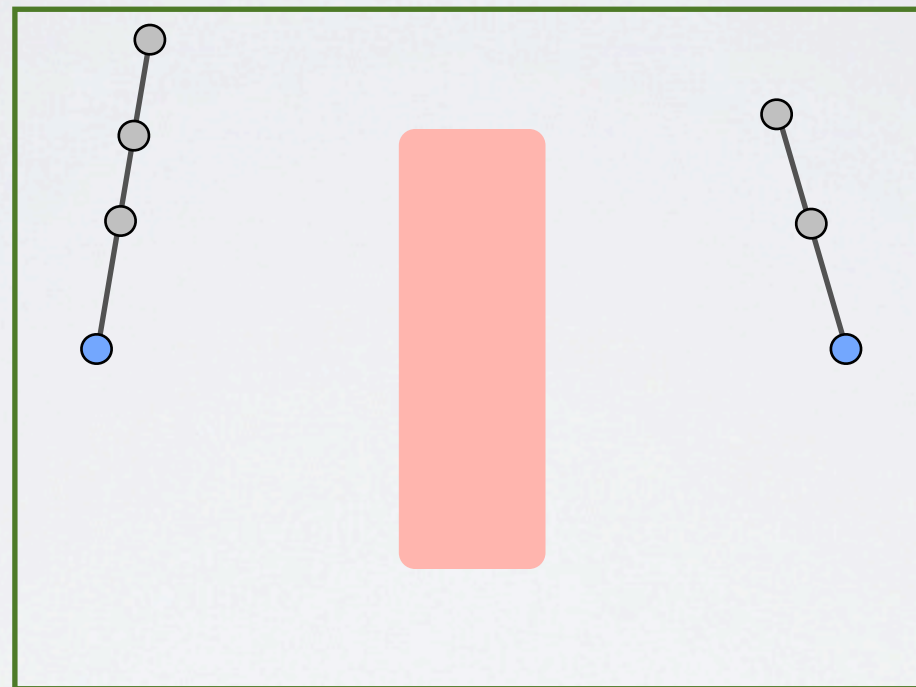
H contains all minimal (s,t) separators and $tw(H) = g(k)$

THE 2-CONNECTED SEPARATOR PROBLEM



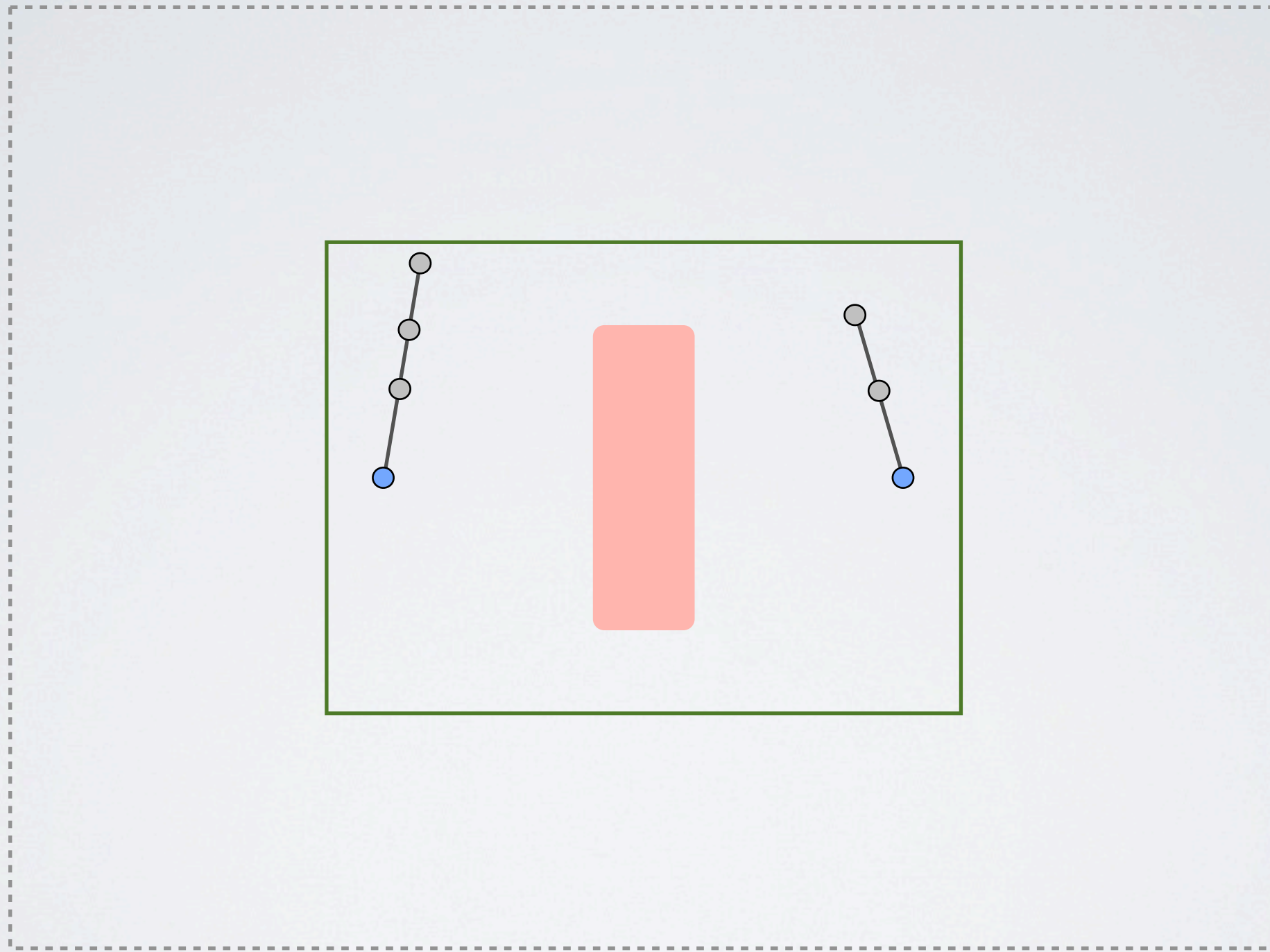
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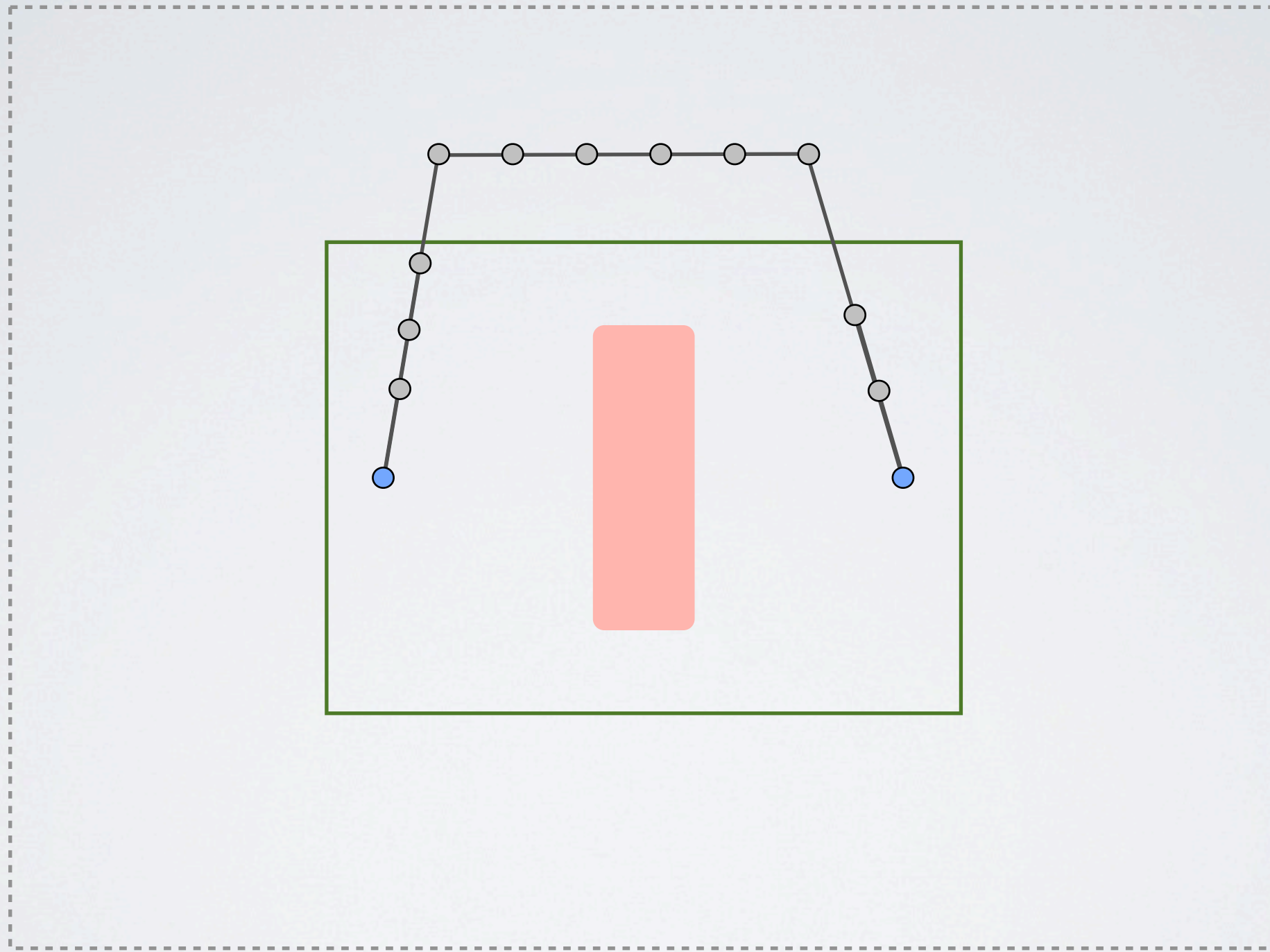
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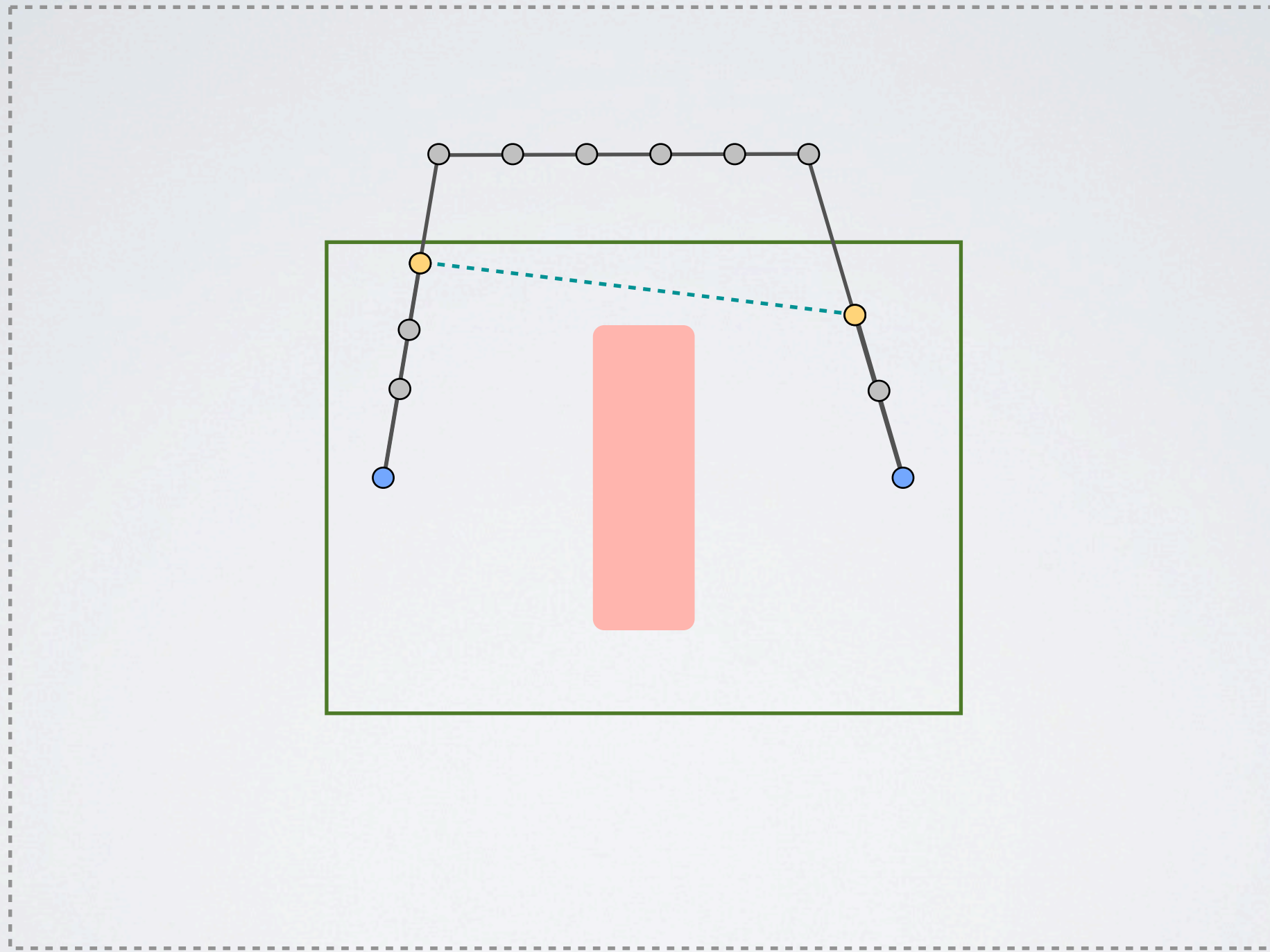
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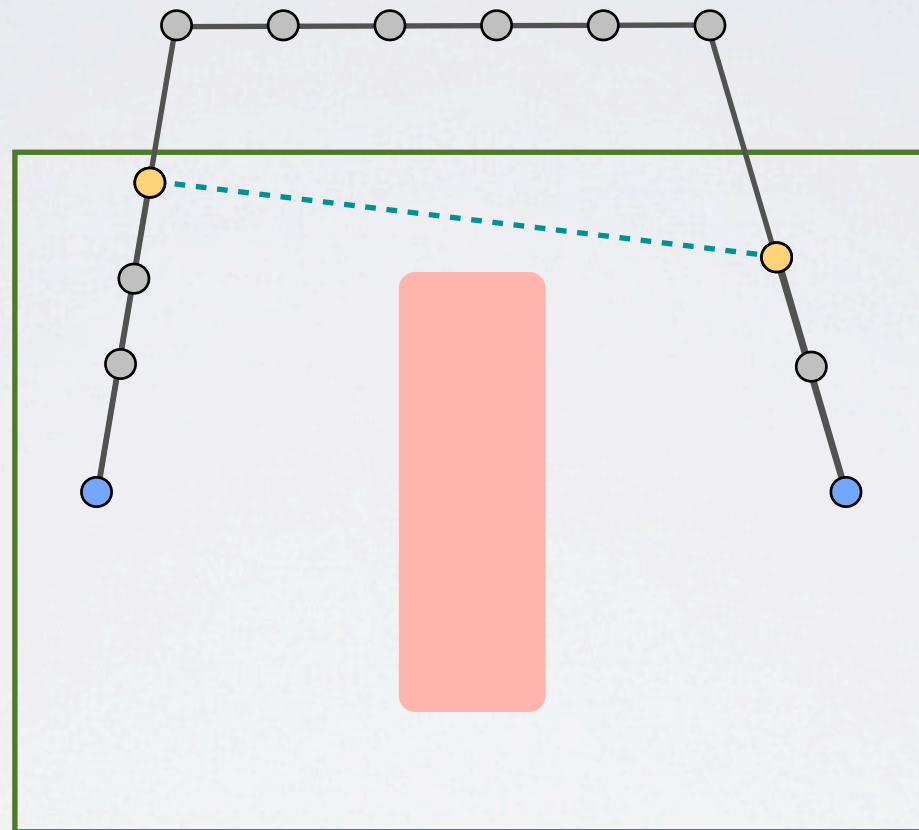
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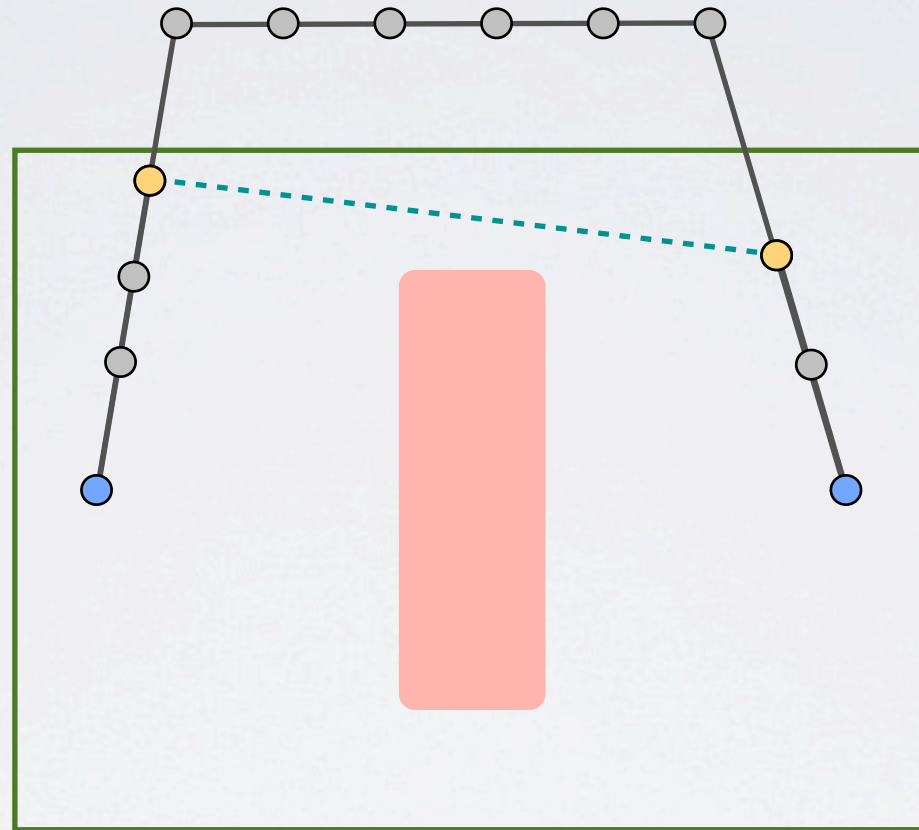
THE 2-CONNECTED SEPARATOR PROBLEM



$G\{H\}$ = torso of H in G

H contains all minimal (s,t) separators and $tw(H) = g(k)$

THE 2-CONNECTED SEPARATOR PROBLEM

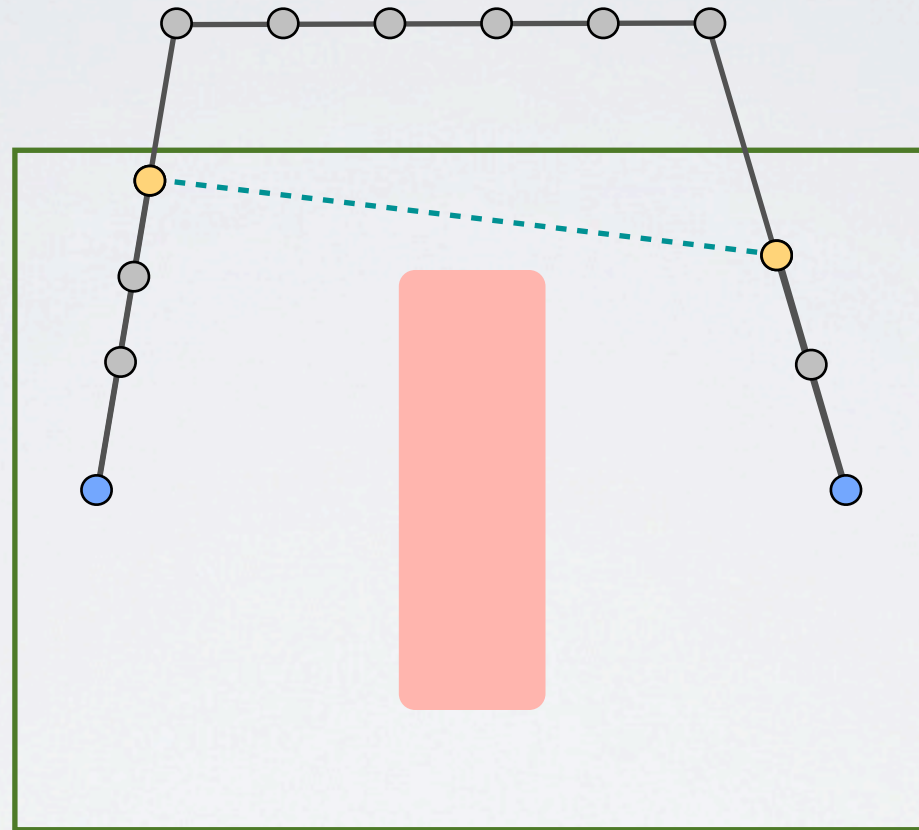


$G\{H\}$ = torso of H in G

2-connected

H contains all minimal $\uparrow$ (s,t) separators and $tw(H) = g(k)$

THE 2-CONNECTED SEPARATOR PROBLEM



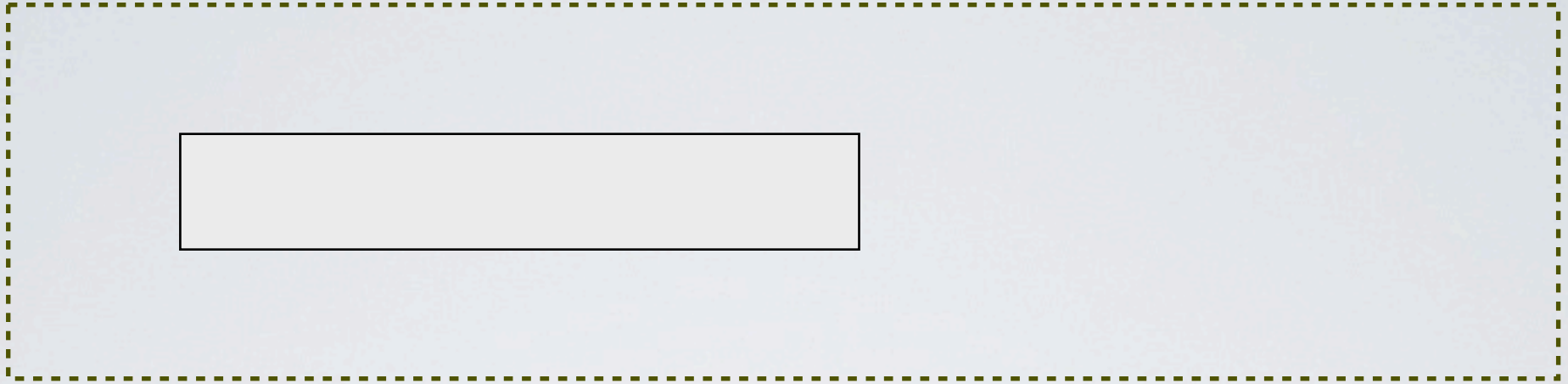
$G\{H\}$ = torso of H in G

witnesses

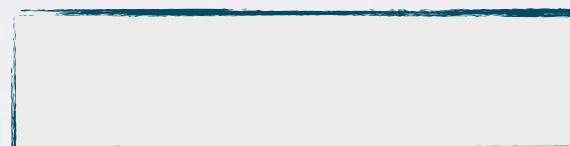
2-connected

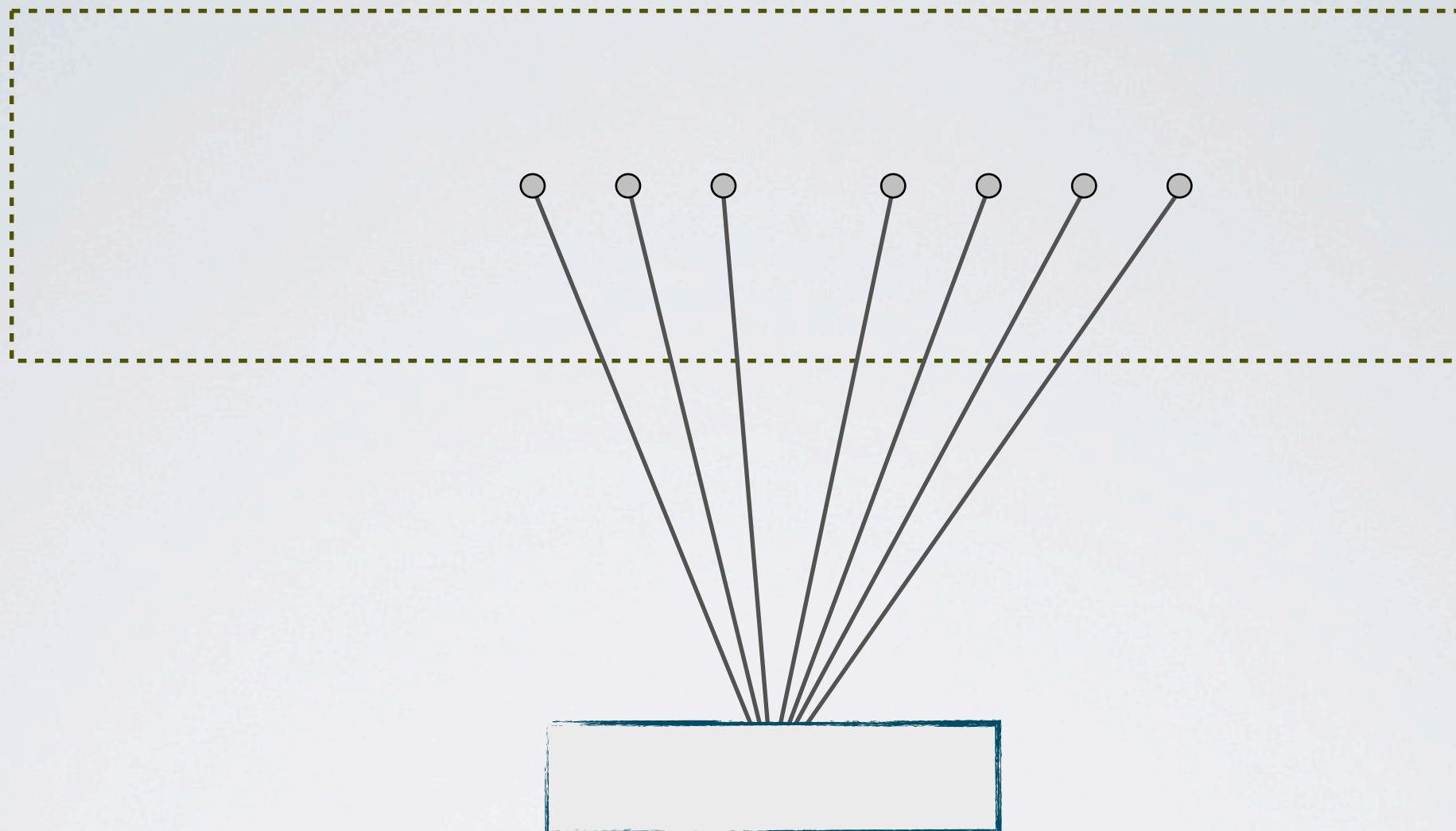
~~H contains~~ all minimal $\uparrow$ (s,t) separators and $tw(H) = g(k)$

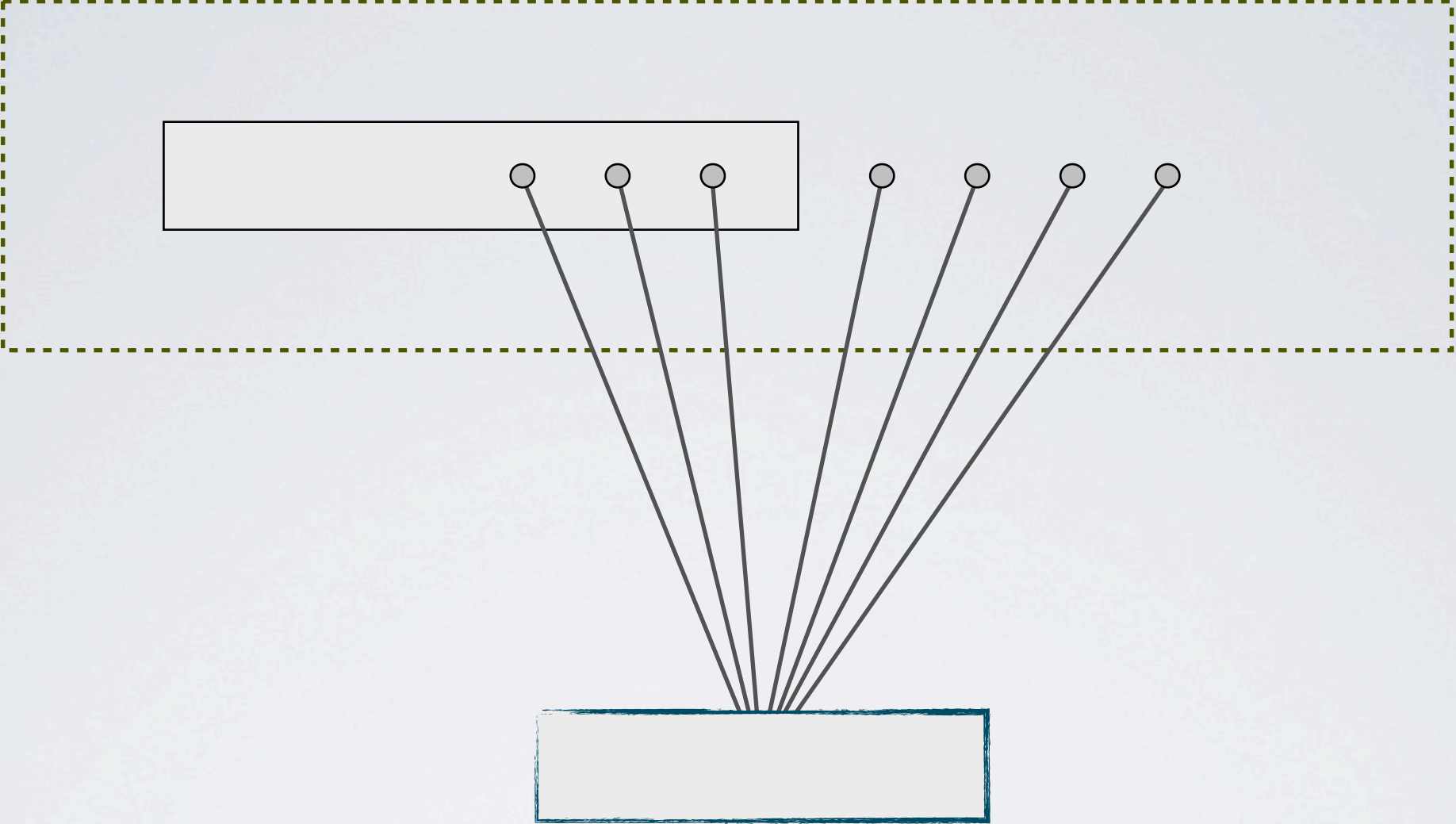


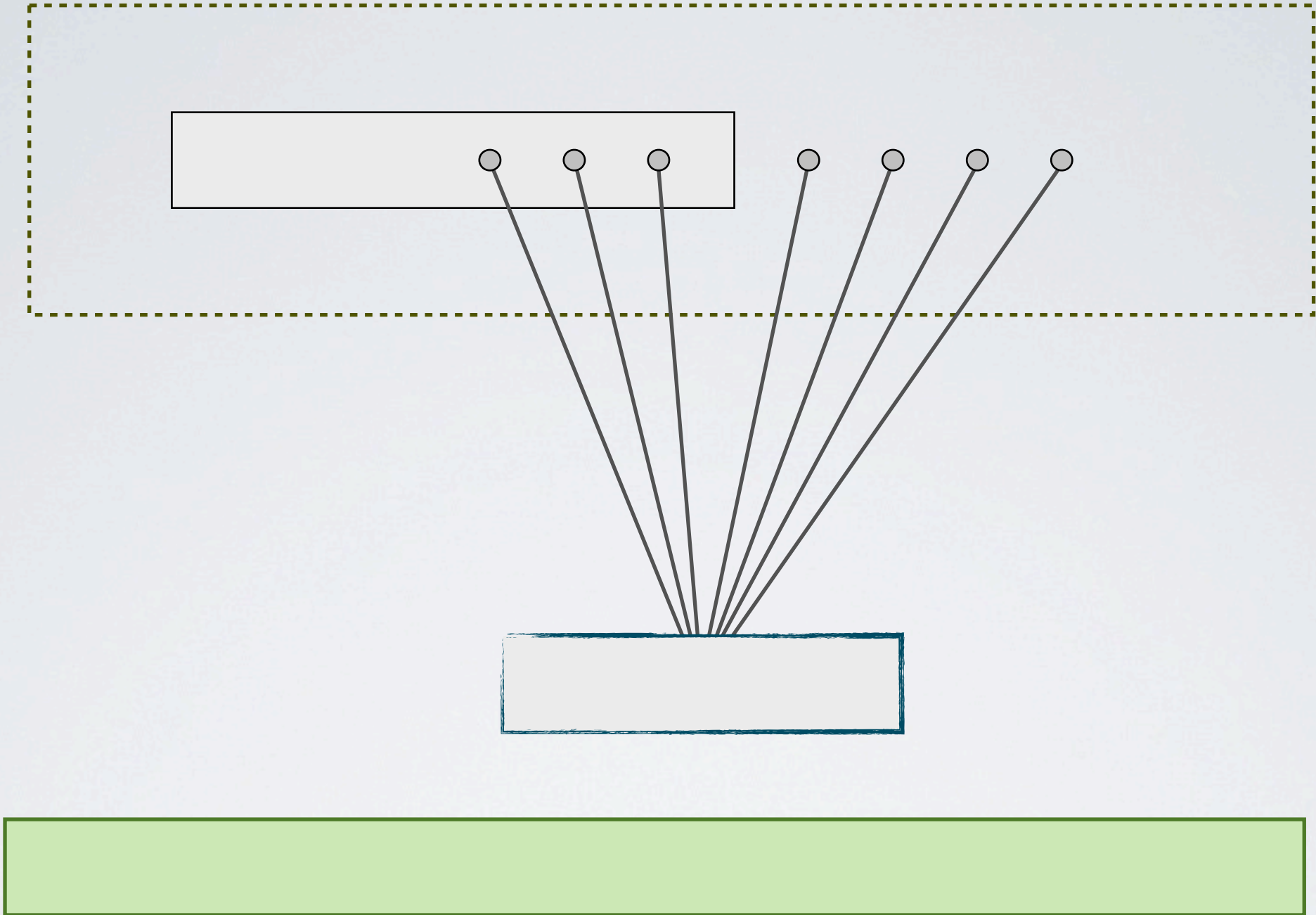


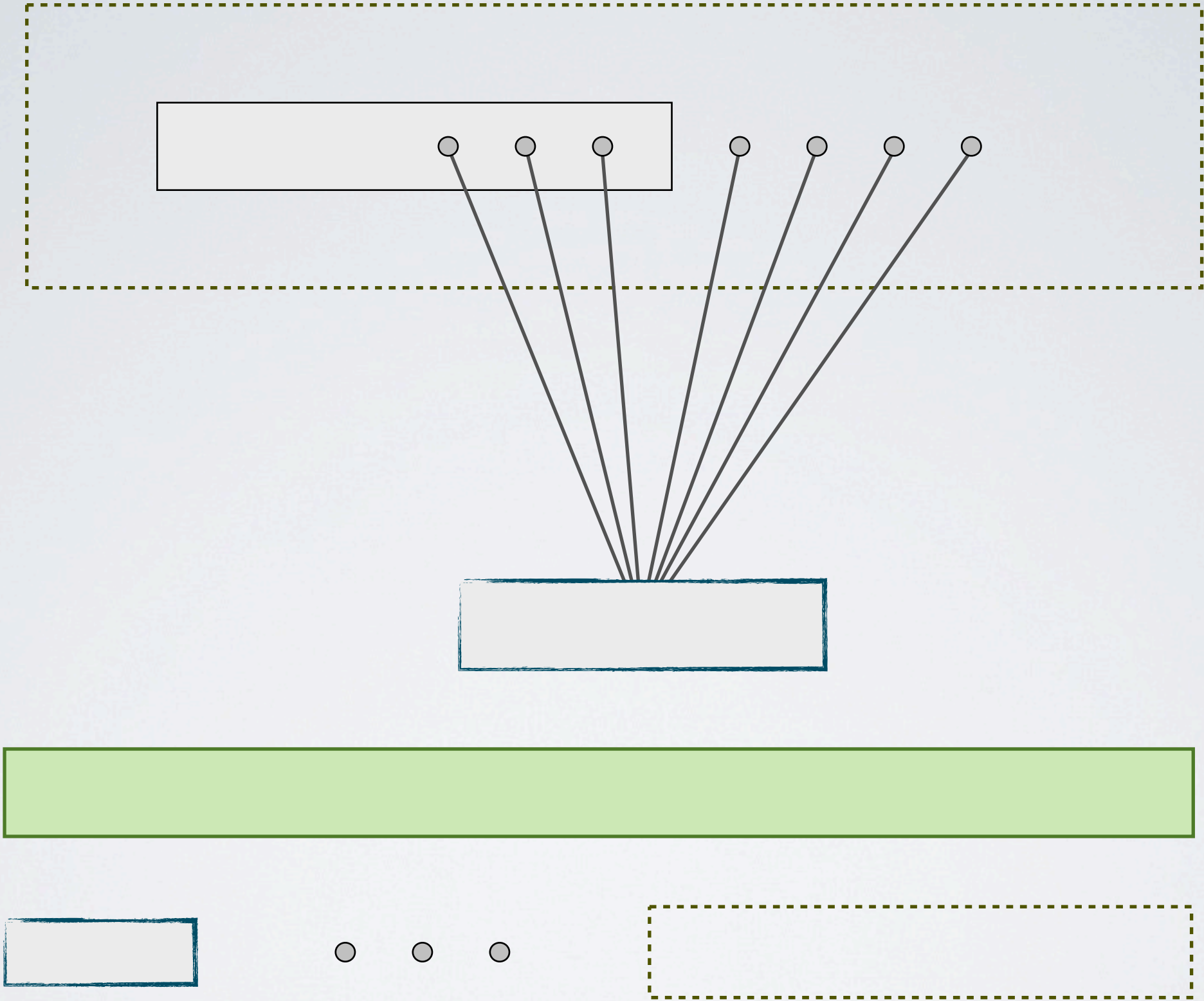


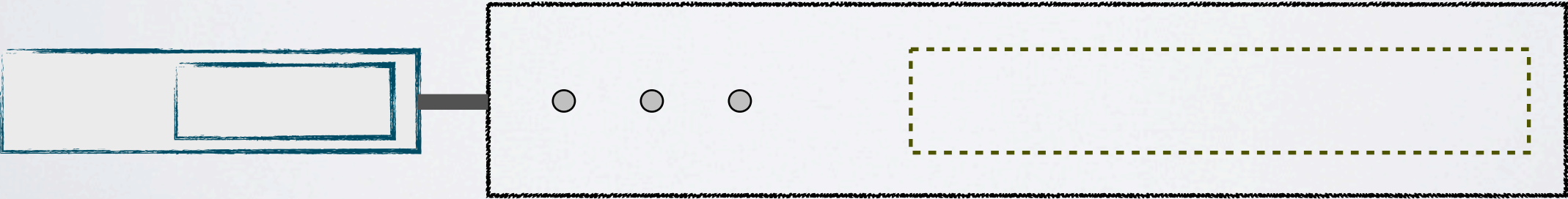
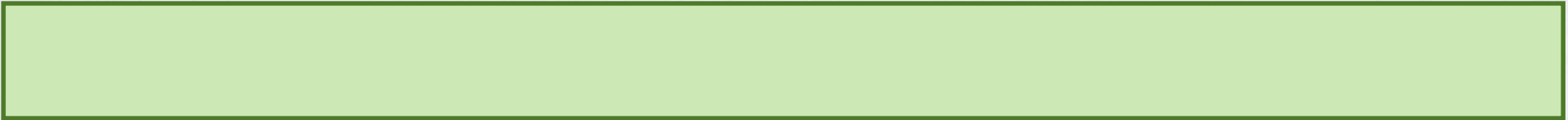
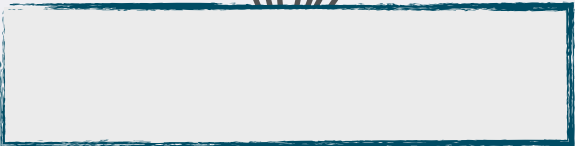
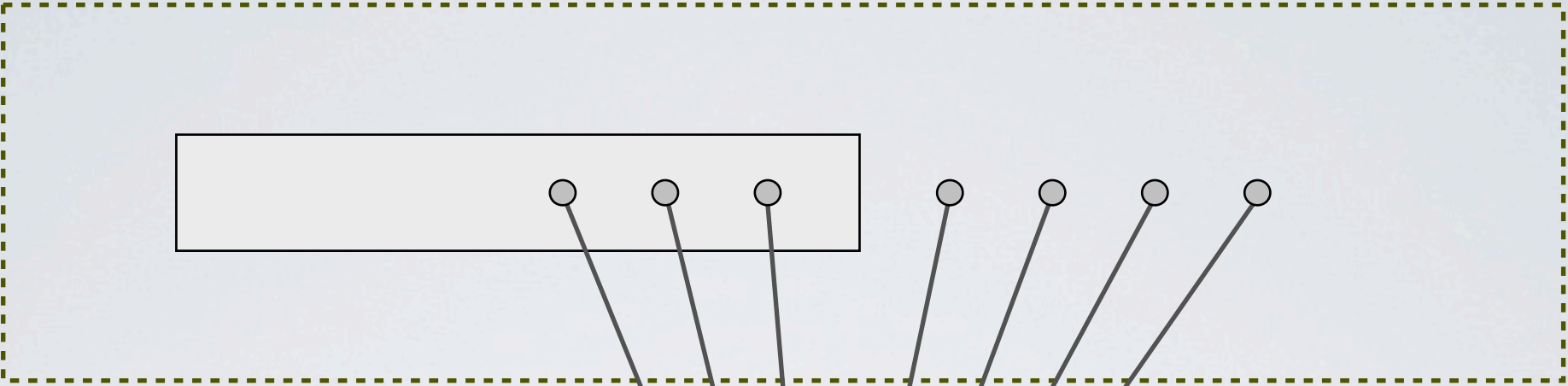




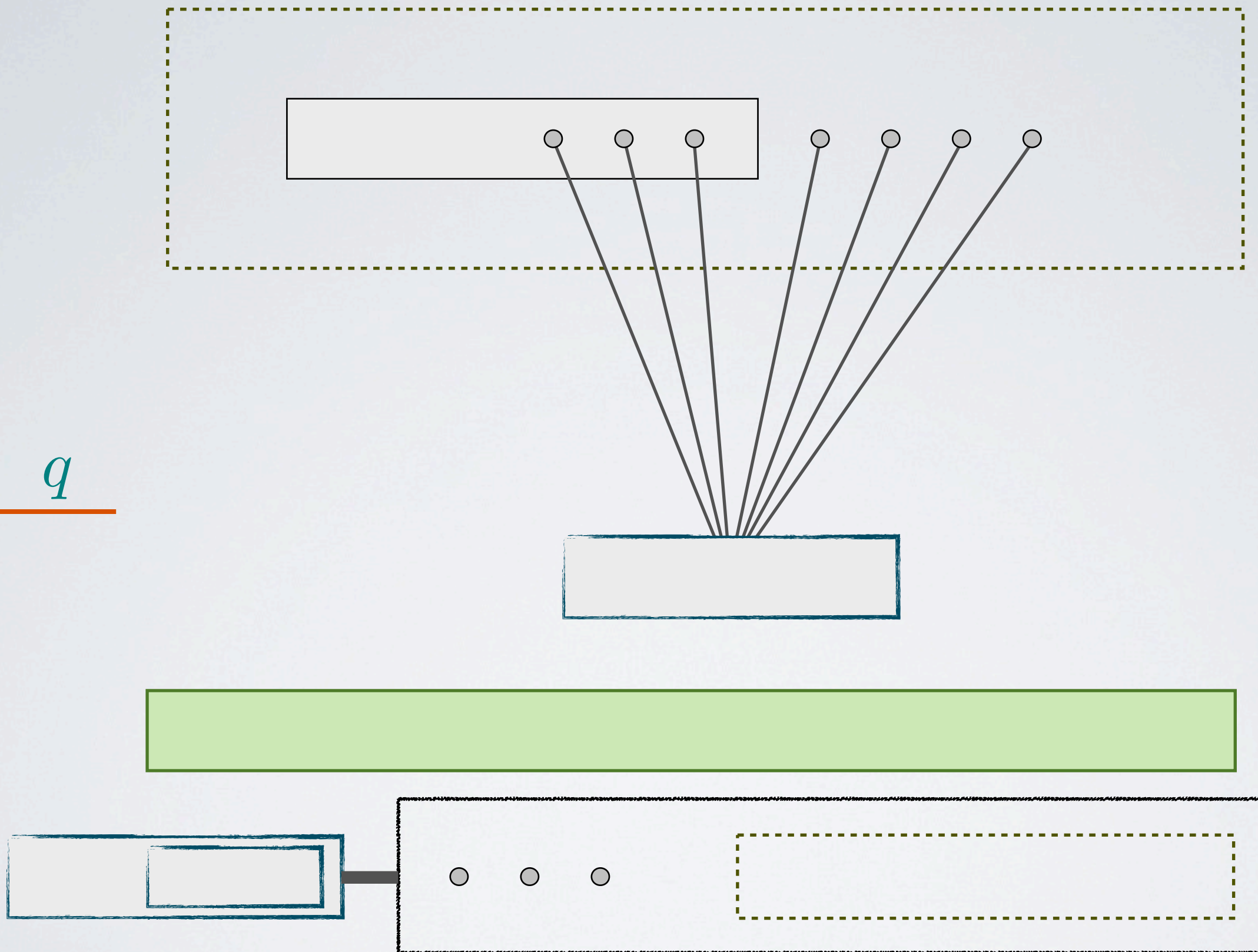




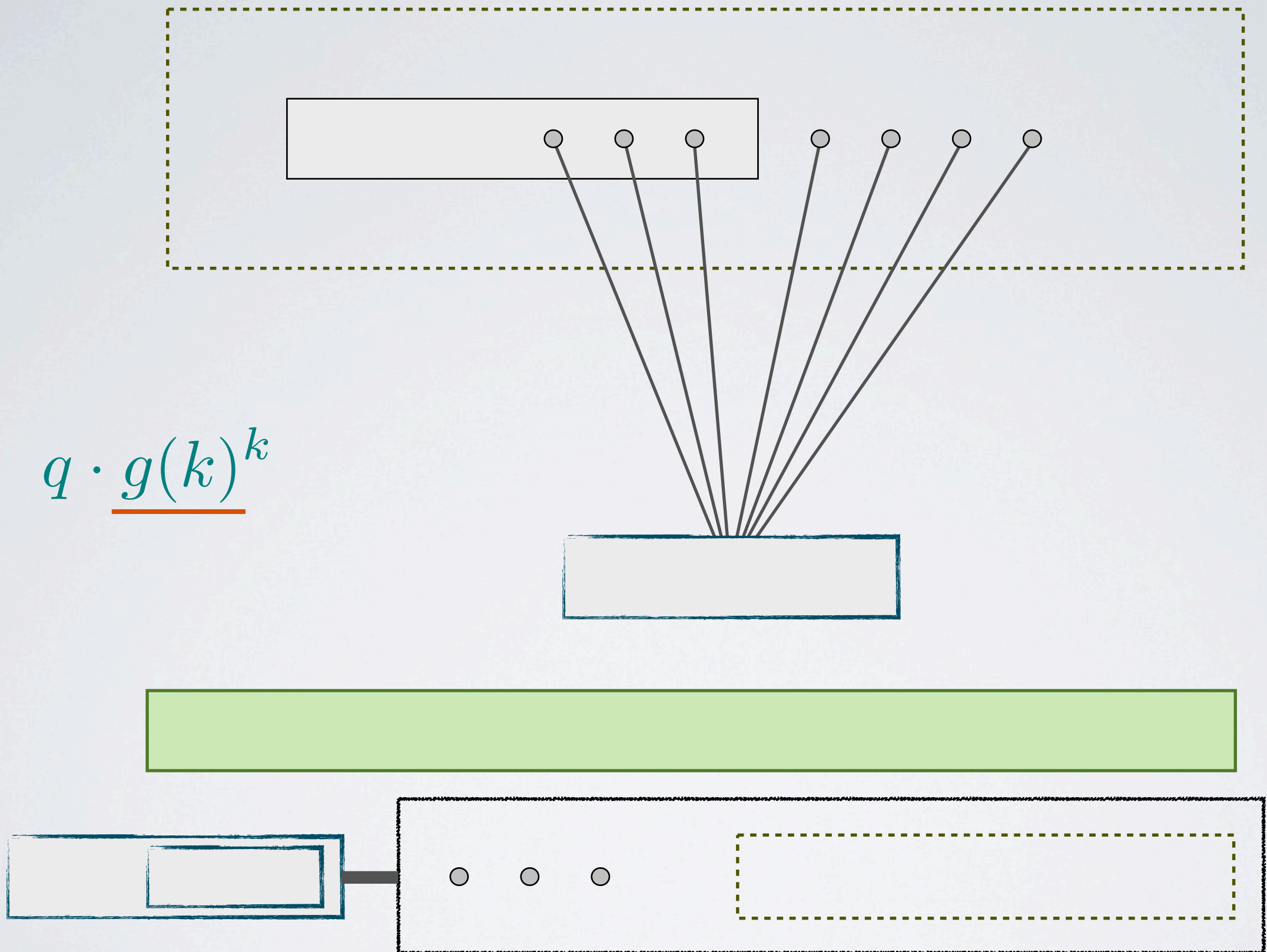


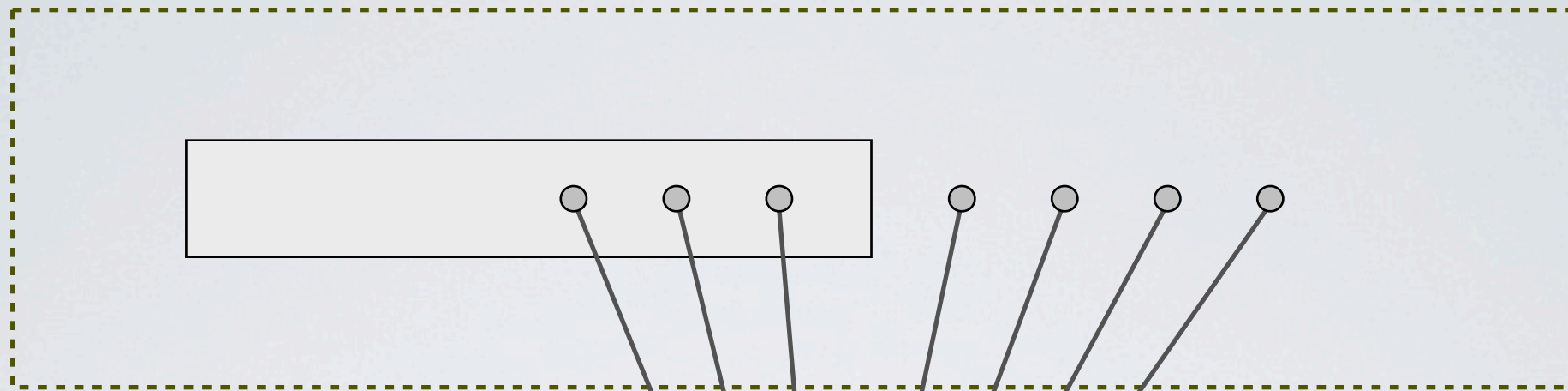


q

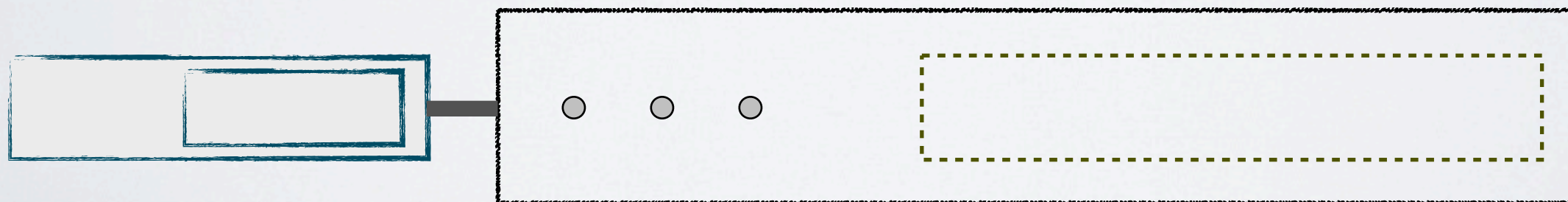
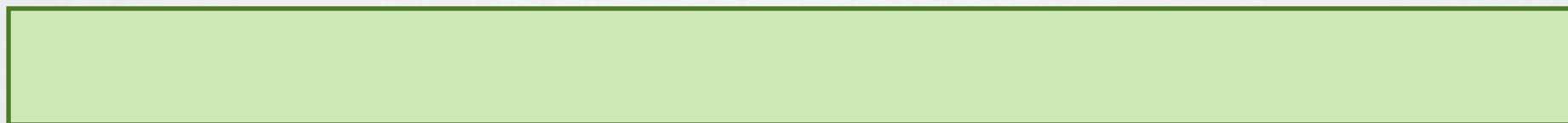
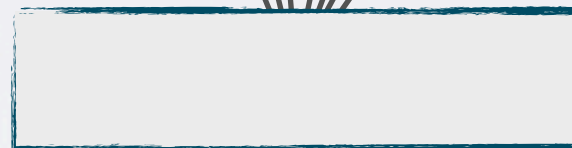


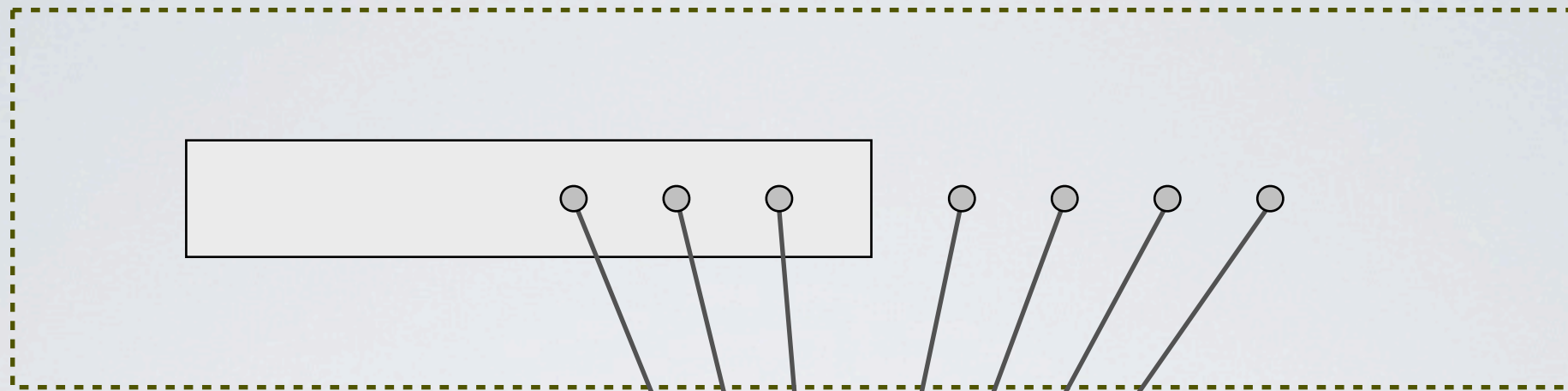
$$q \cdot \underline{g(k)^k}$$



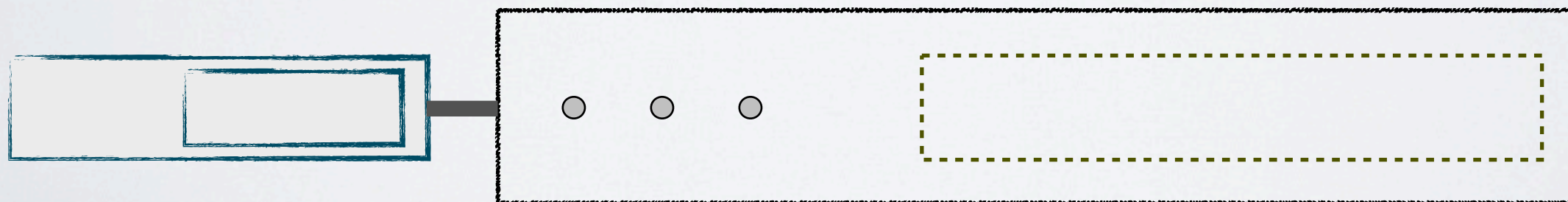
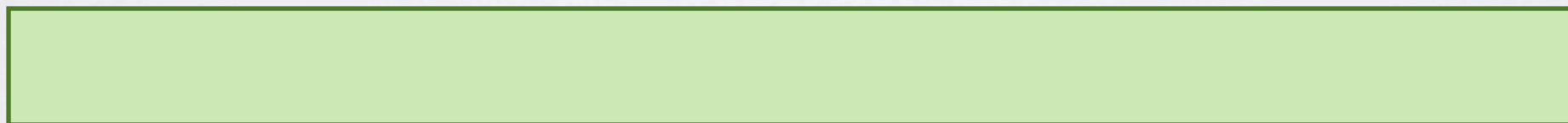
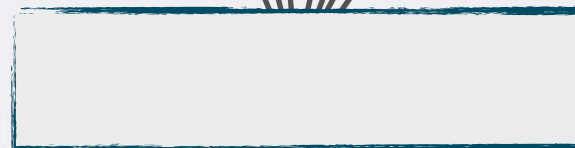


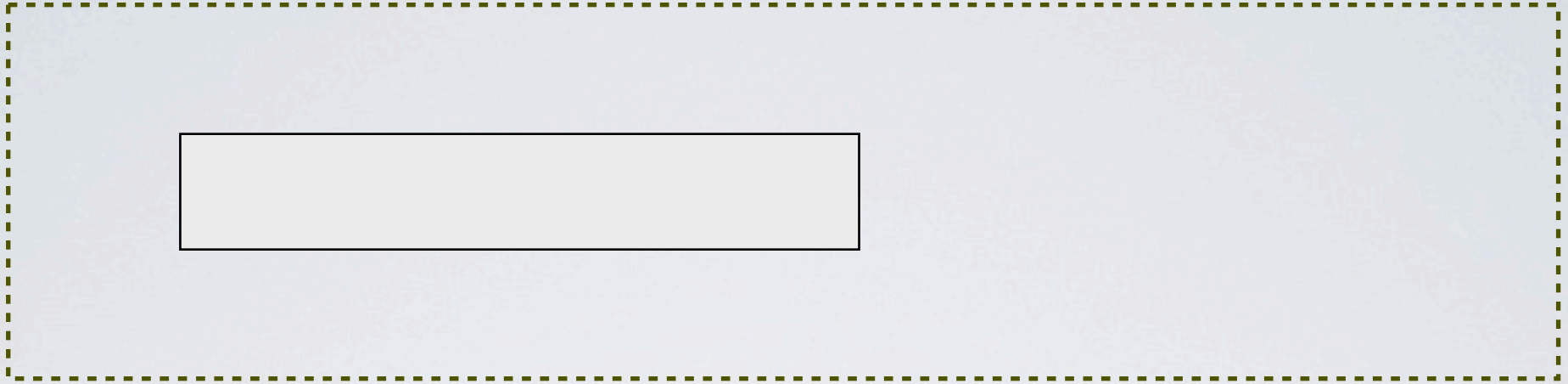
$$q \cdot g(k)^k \cdot \underline{2^{k^2}}$$

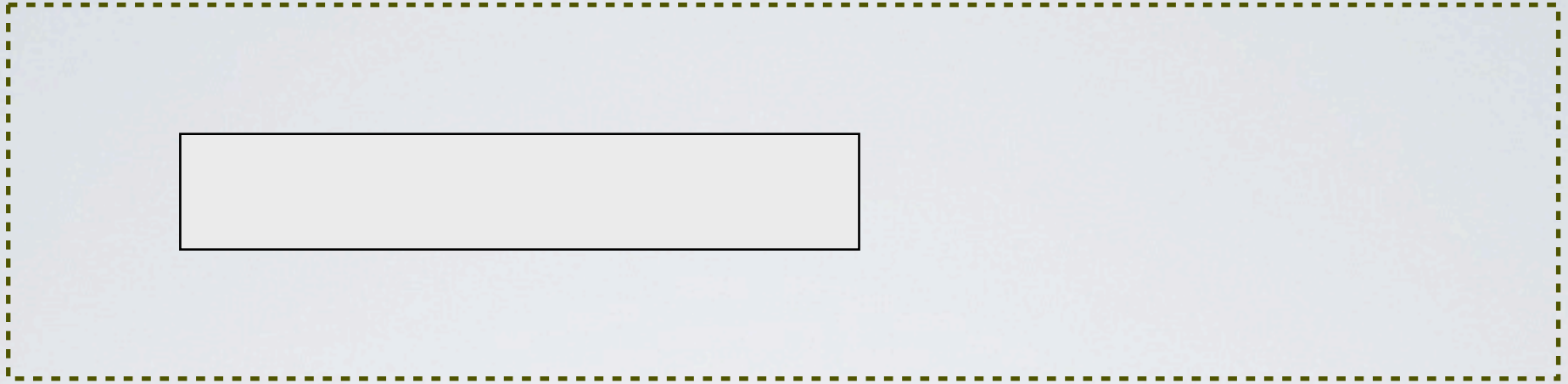


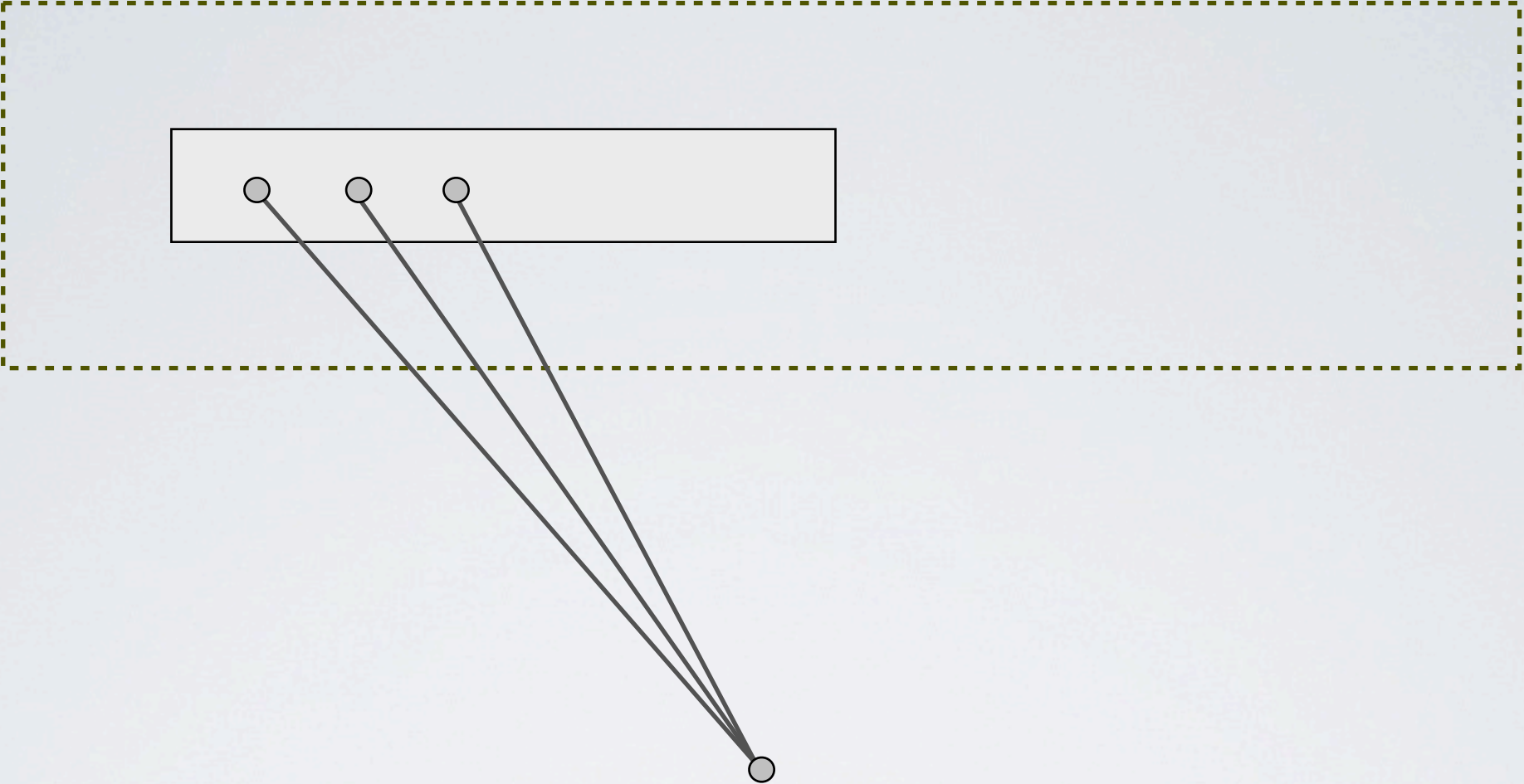


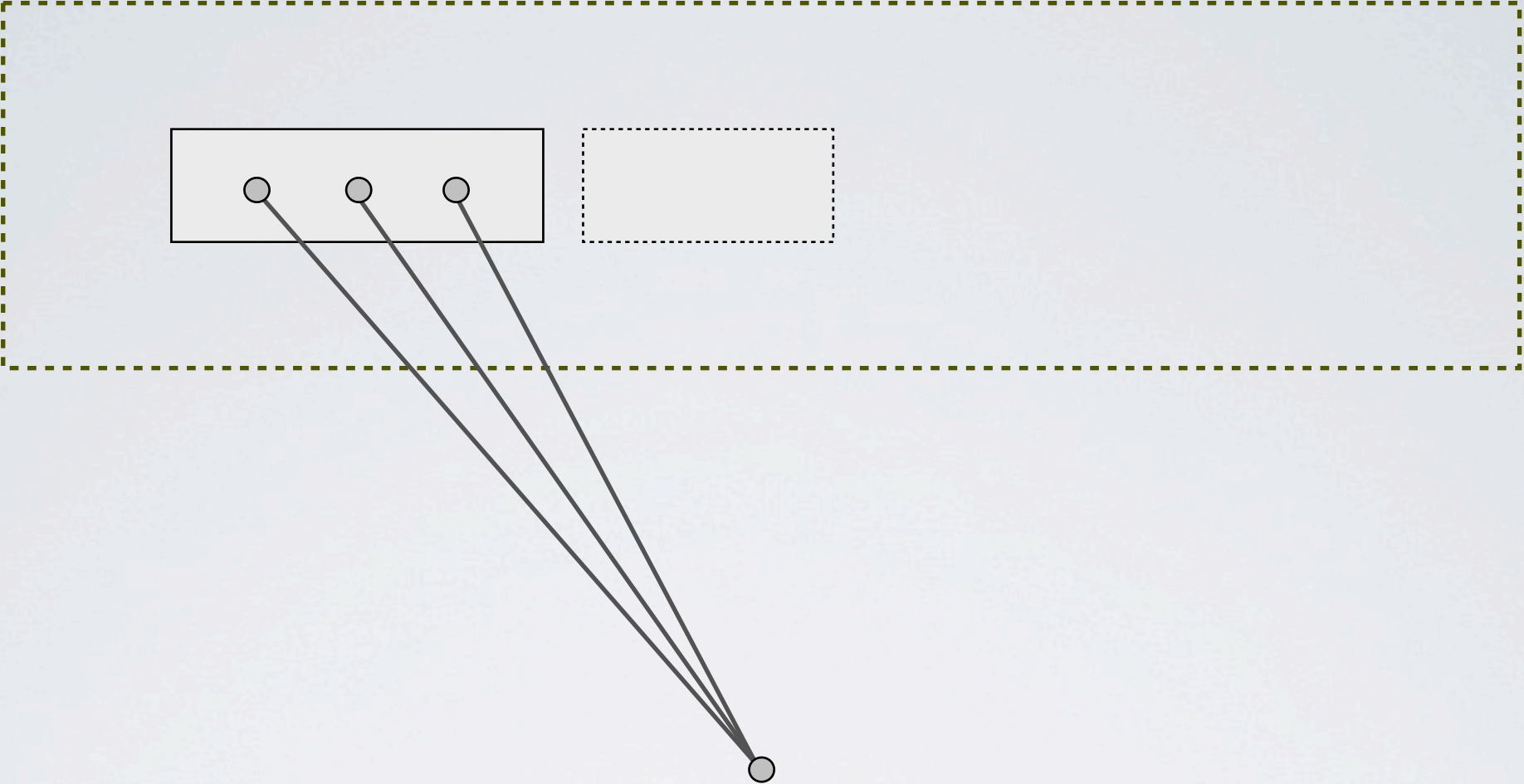
$$q \cdot g(k)^k \cdot 2^{k^2}$$

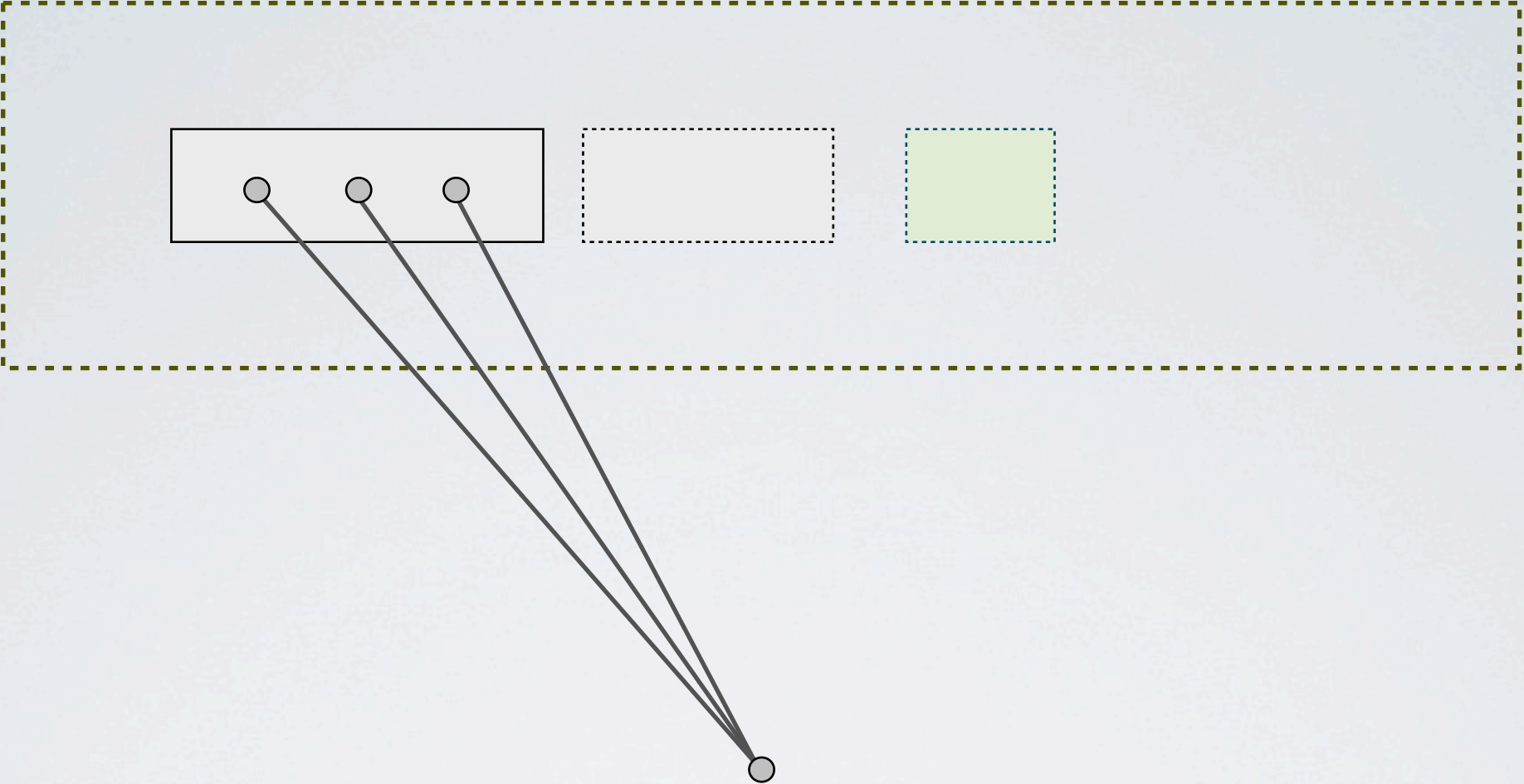


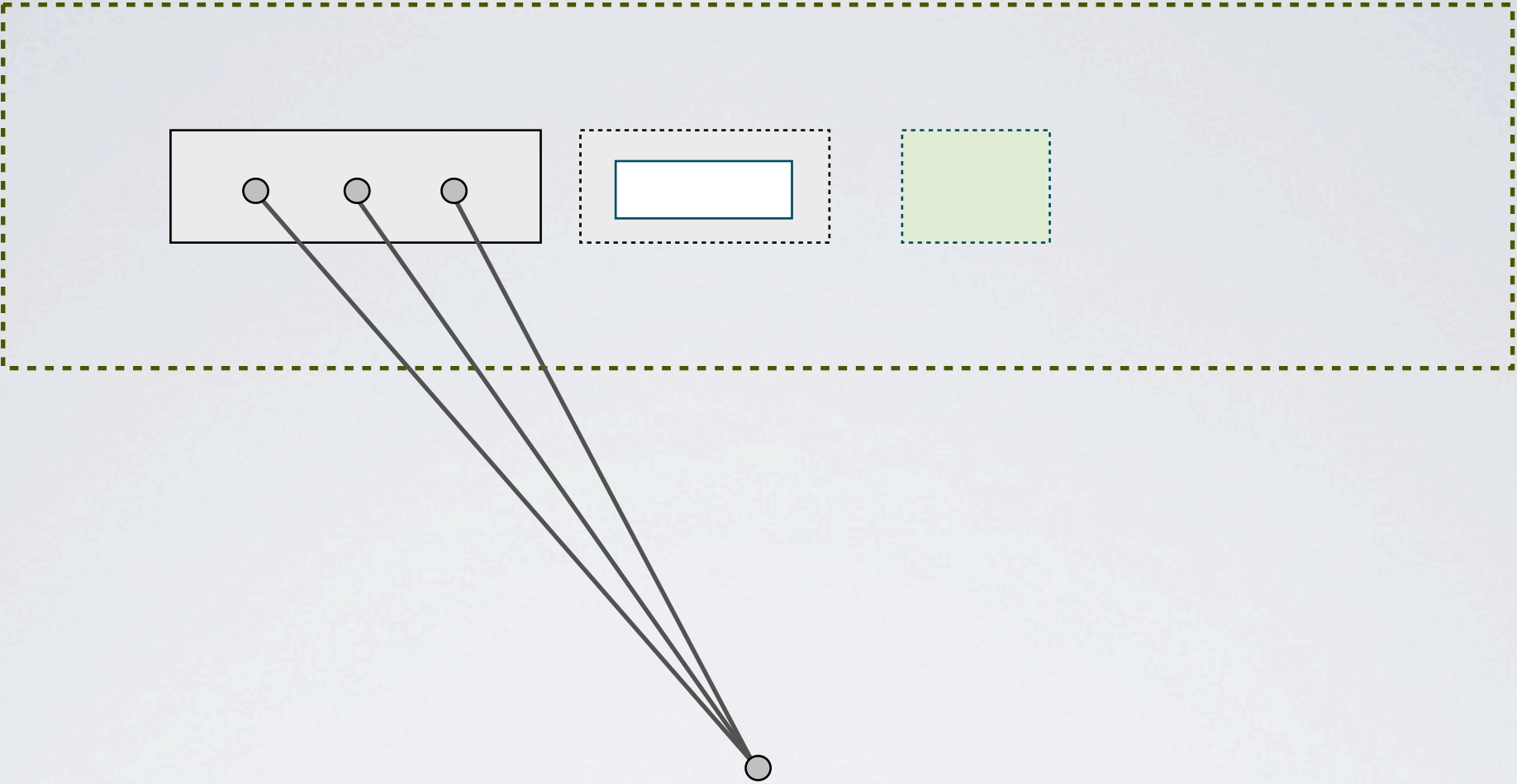


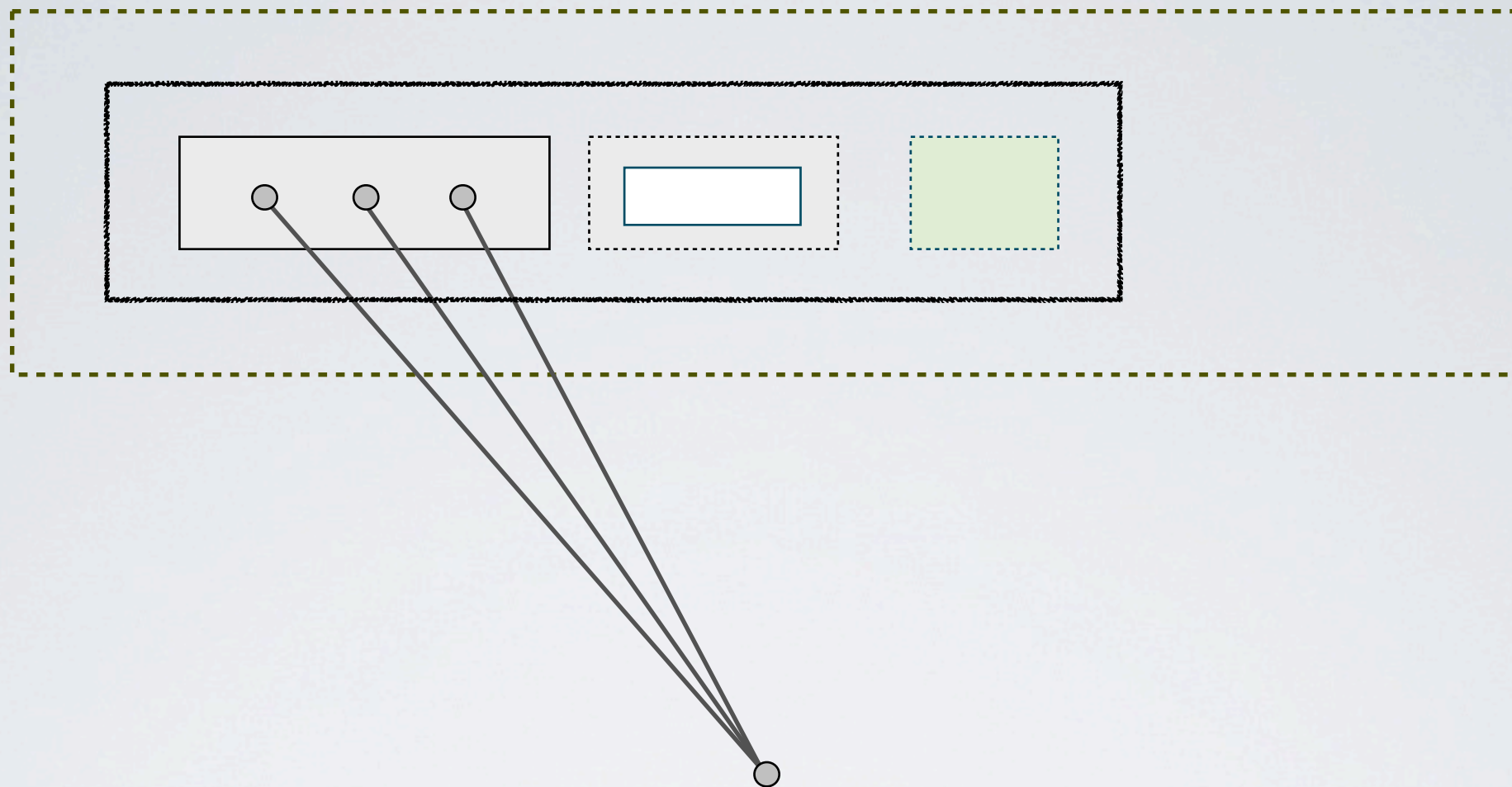




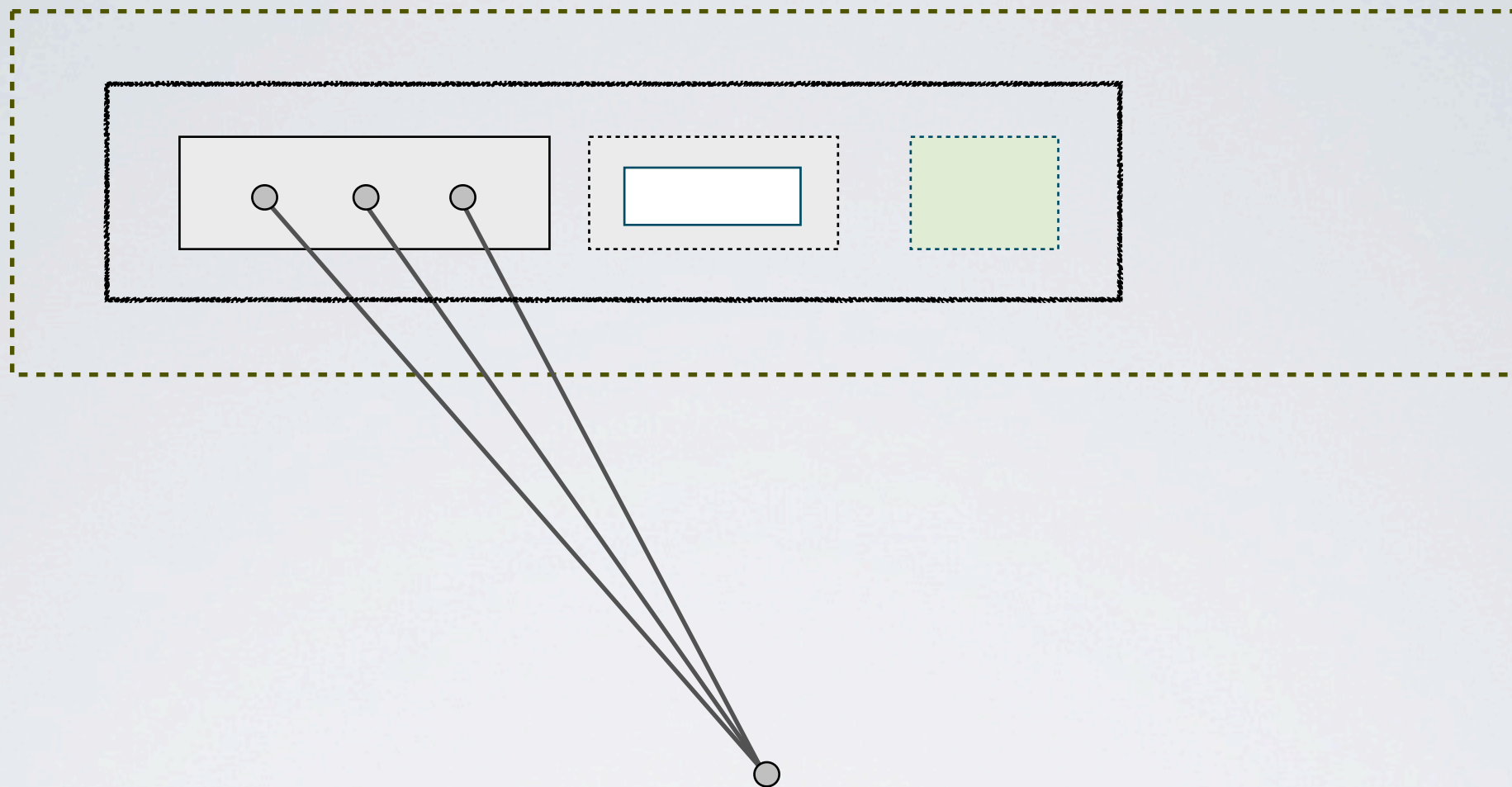




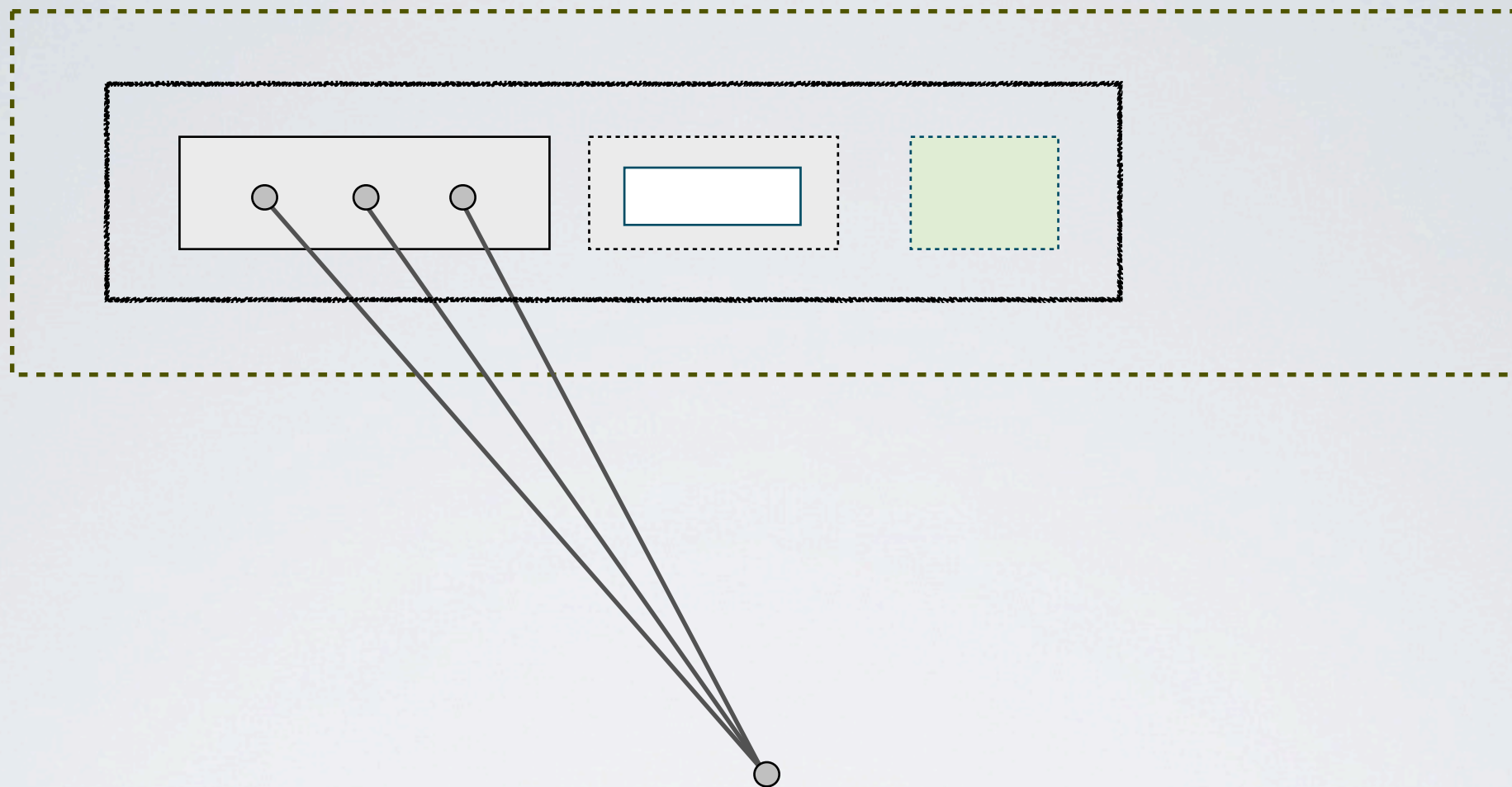




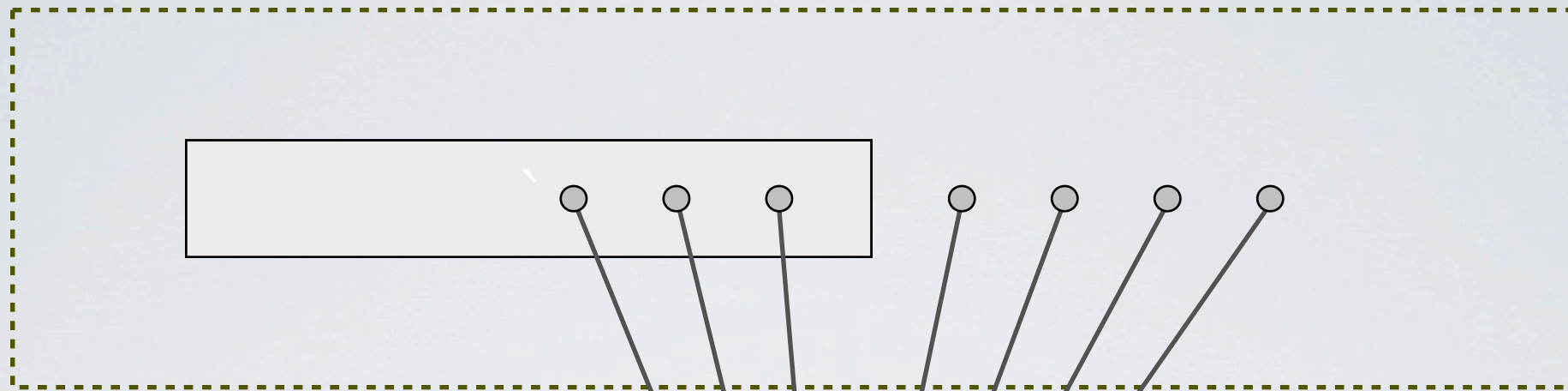
2-connected
 (s,t) separator



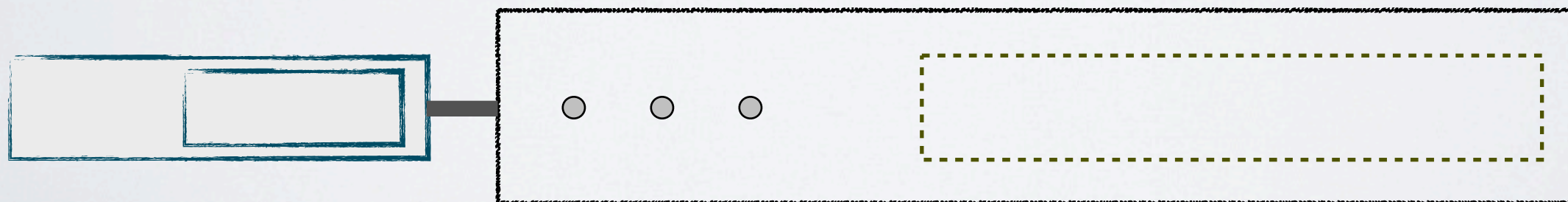
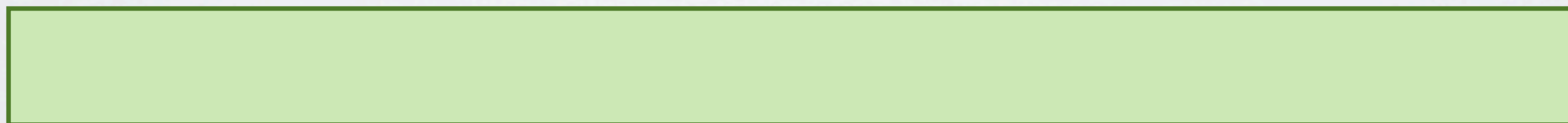
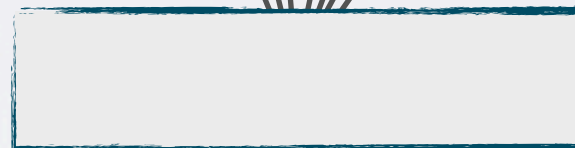
2-connected
by construction
(s,t) separator

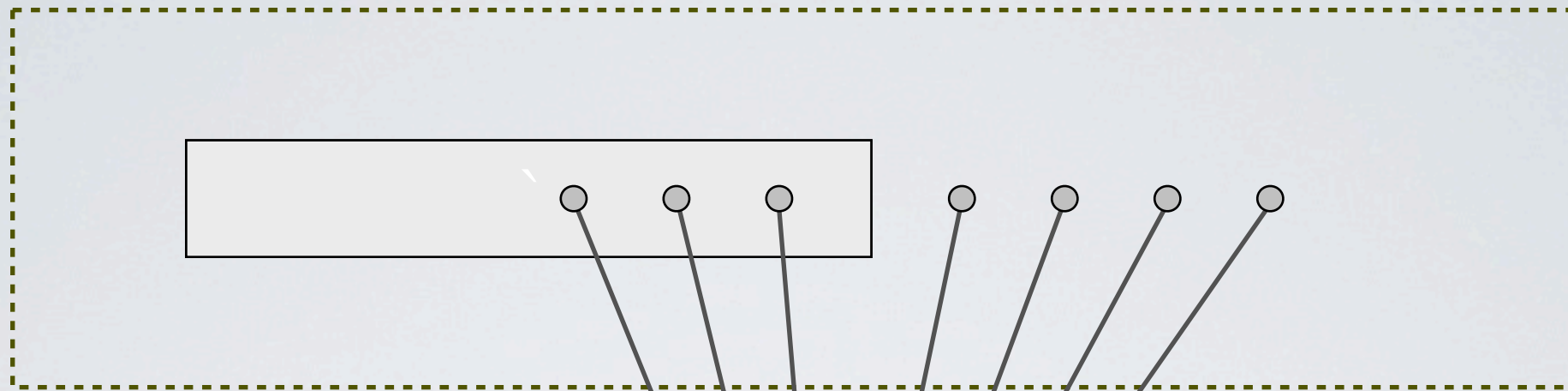


2-connected
by construction
(s,t) separator
contains a minimal separator

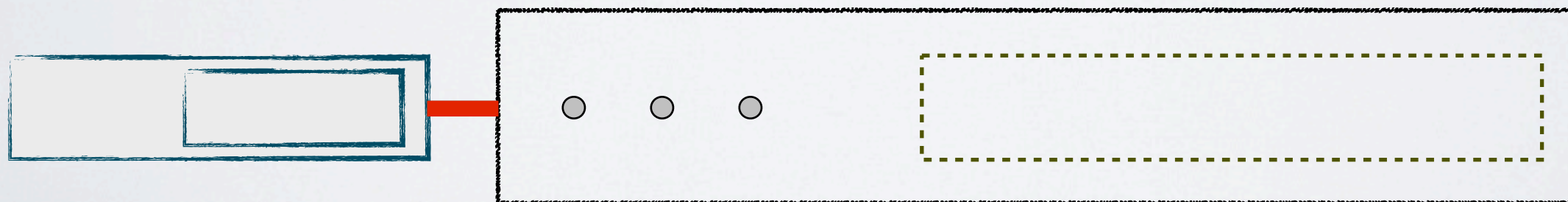
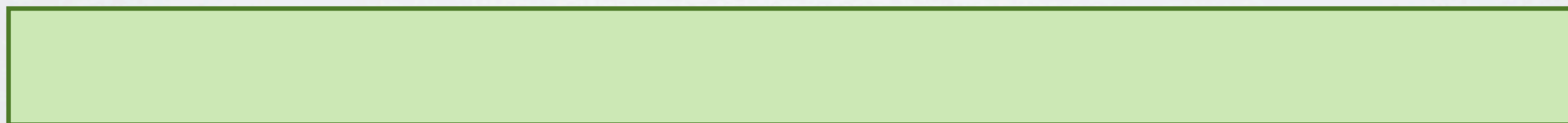
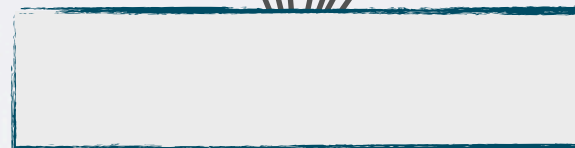


$$q \cdot g(k)^k \cdot 2^{k^2}$$





$$q \cdot g(k)^k \cdot 2^{k^2}$$







2-CONNECTED STEINER TREE

THE 2-CONNECTED STEINER TREE PROBLEM

SOME STRUCTURAL DISCOVERIES

AN ALGORITHM

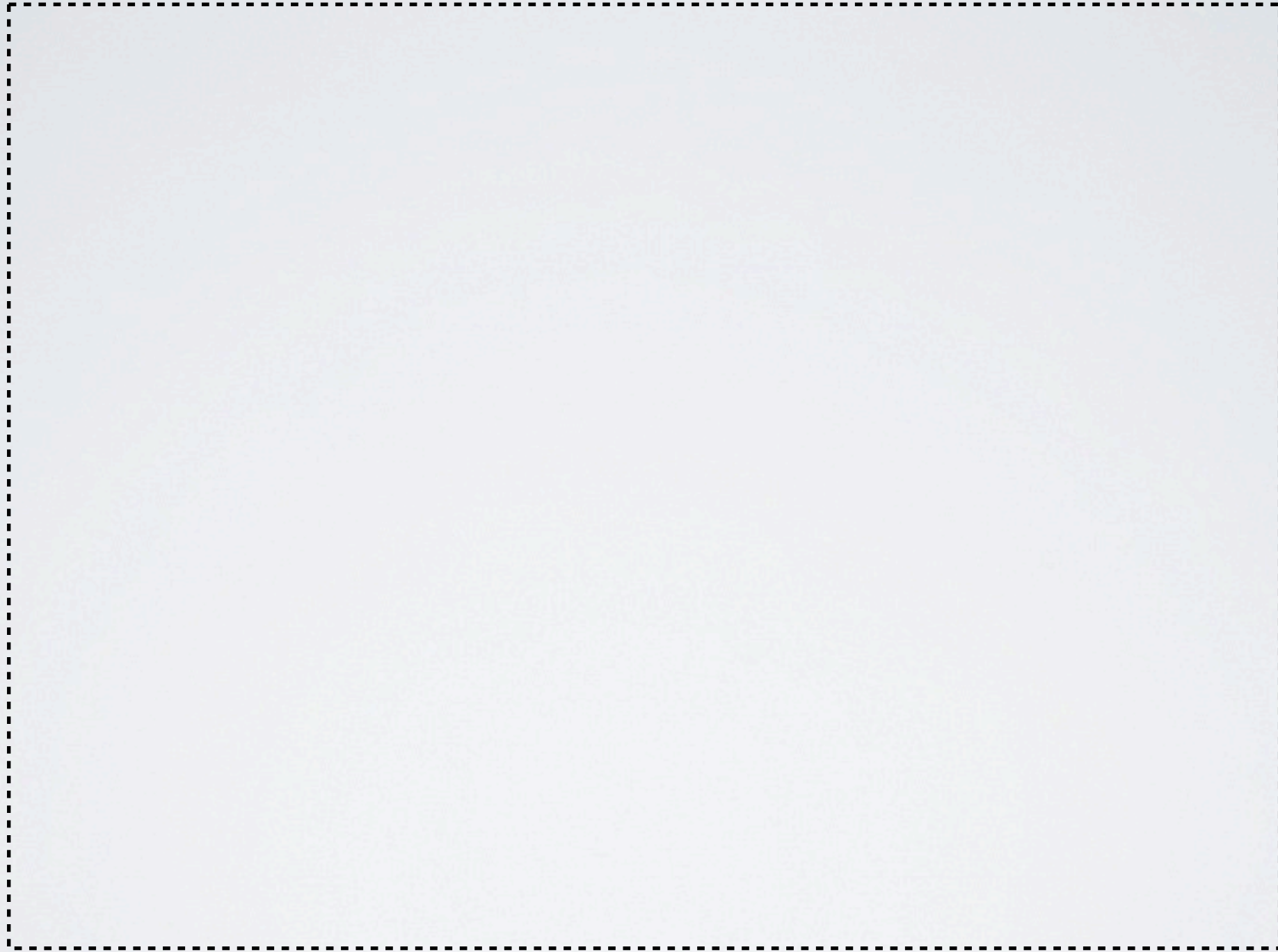
THE 2-CONNECTED STEINER TREE PROBLEM

by
S. S. RAO

THE 2-CONNECTED STEINER TREE PROBLEM

by
S. S. Ravi, P. M. Tamassia, and
R. Tamassia

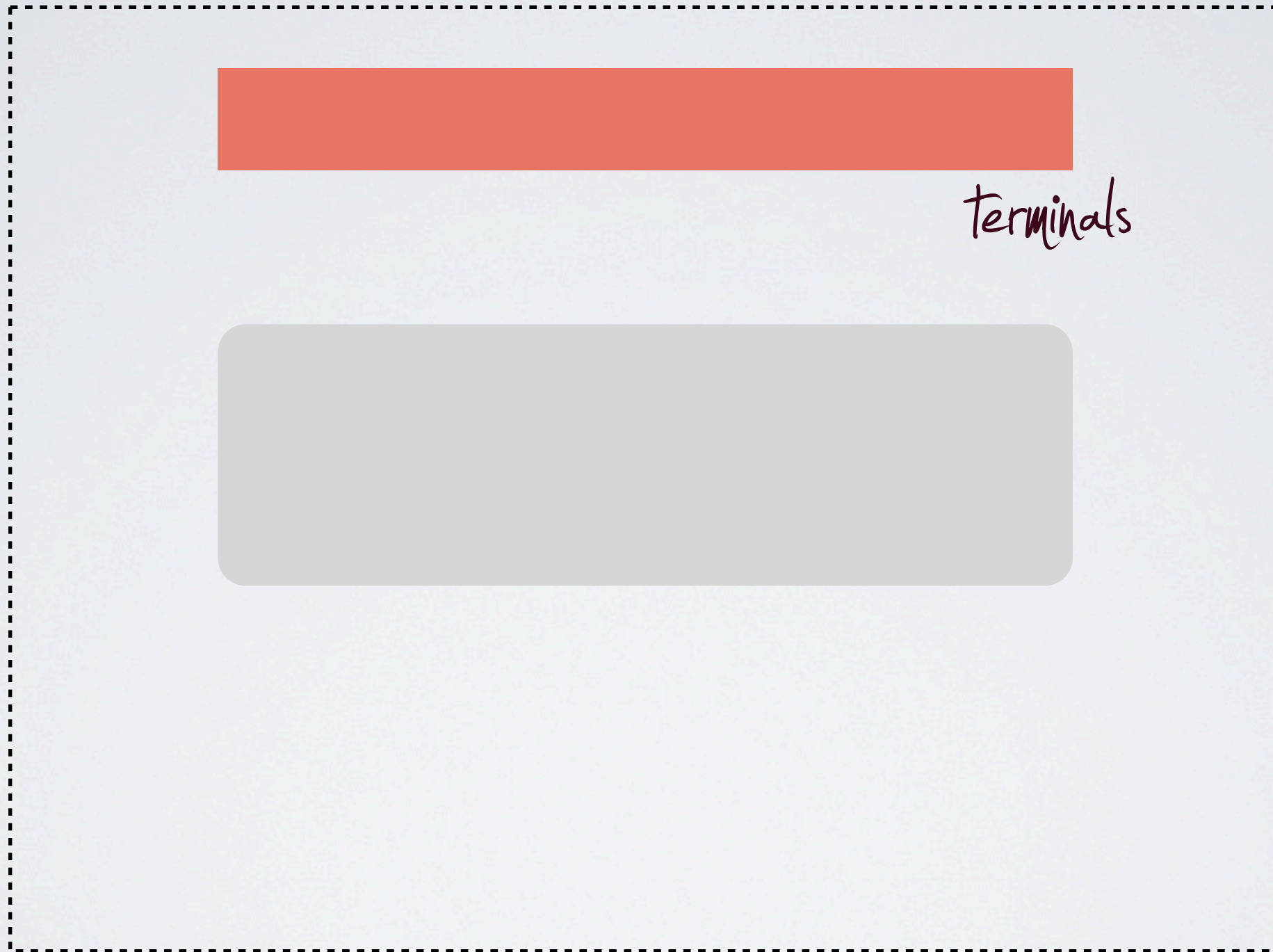
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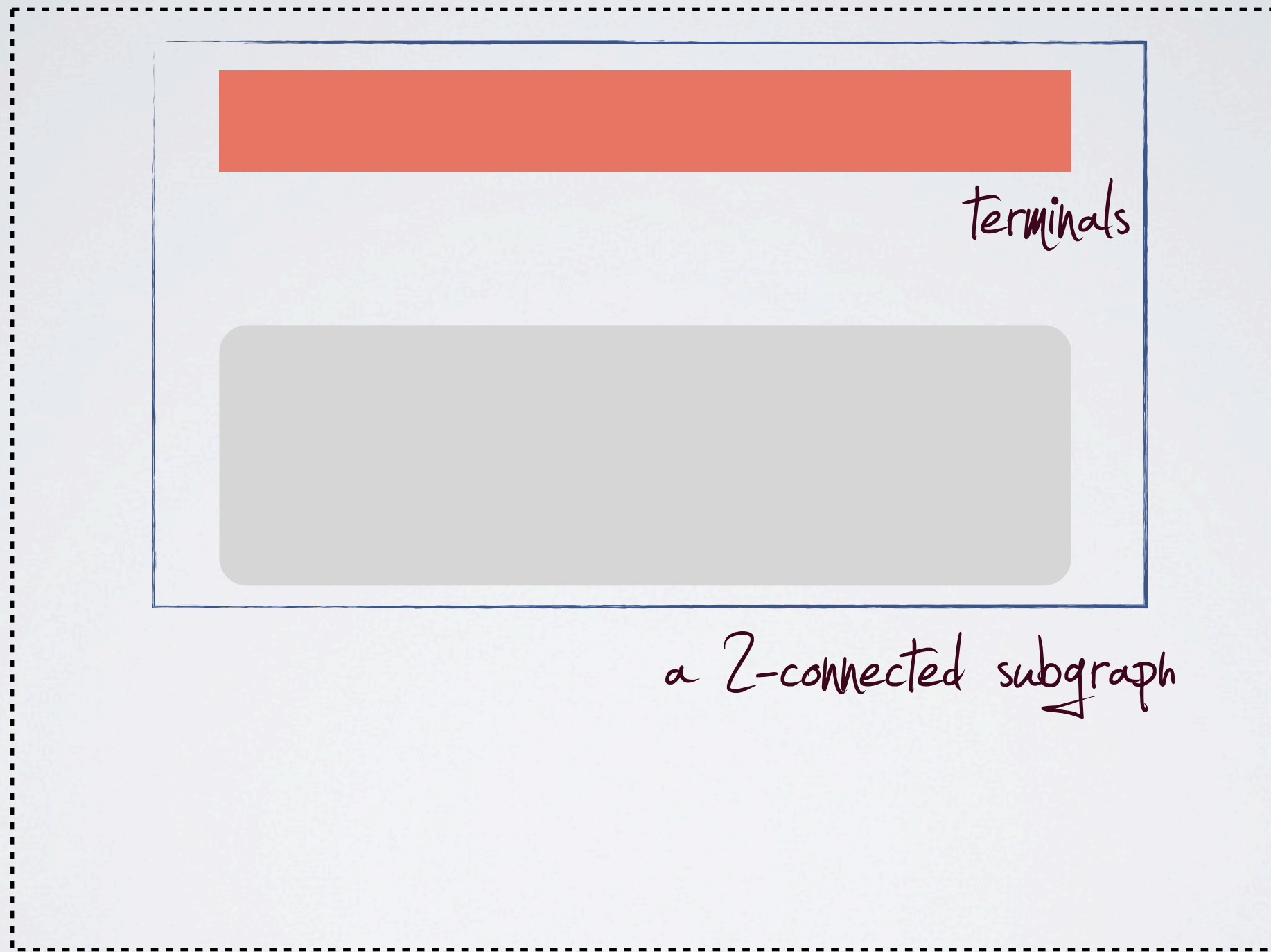
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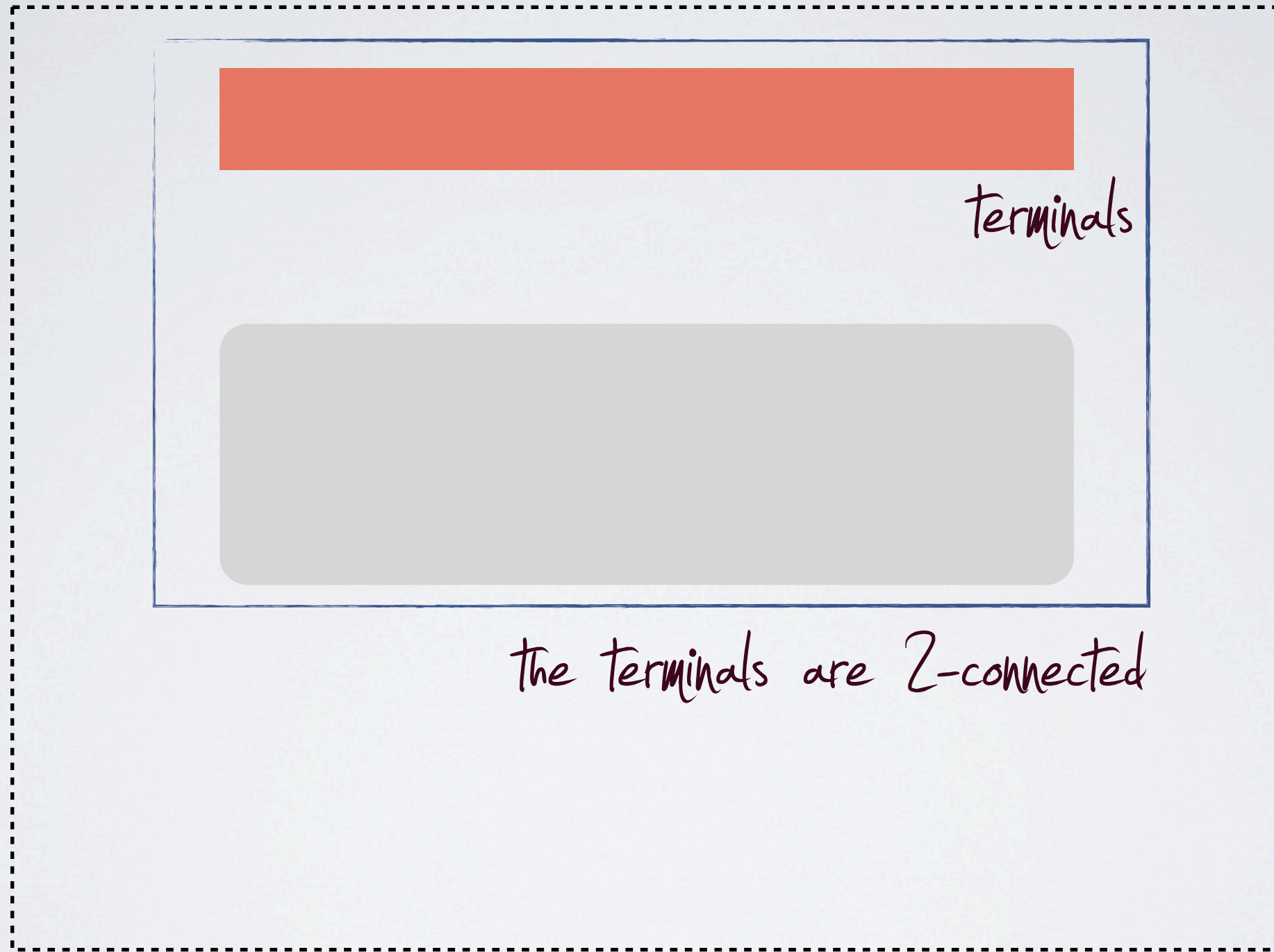
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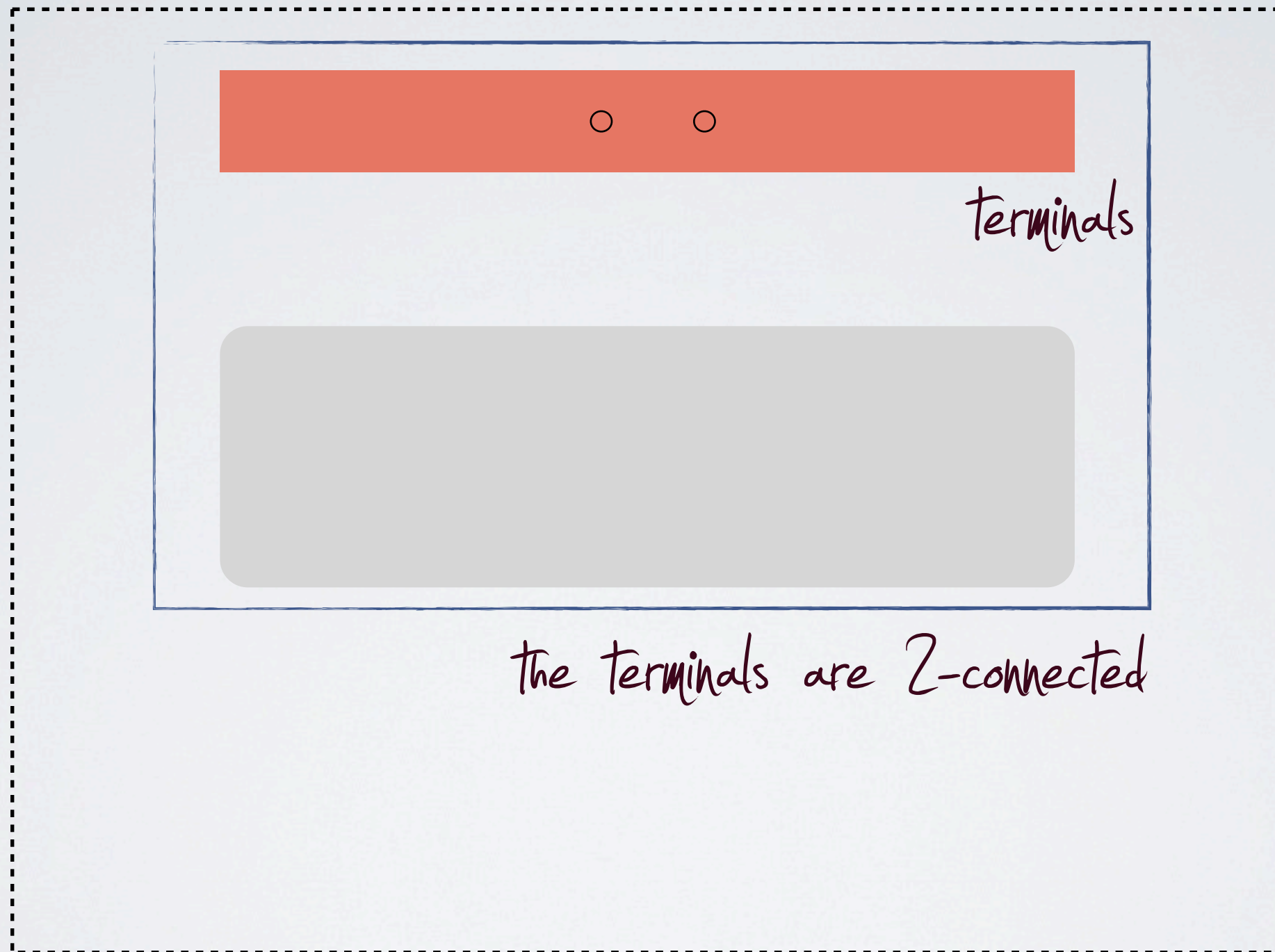
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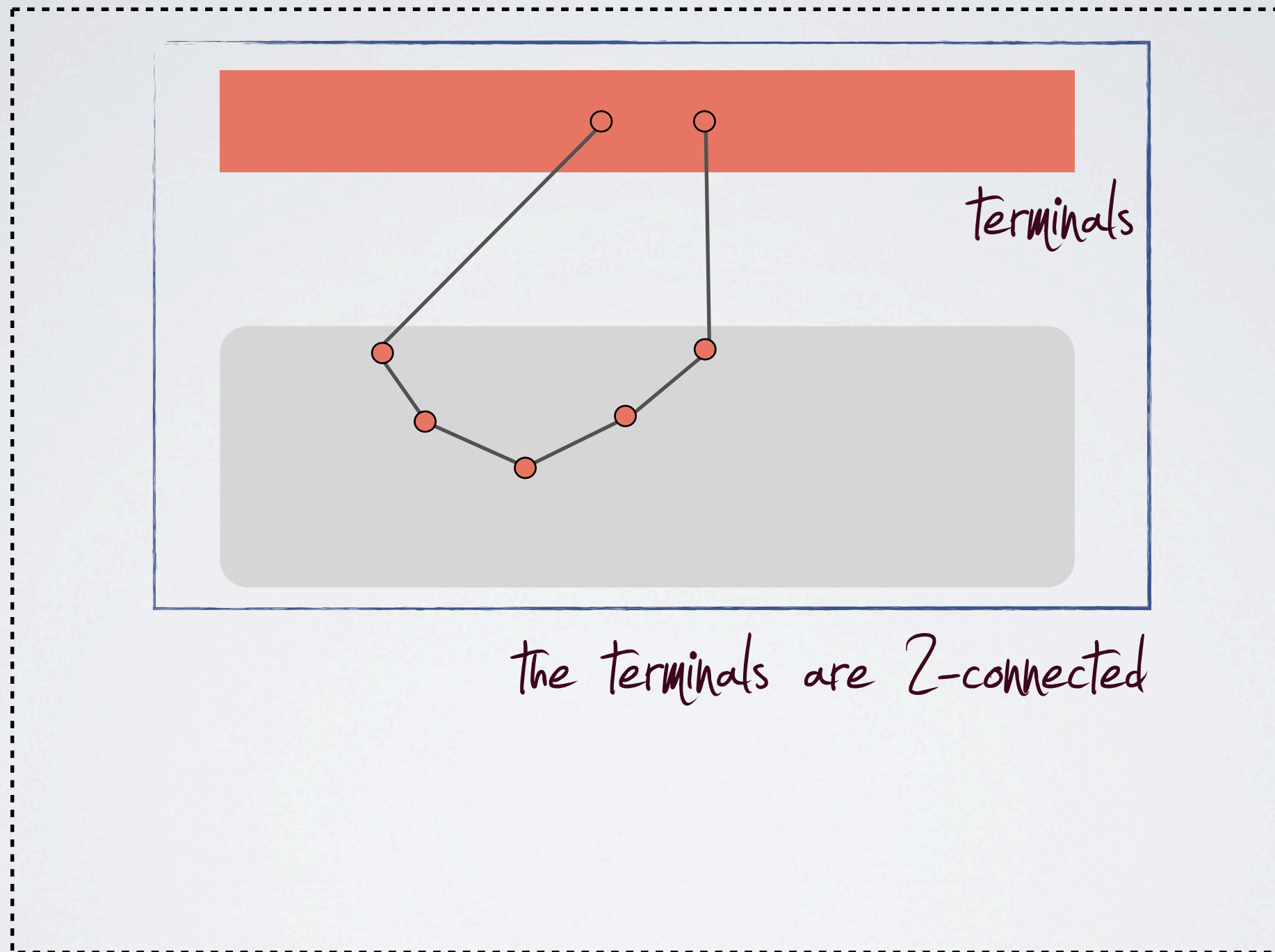
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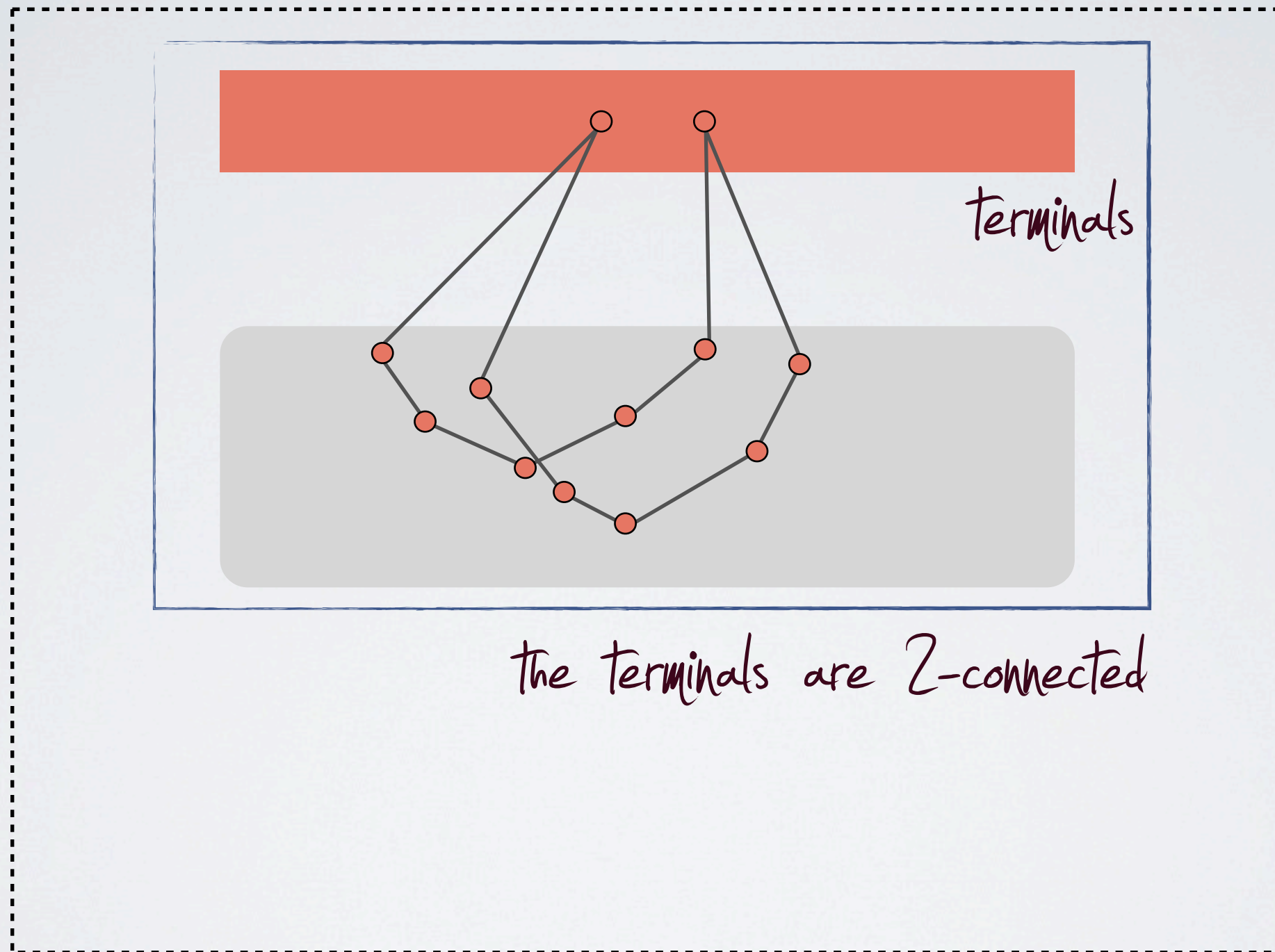
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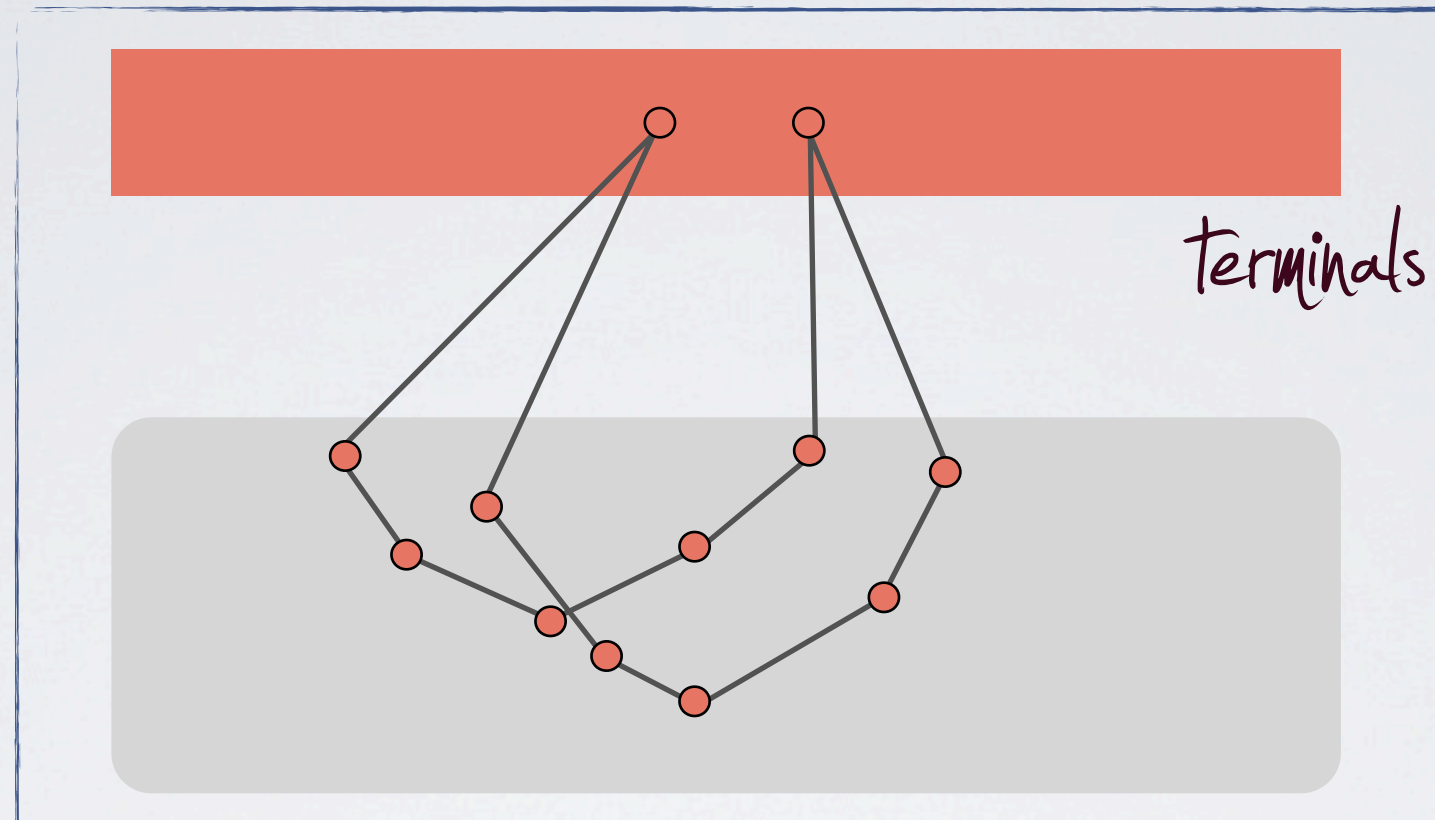
THE 2-CONNECTED STEINER TREE PROBLEM



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THE 2-CONNECTED STEINER TREE PROBLEM

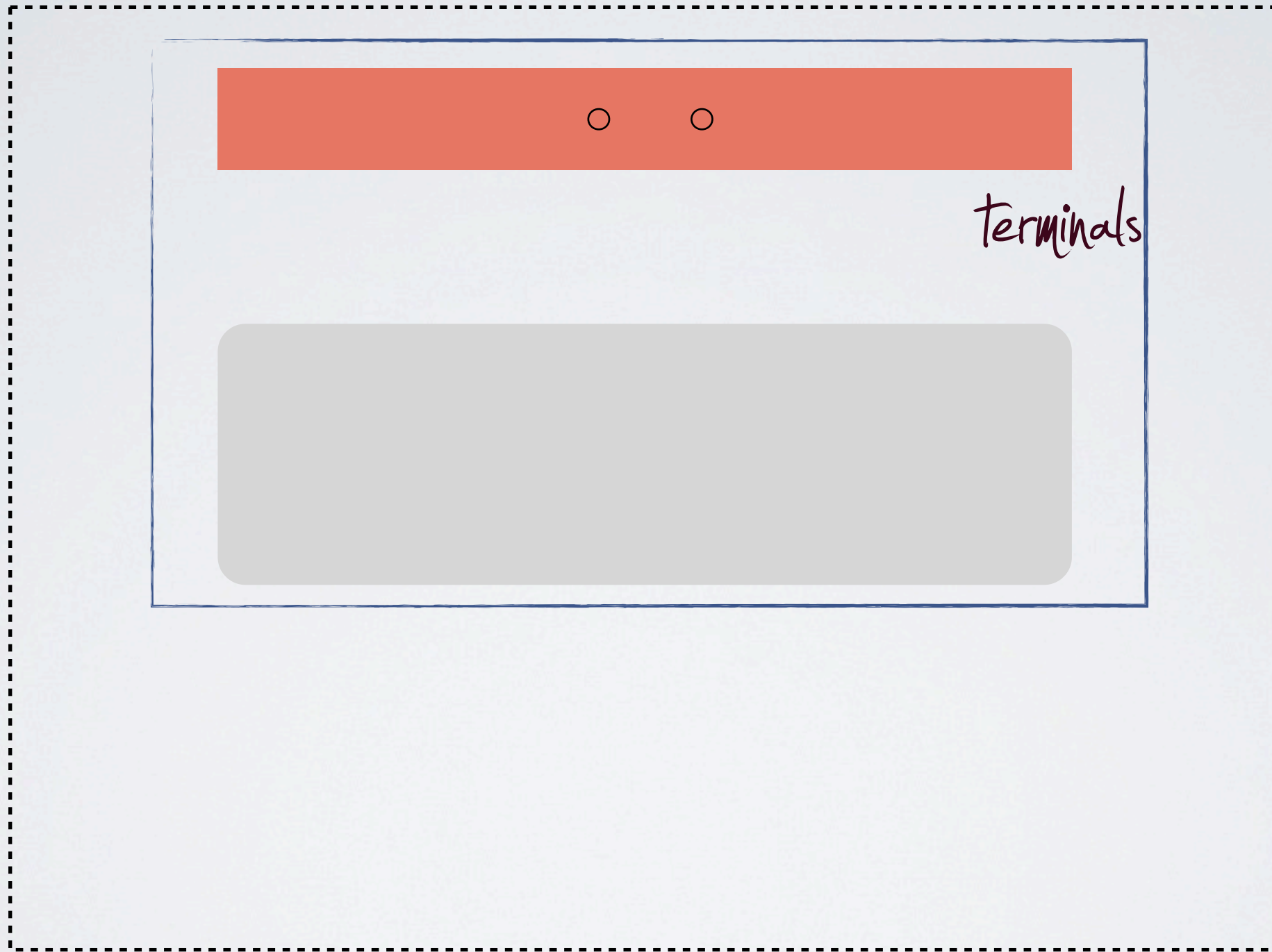


the terminals are 2-connected

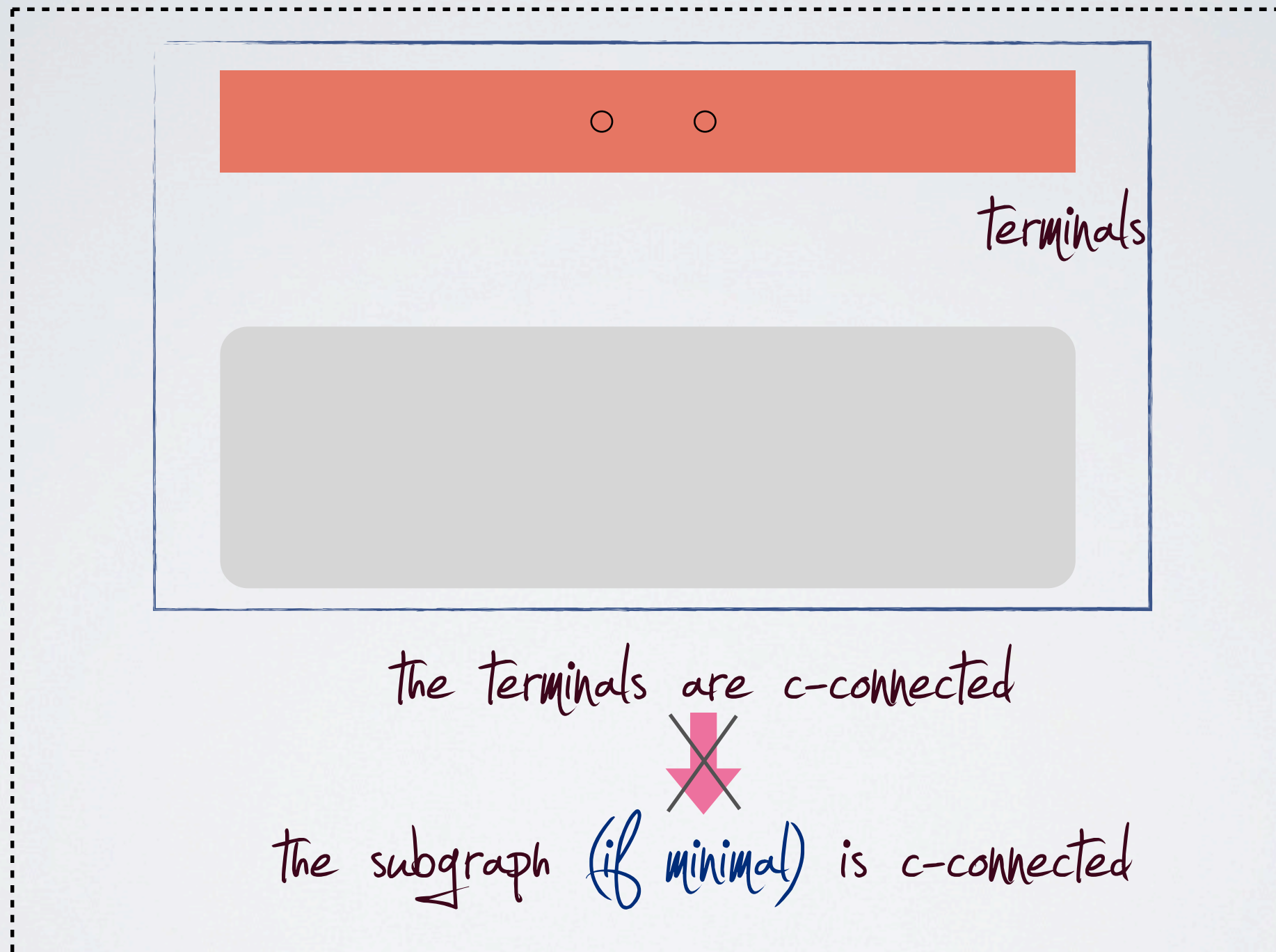


the subgraph (if minimal) is 2-connected

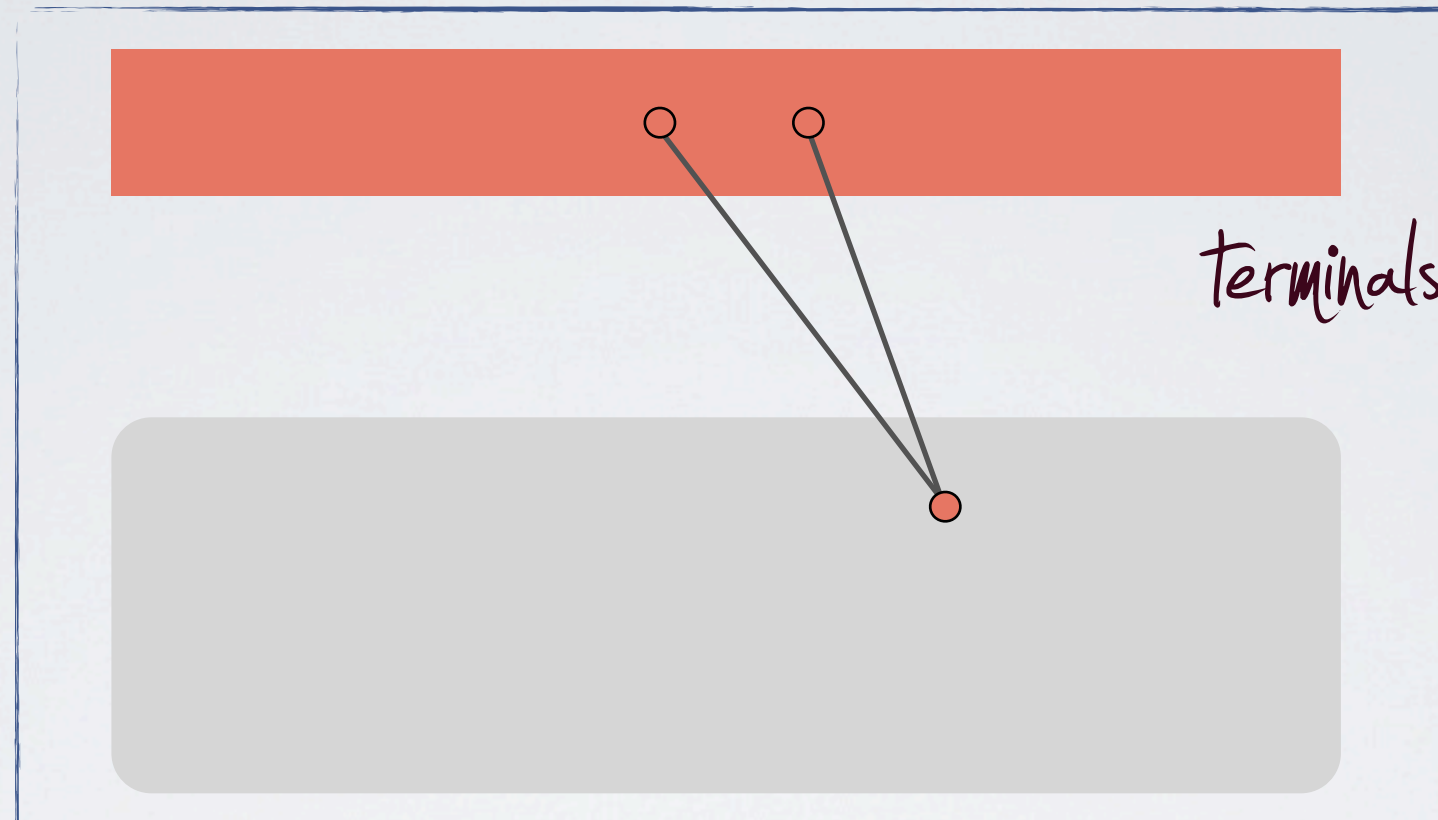
THE 2-CONNECTED STEINER TREE PROBLEM



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THE 2-CONNECTED STEINER TREE PROBLEM

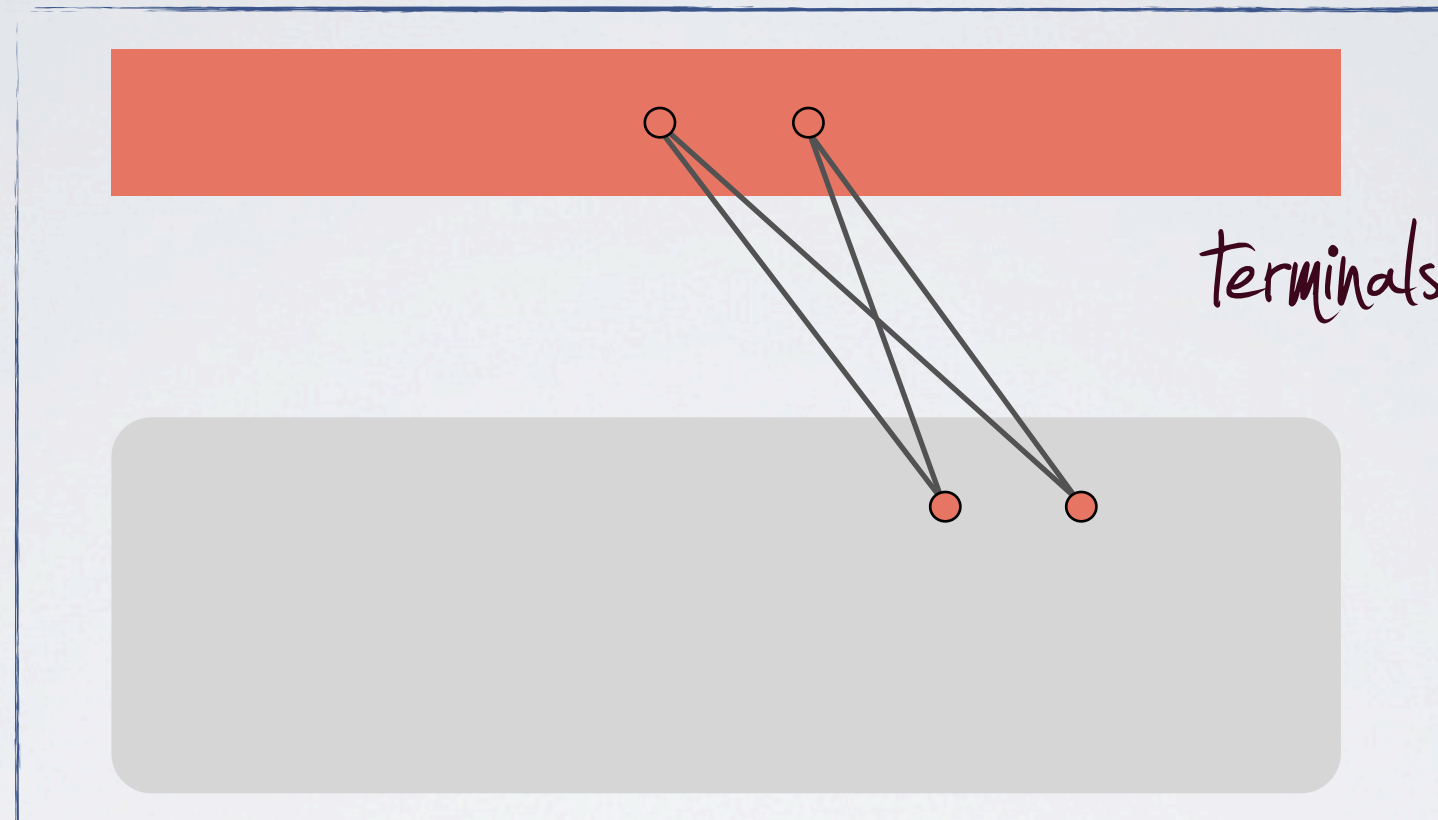


the terminals are c -connected



the subgraph (if minimal) is c -connected

THE 2-CONNECTED STEINER TREE PROBLEM

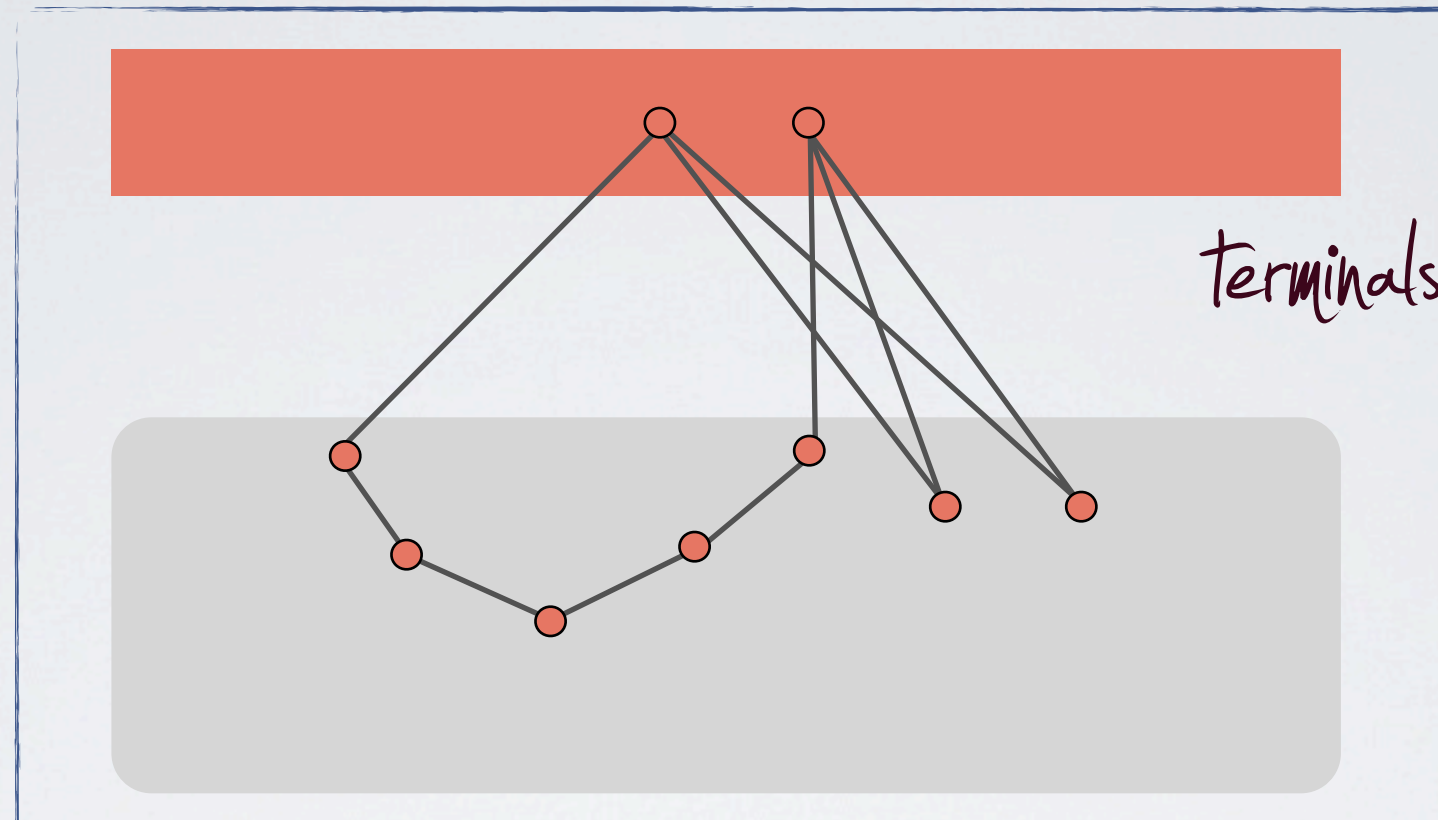


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the subgraph (if minimal) is c -connected

THE 2-CONNECTED STEINER TREE PROBLEM

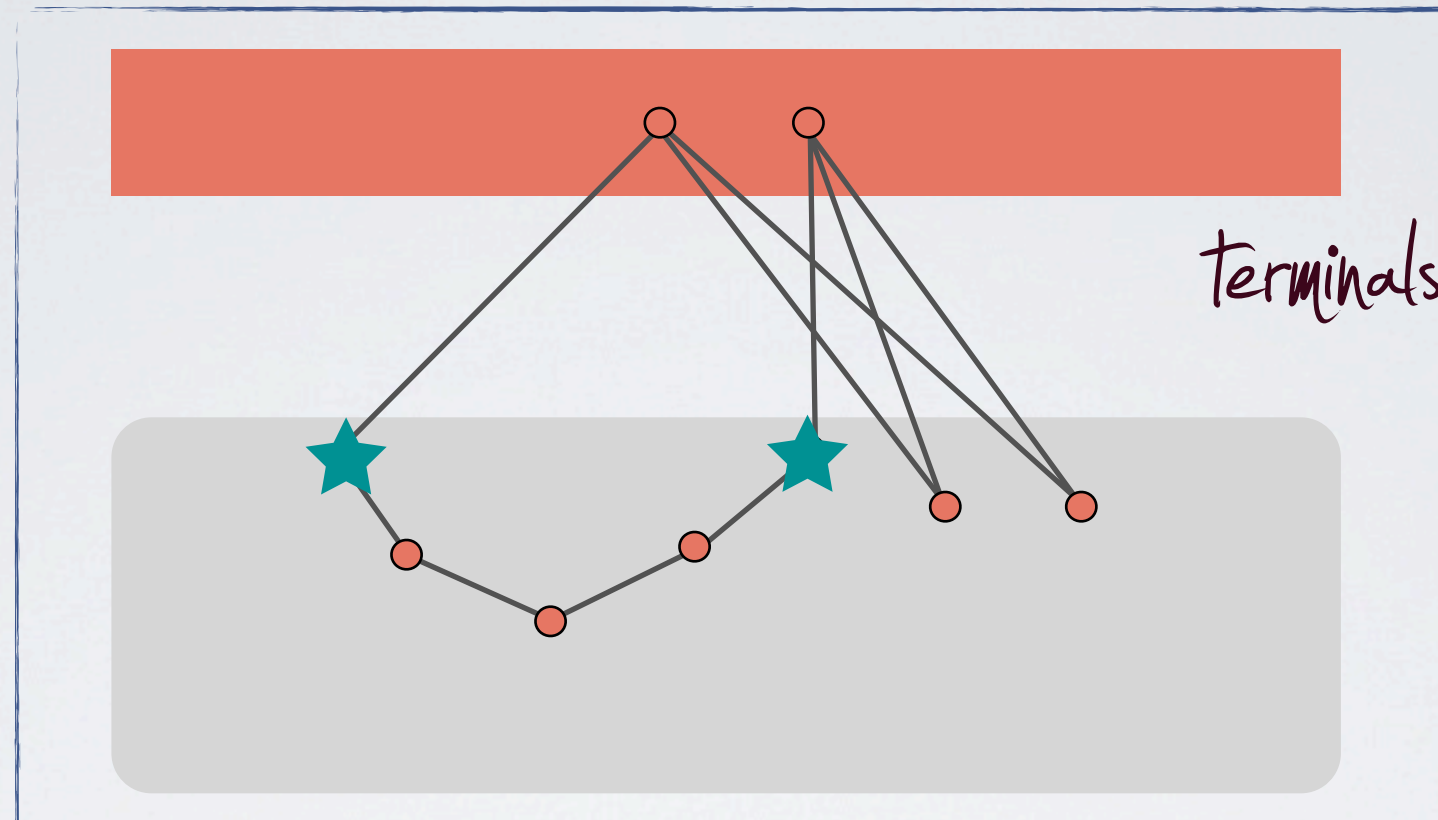


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the subgraph (if minimal) is c -connected

THE 2-CONNECTED STEINER TREE PROBLEM

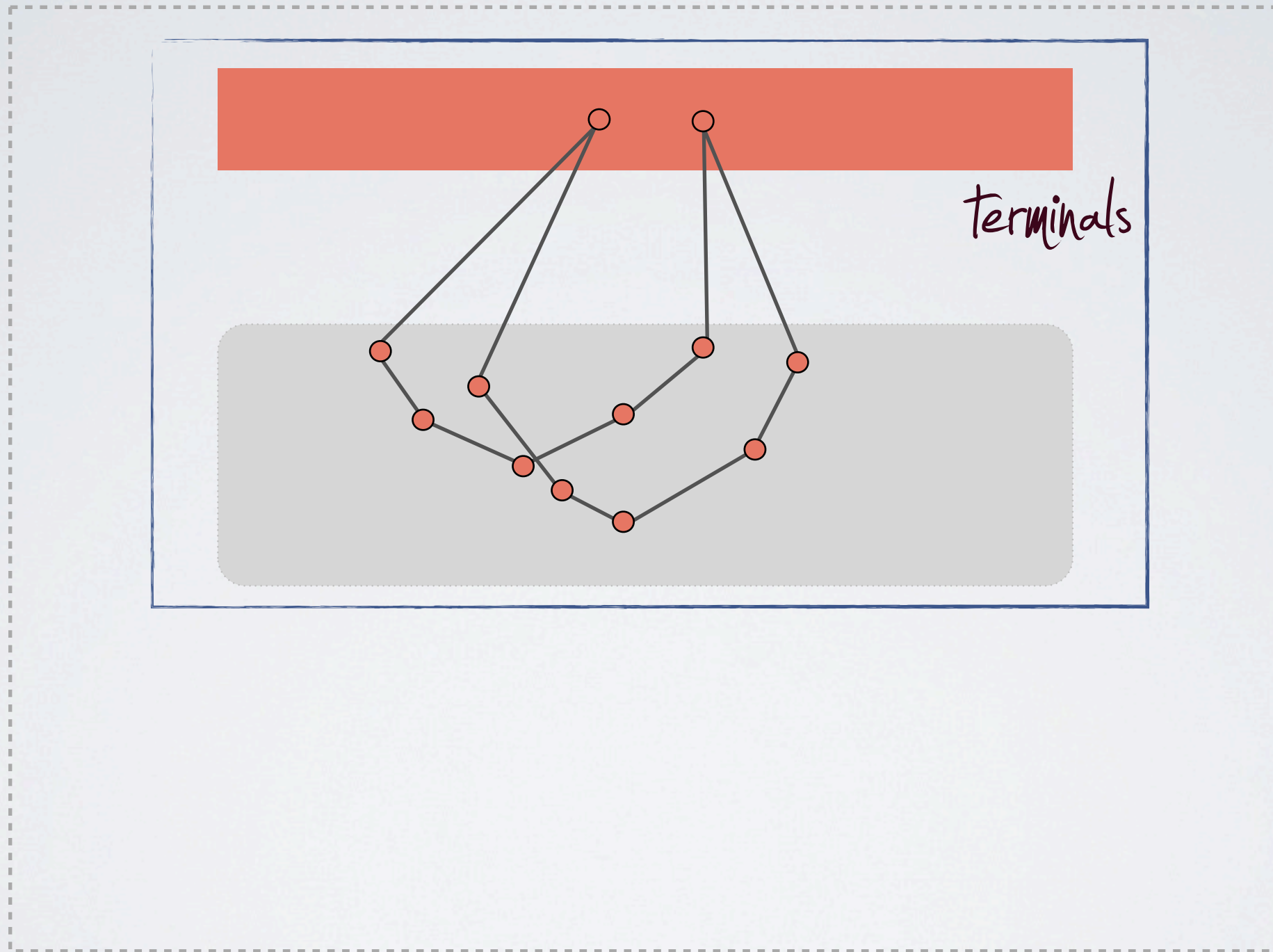


the terminals are c -connected

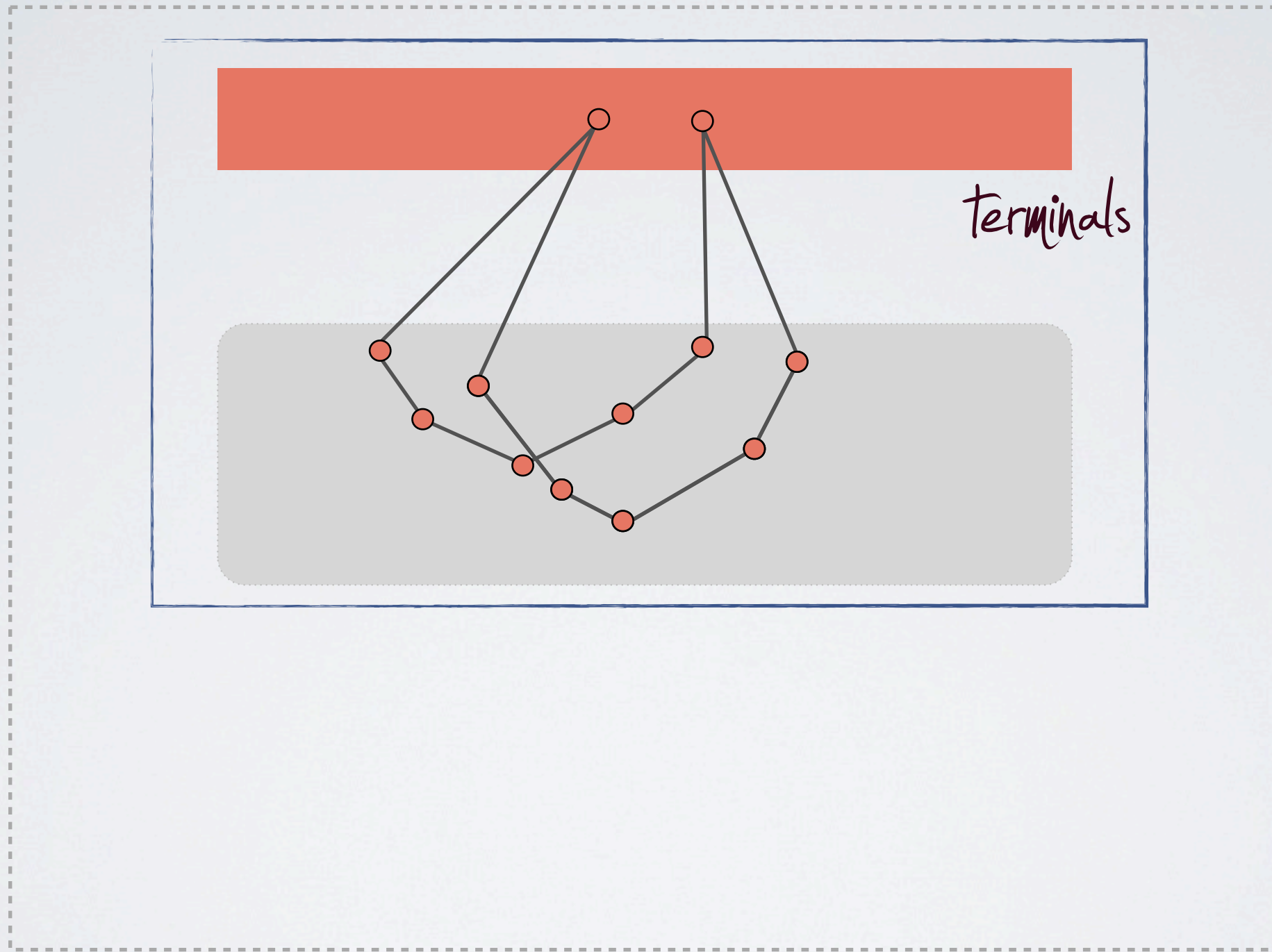


the subgraph (if minimal) is c -connected

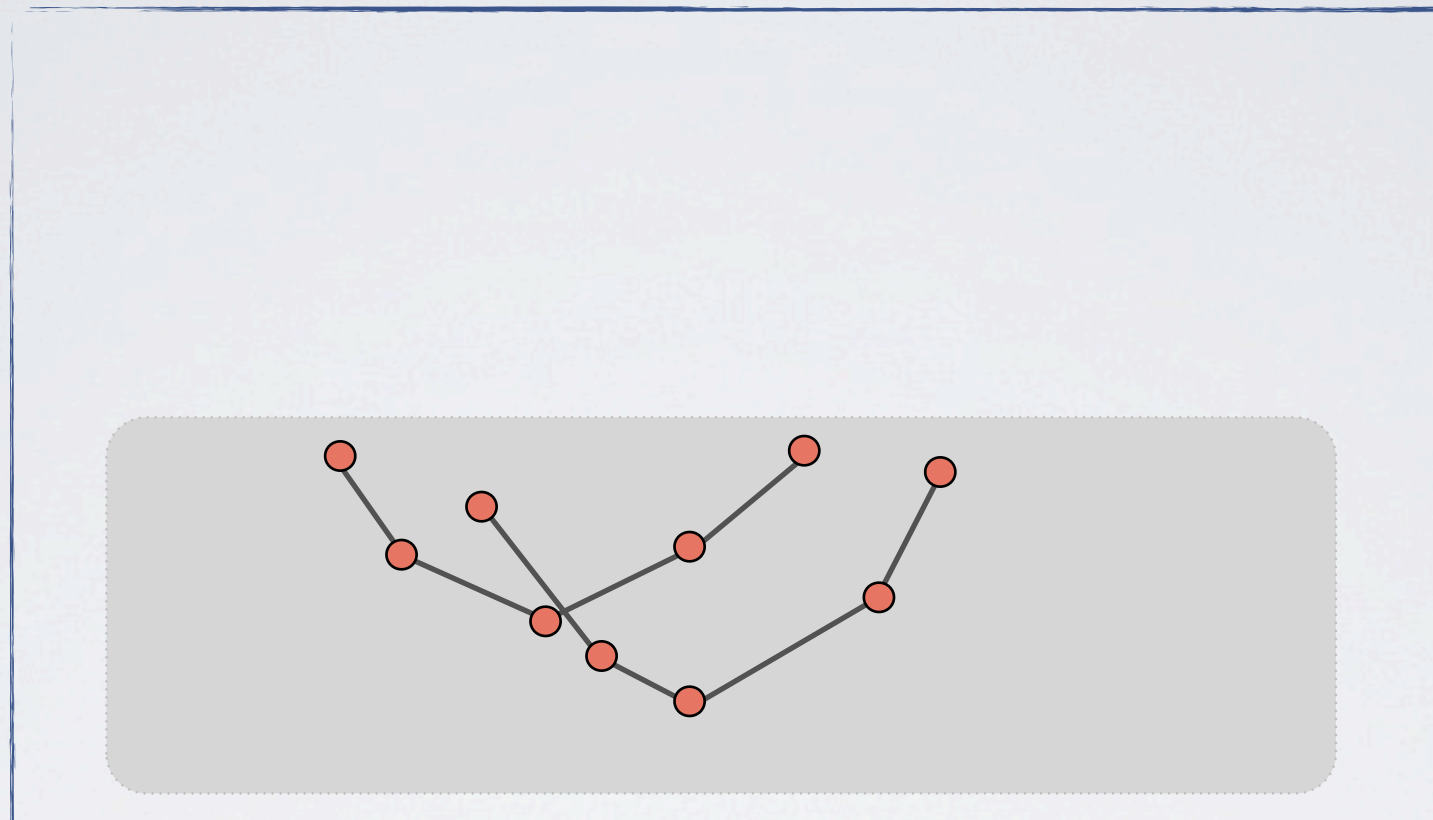
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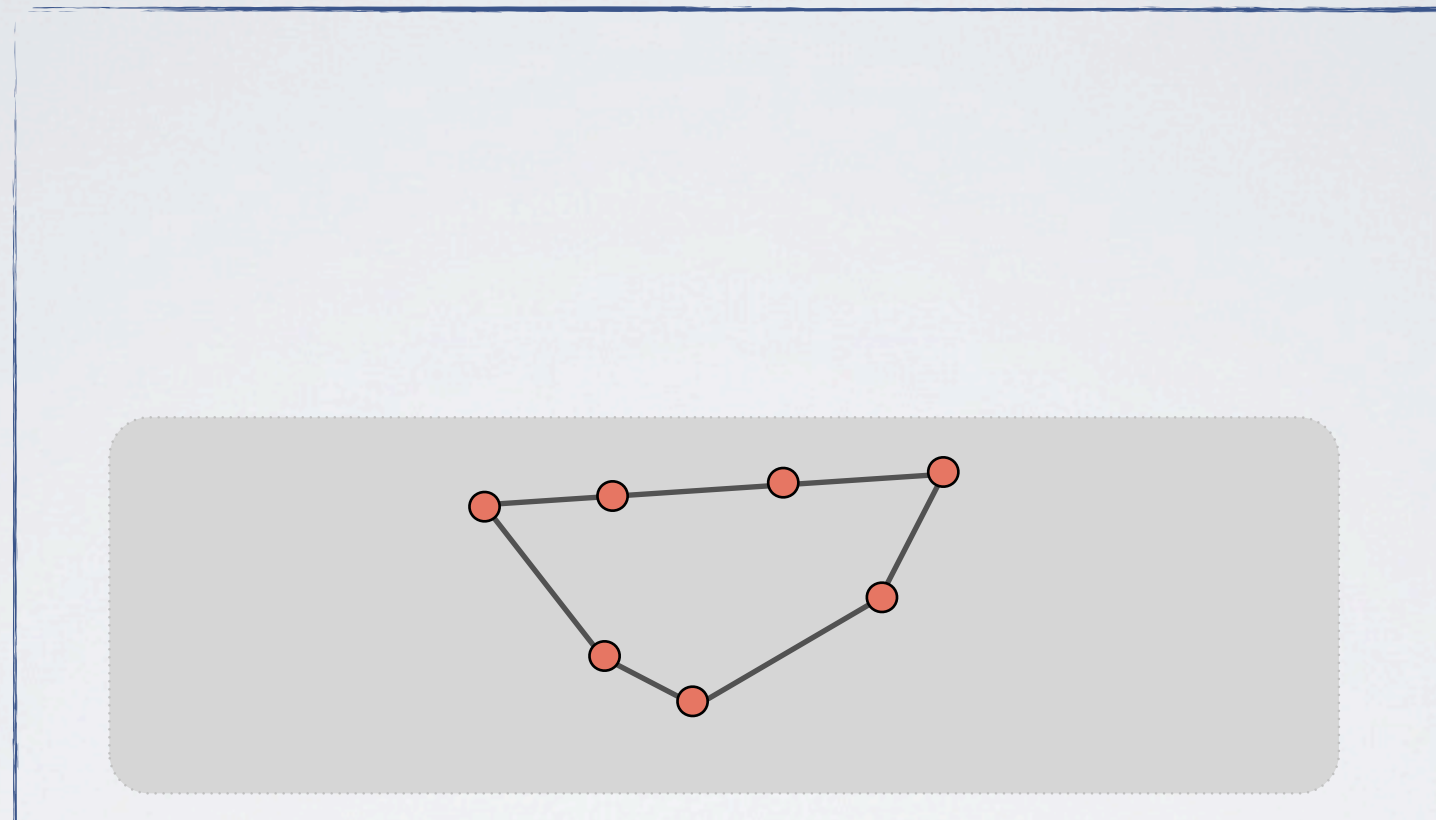


THE 2-CONNECTED STEINER TREE PROBLEM



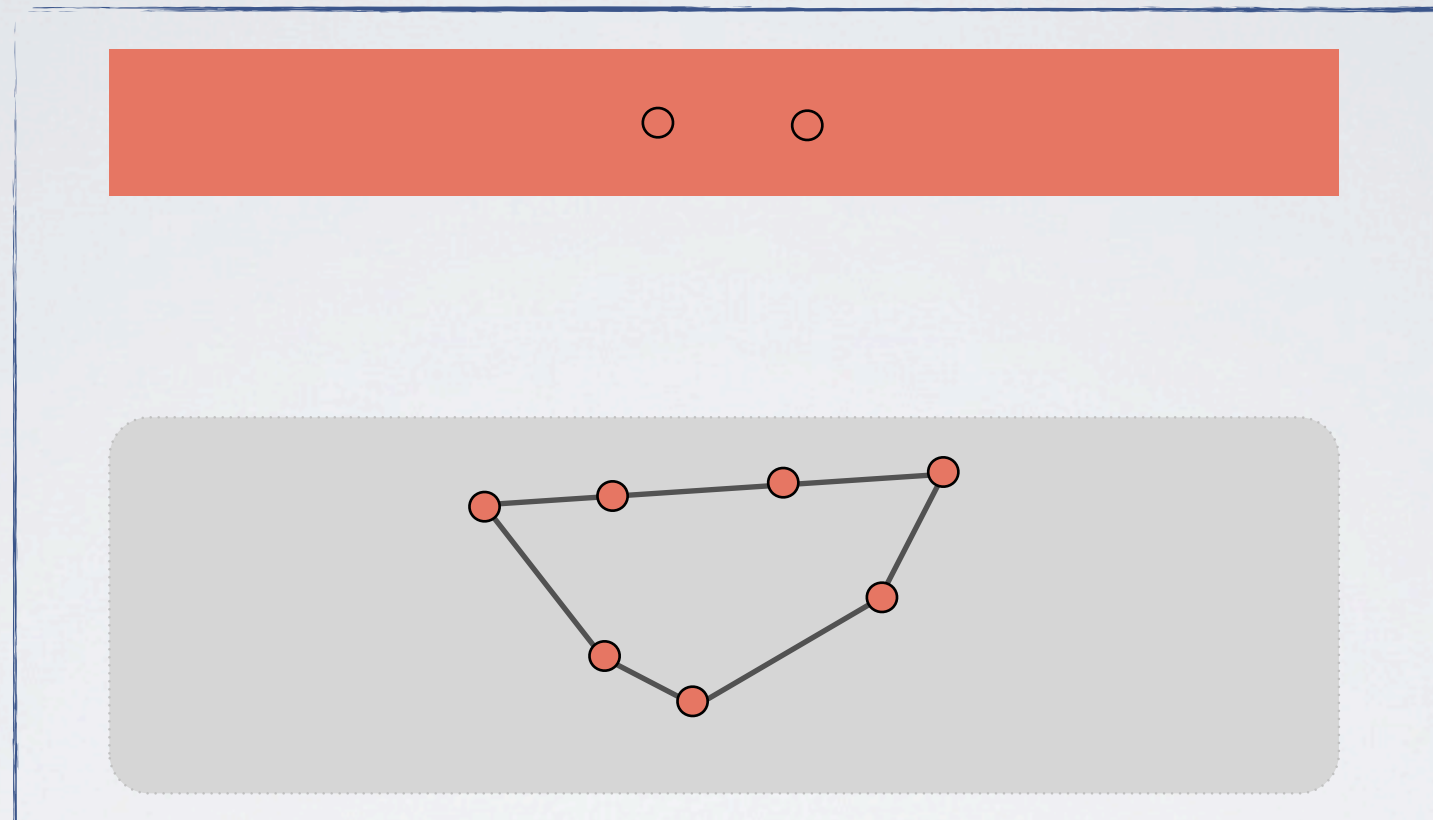
Claim: $H \setminus T$ is a forest.

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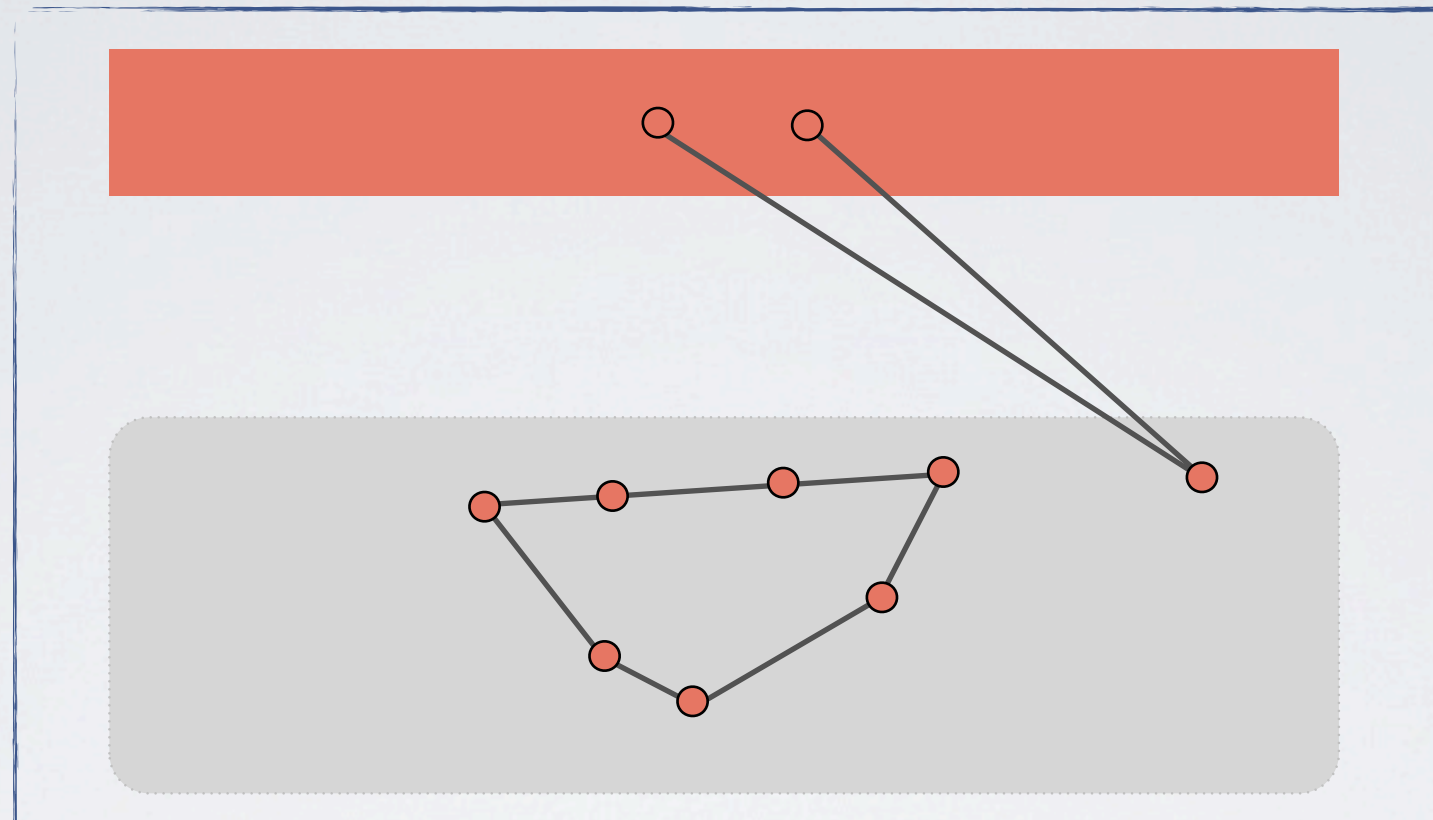
THE 2-CONNECTED STEINER TREE PROBLEM



Case 1: When C does not separate T .

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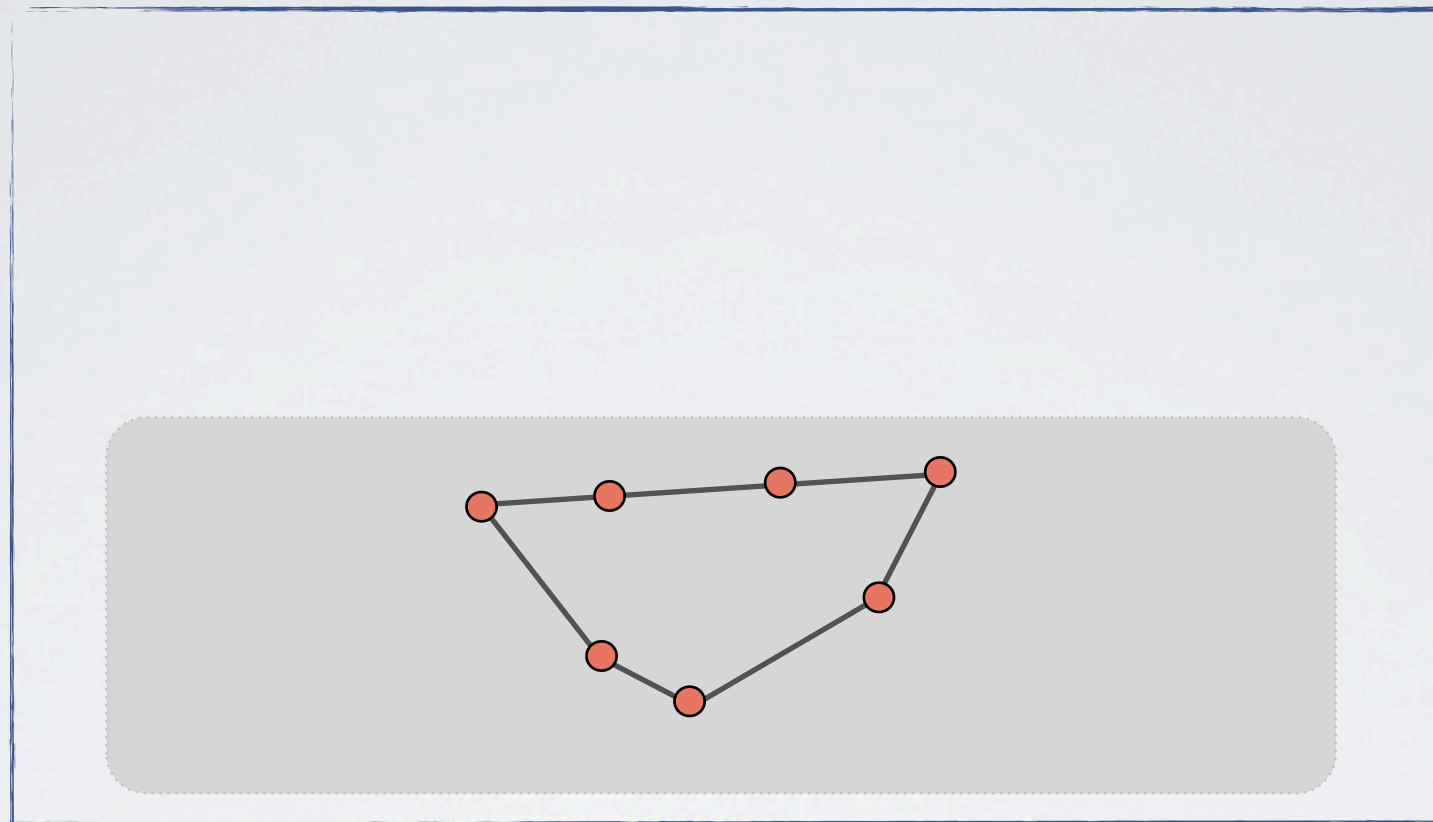
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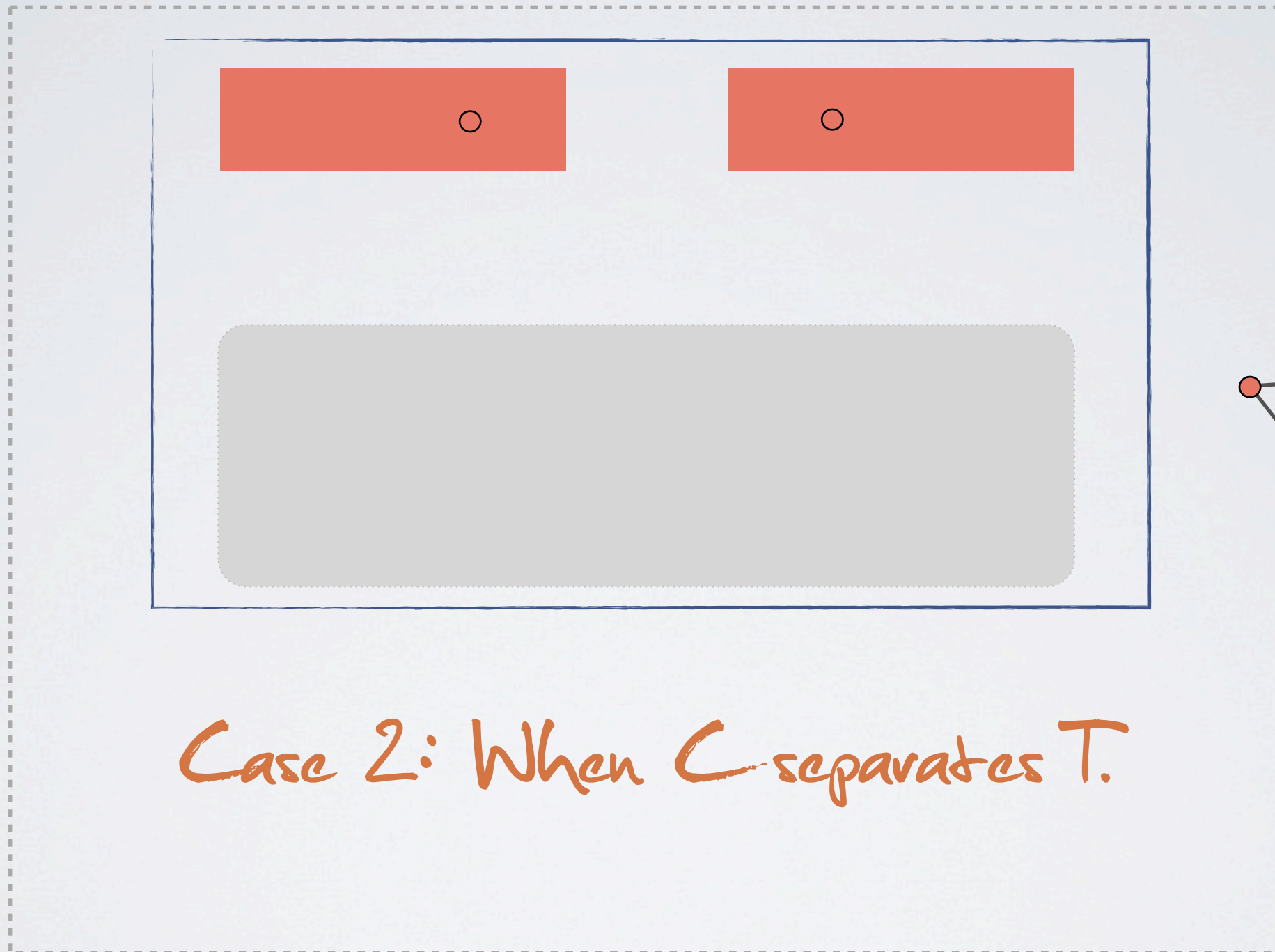
THE 2-CONNECTED STEINER TREE PROBLEM



Case 2: When C separates T .

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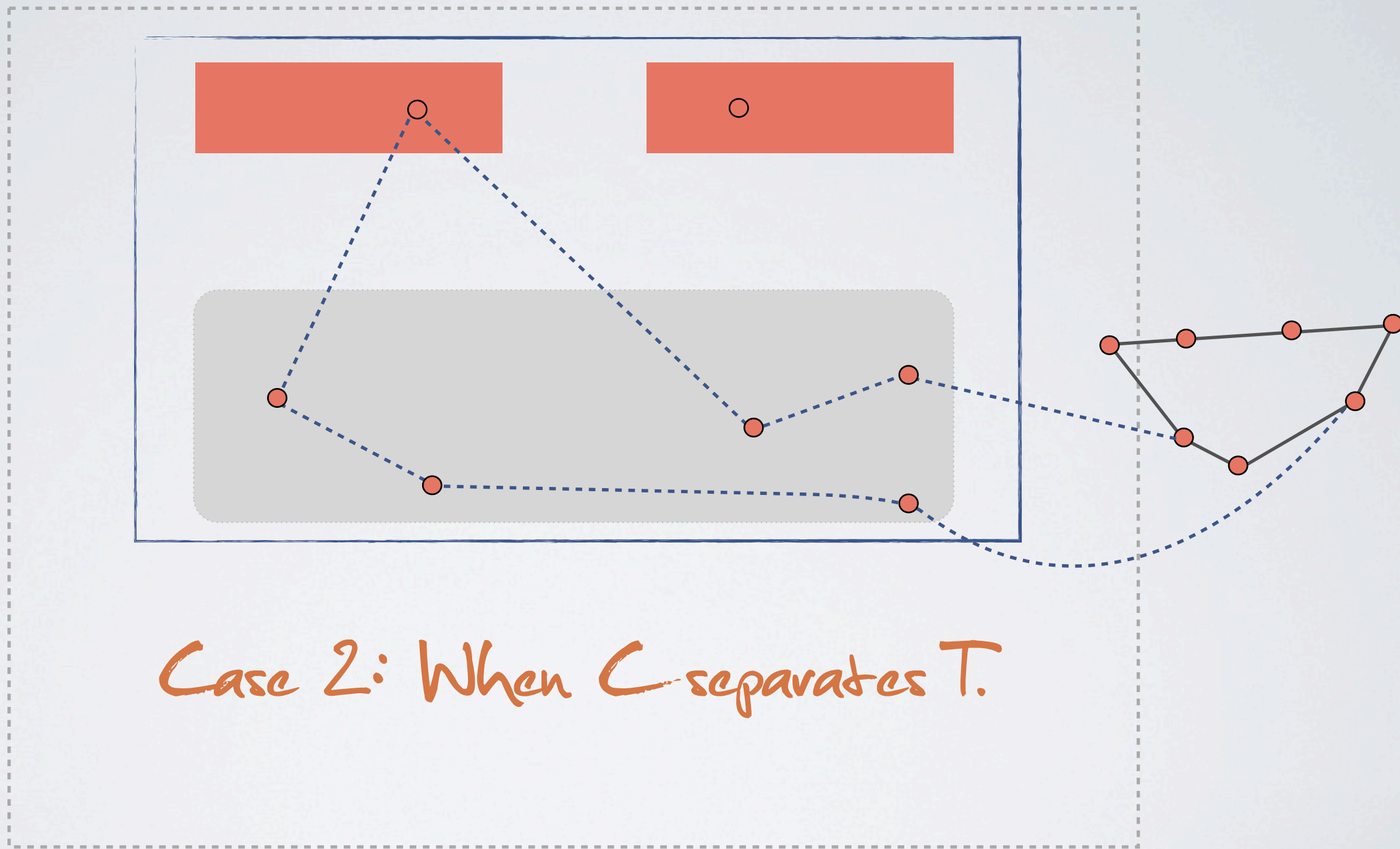
THE 2-CONNECTED STEINER TREE PROBLEM



Case 2: When C separates T .

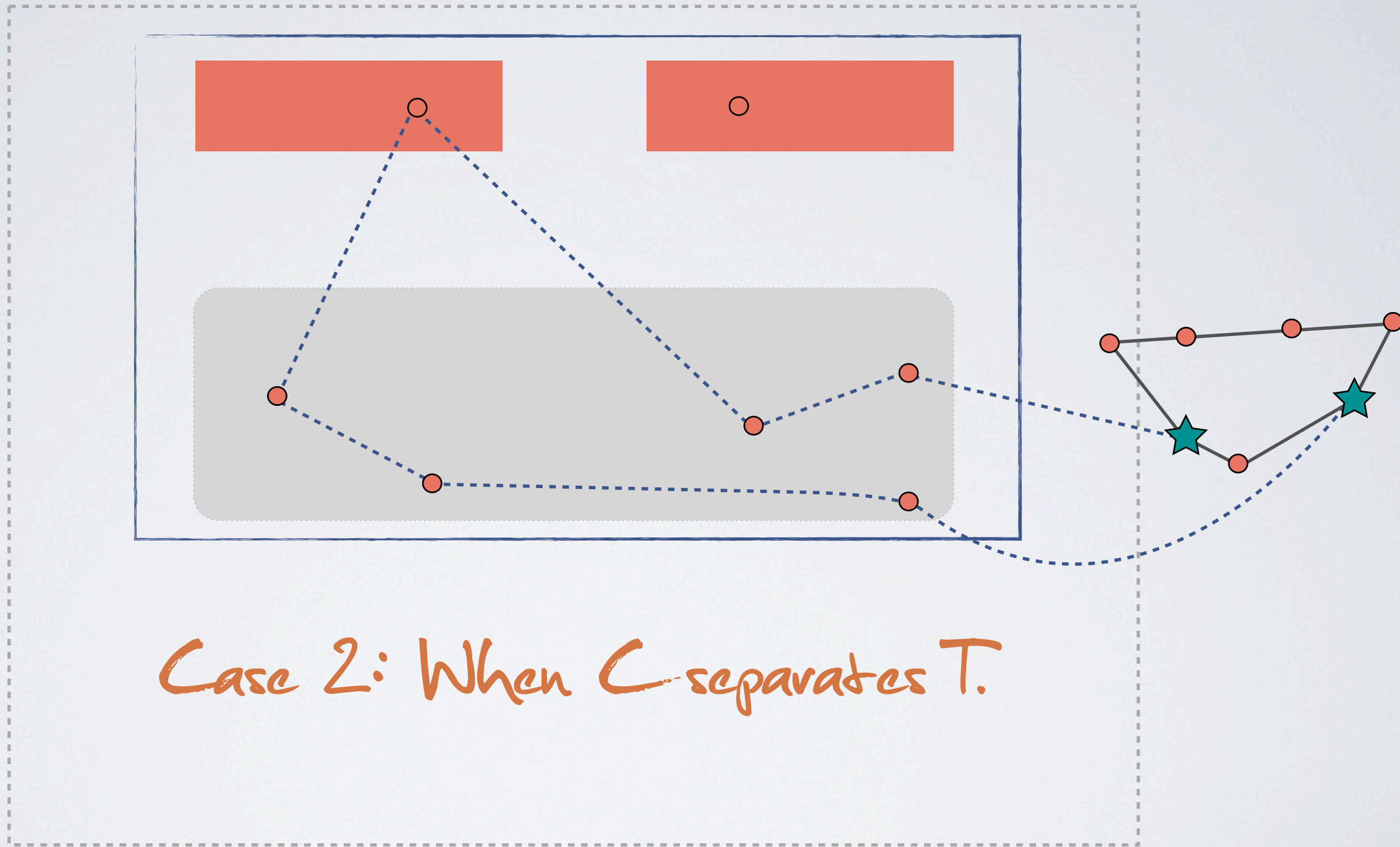
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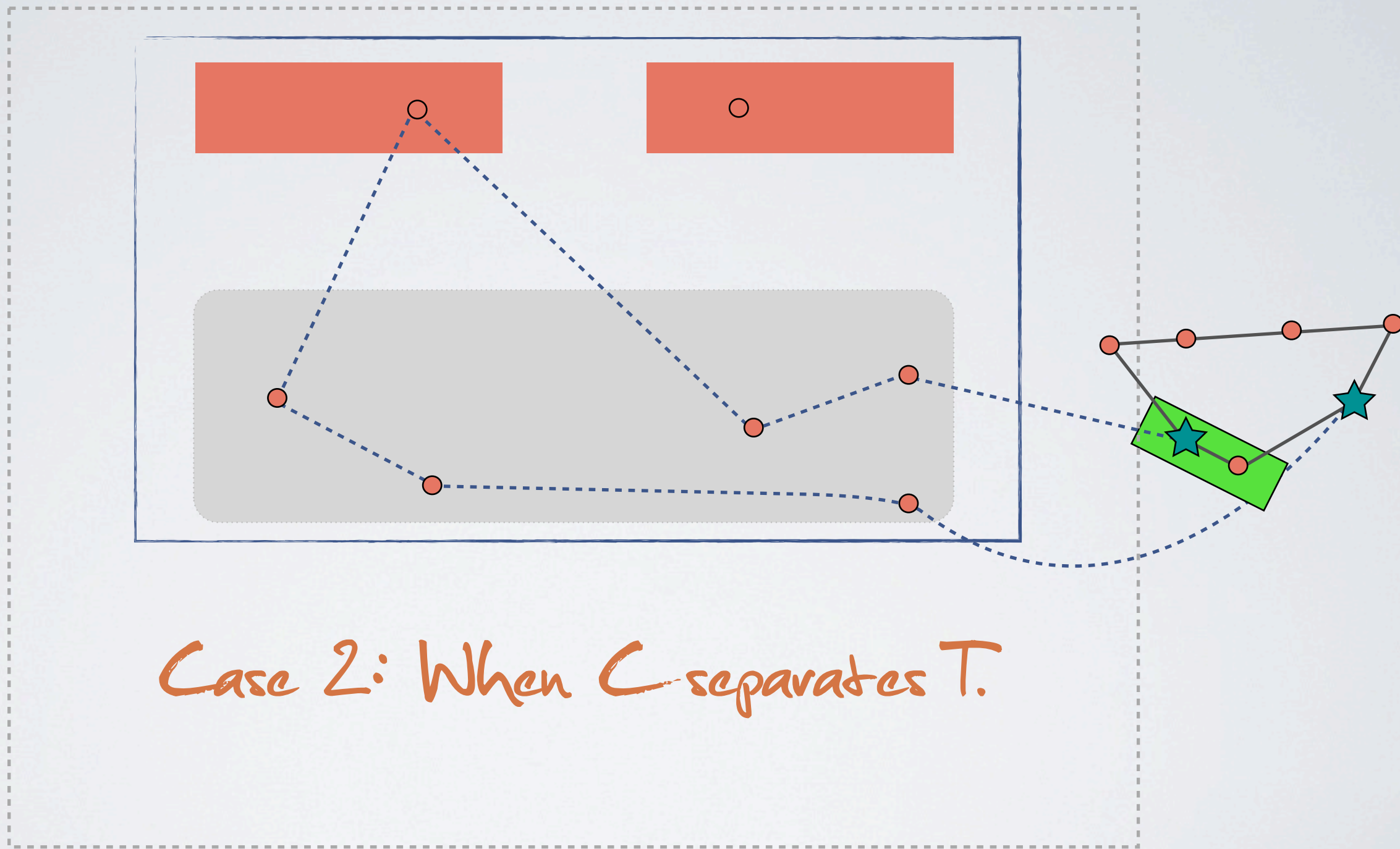
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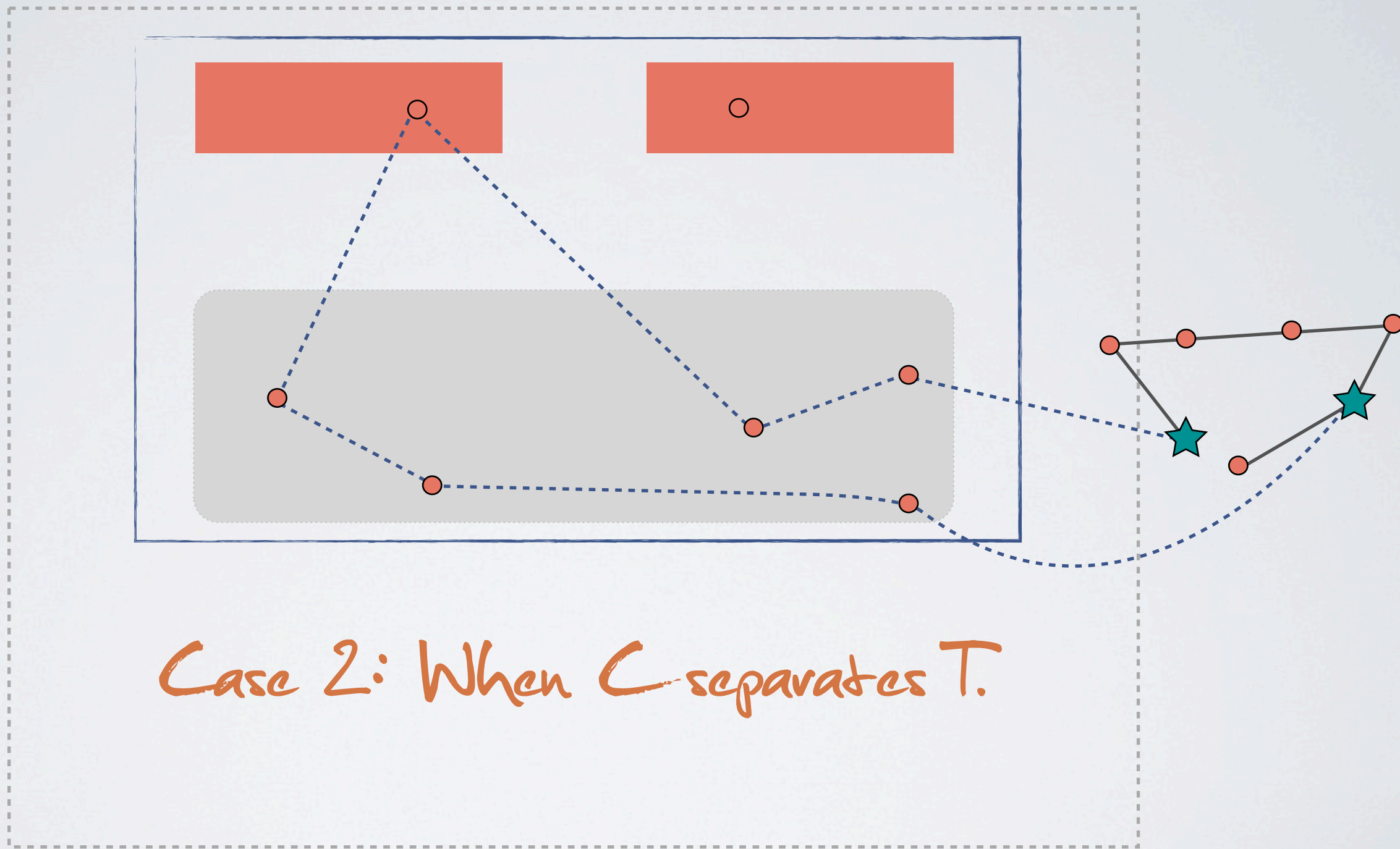
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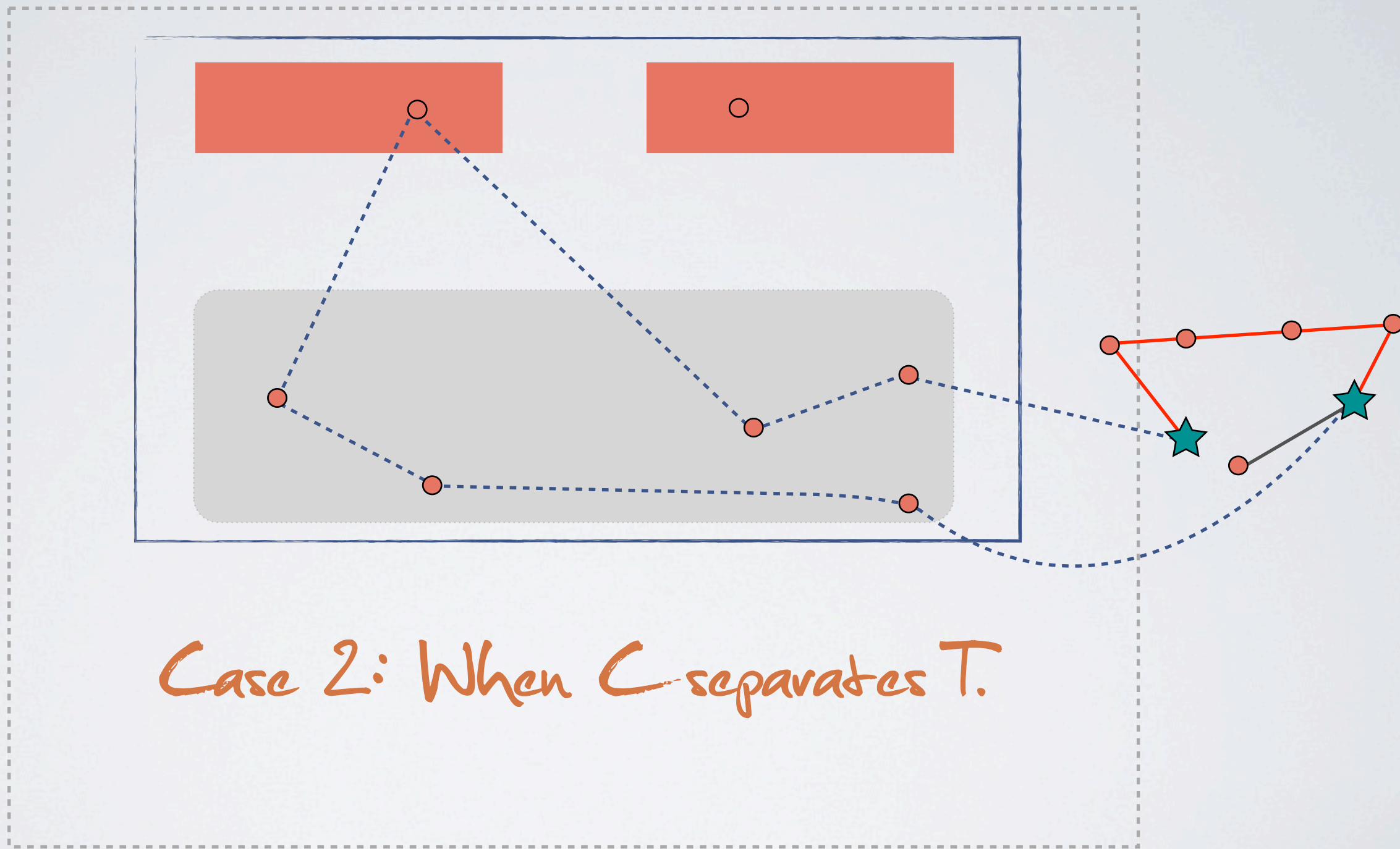
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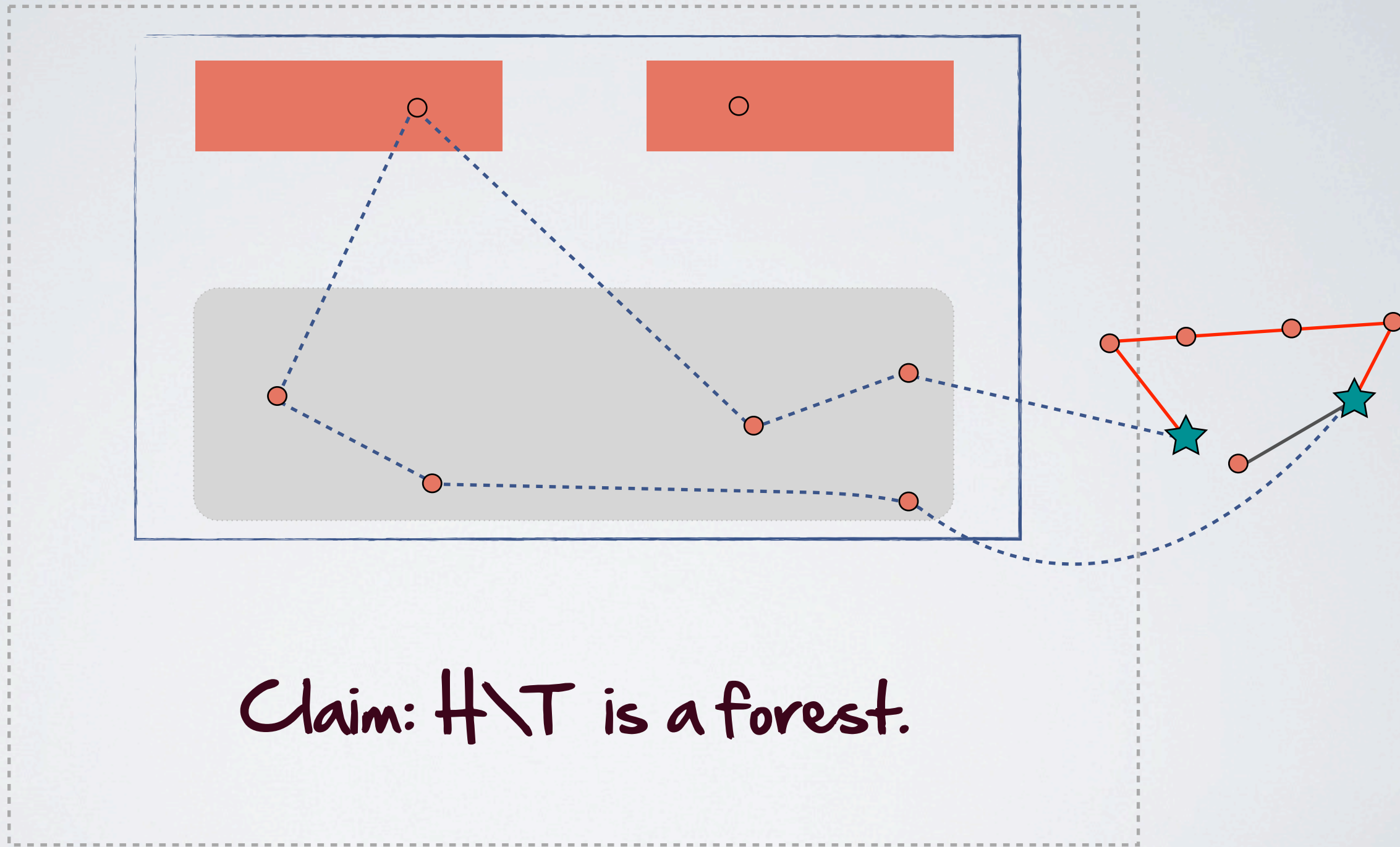
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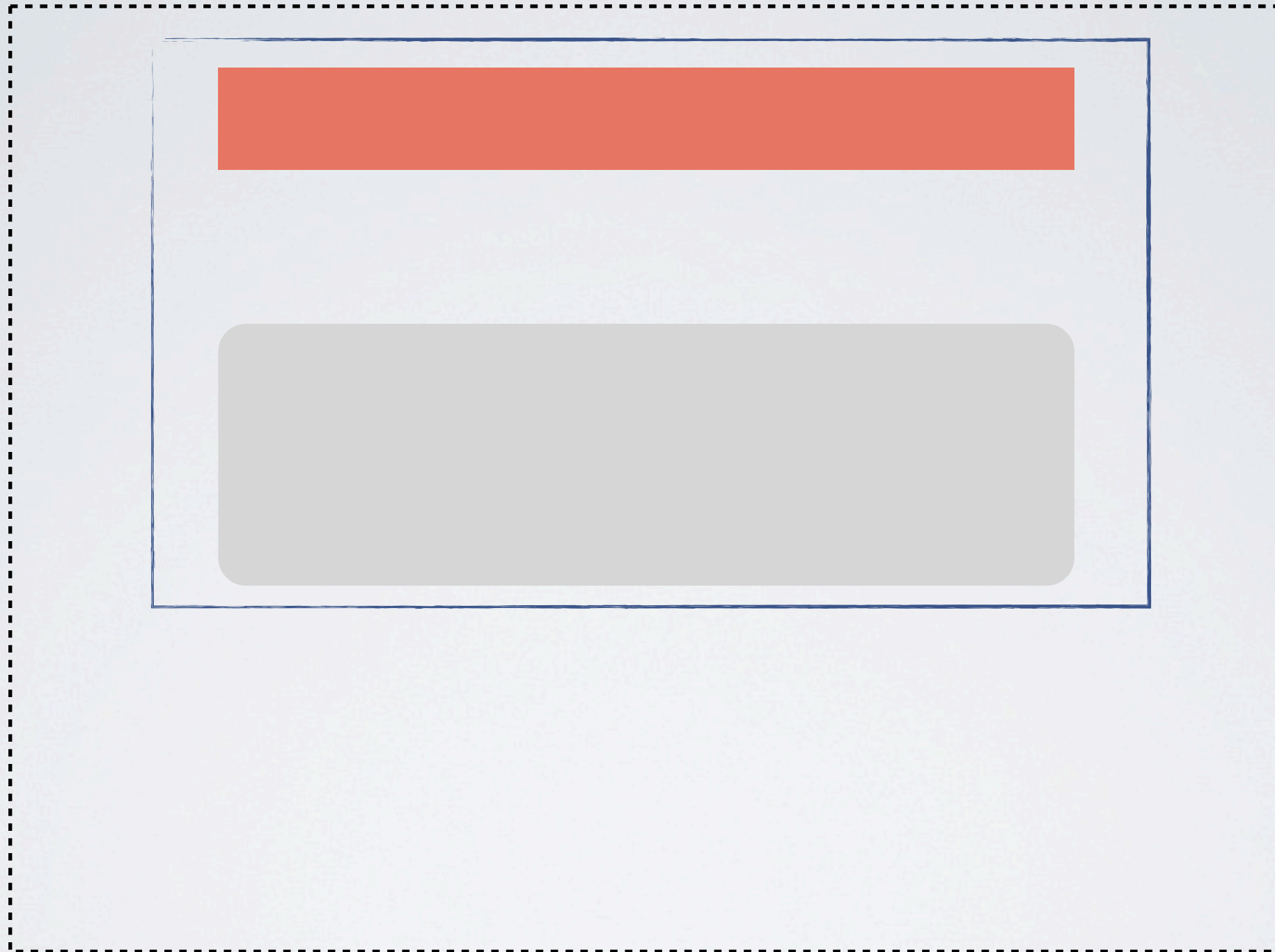


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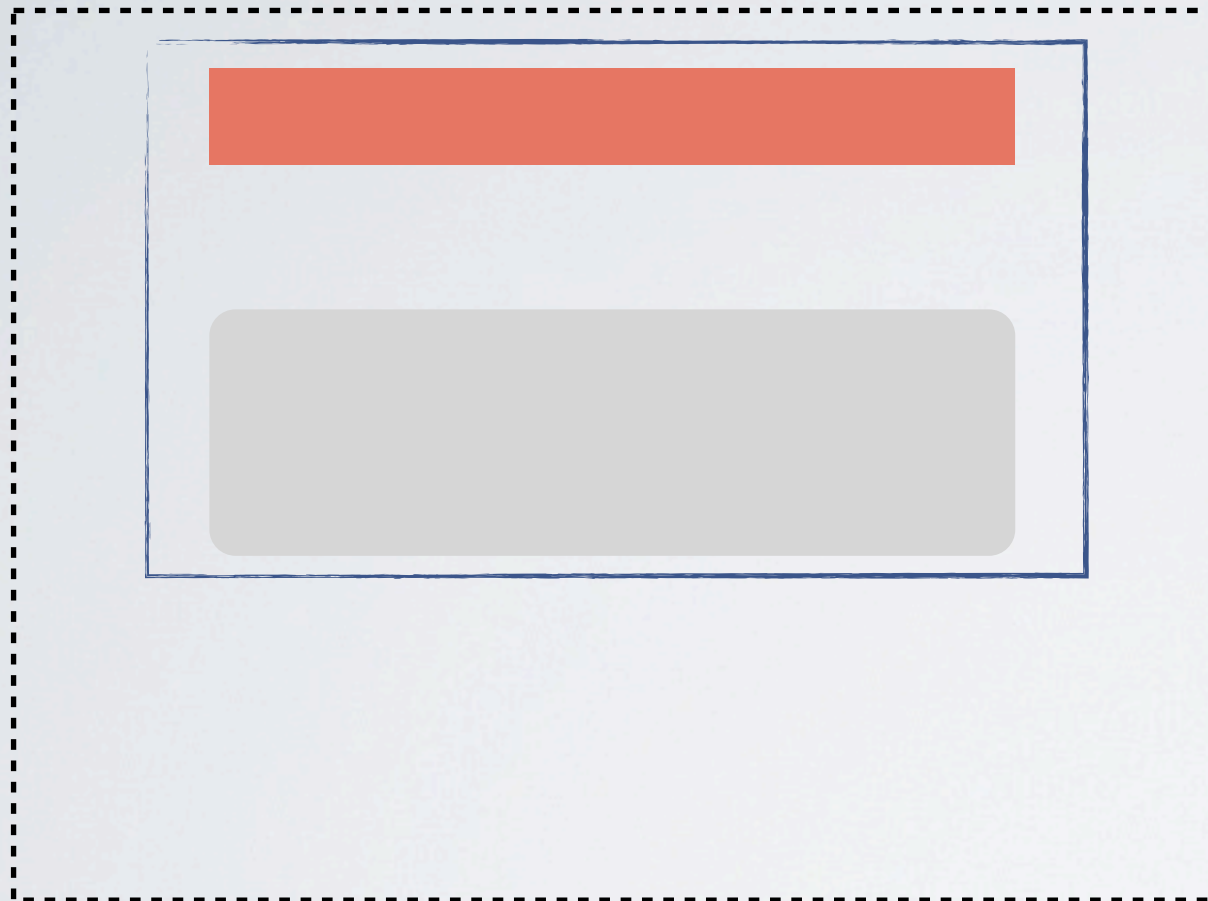
THE 2-CONNECTED STEINER TREE PROBLEM



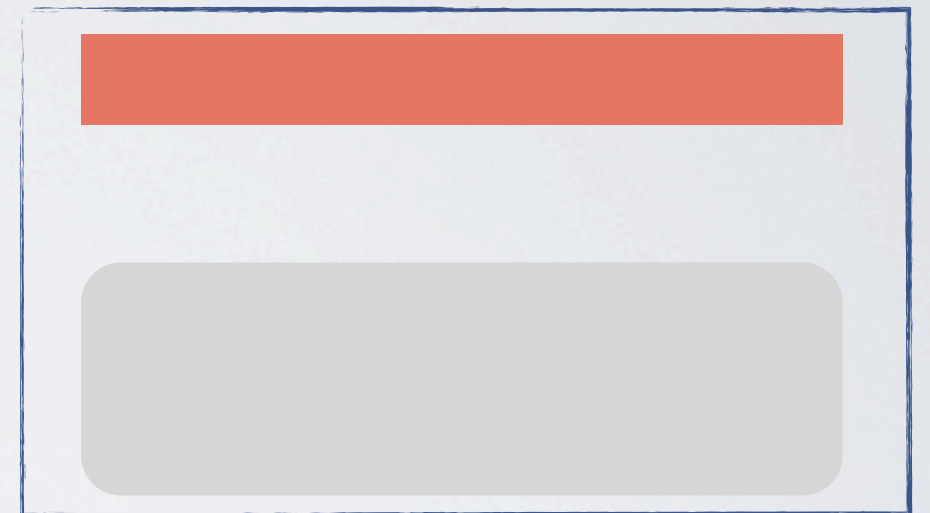
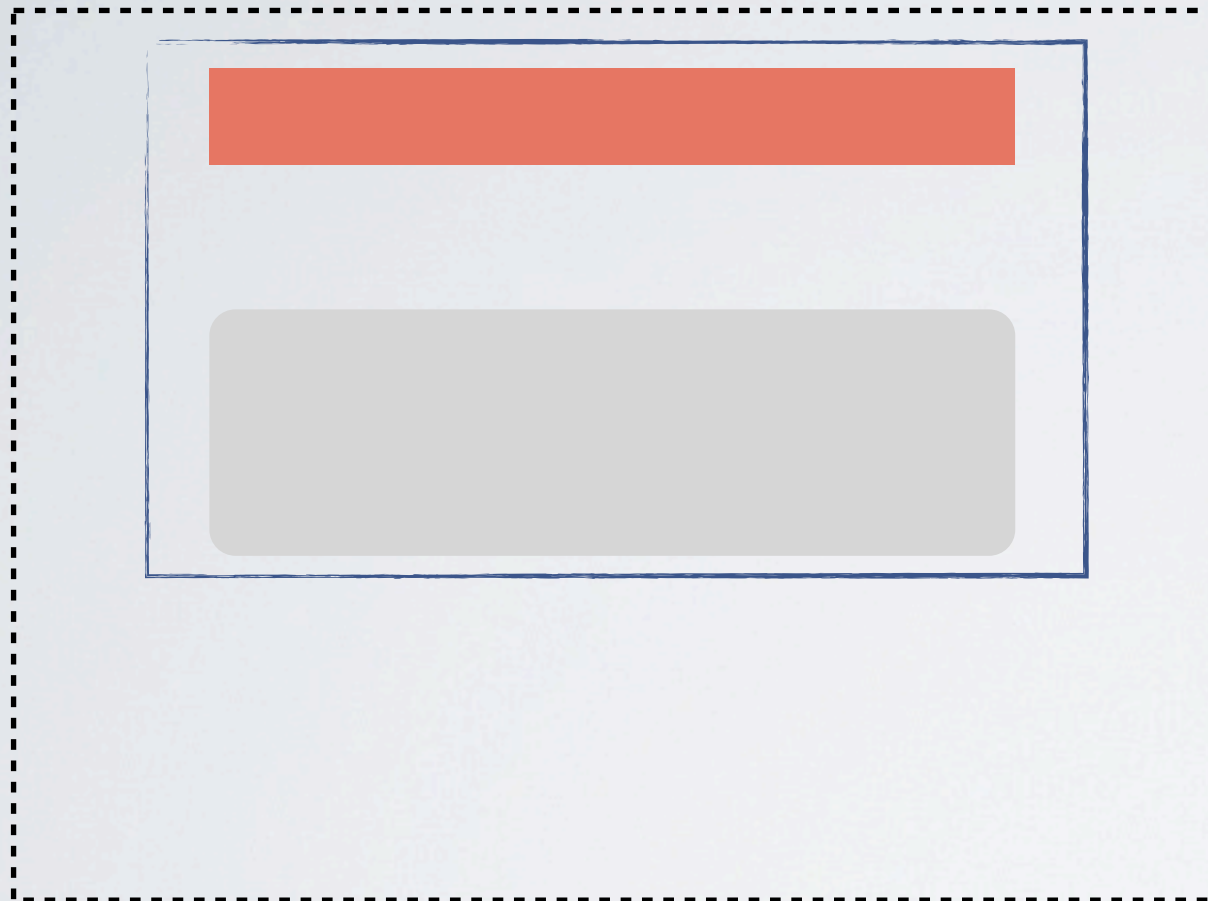
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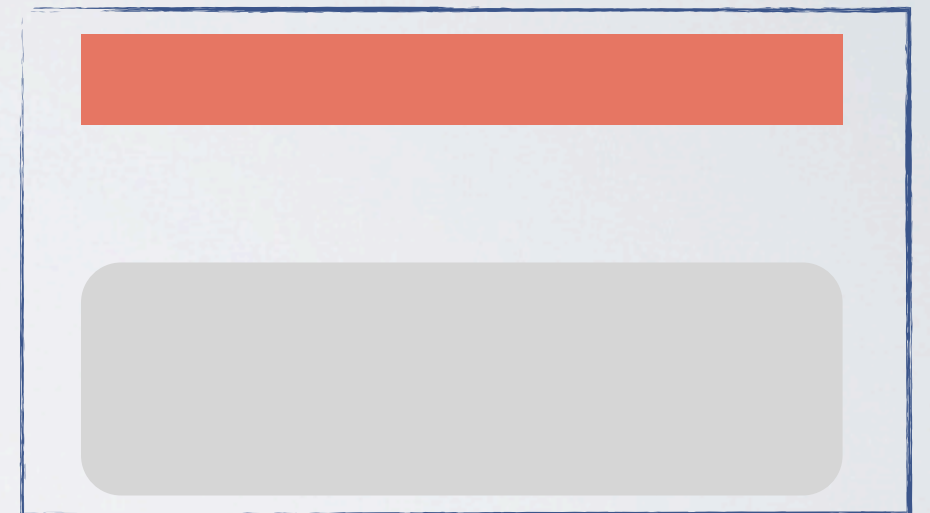
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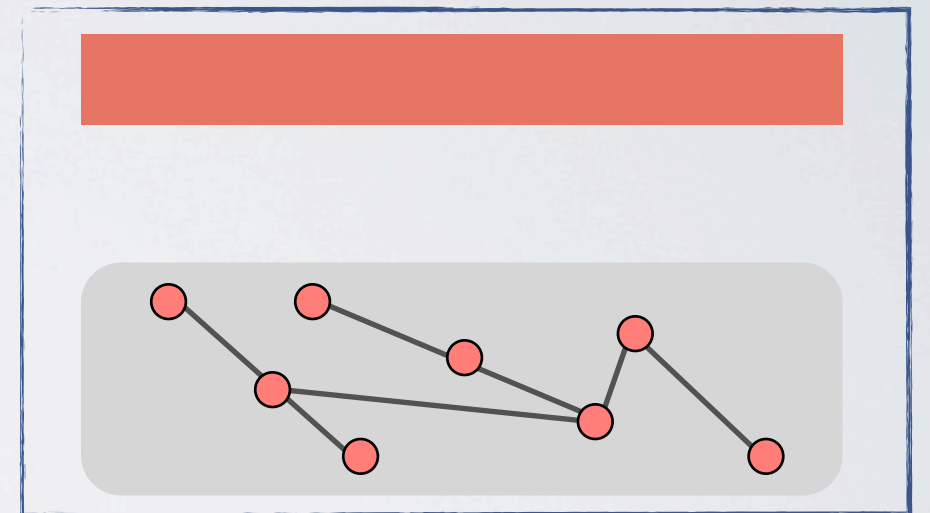
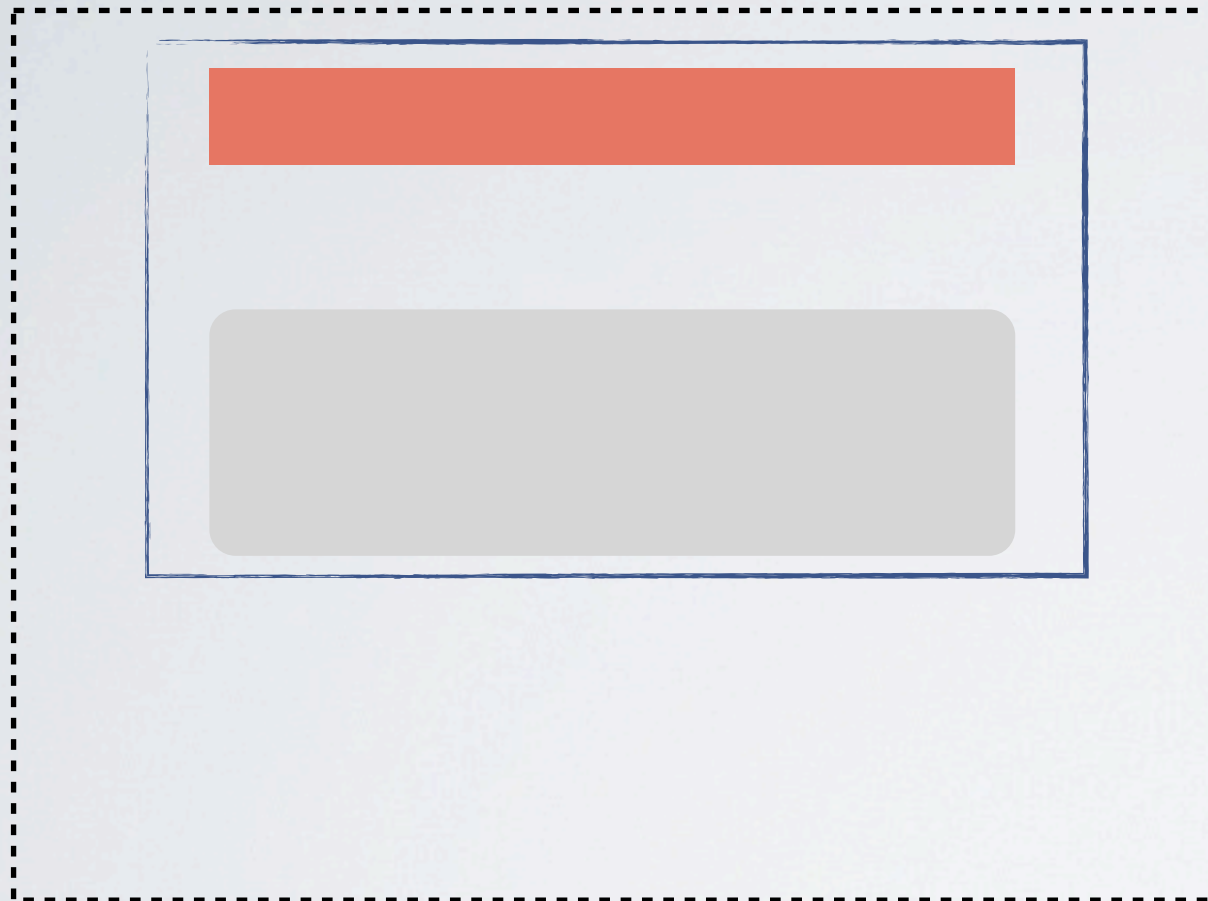
THE 2-CONNECTED STEINER TREE PROBLEM



Guess the structure of H .

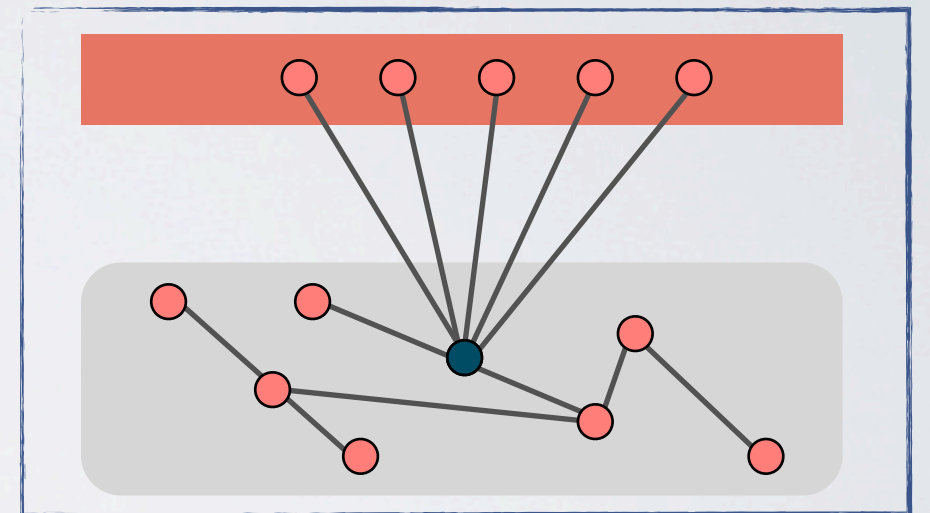
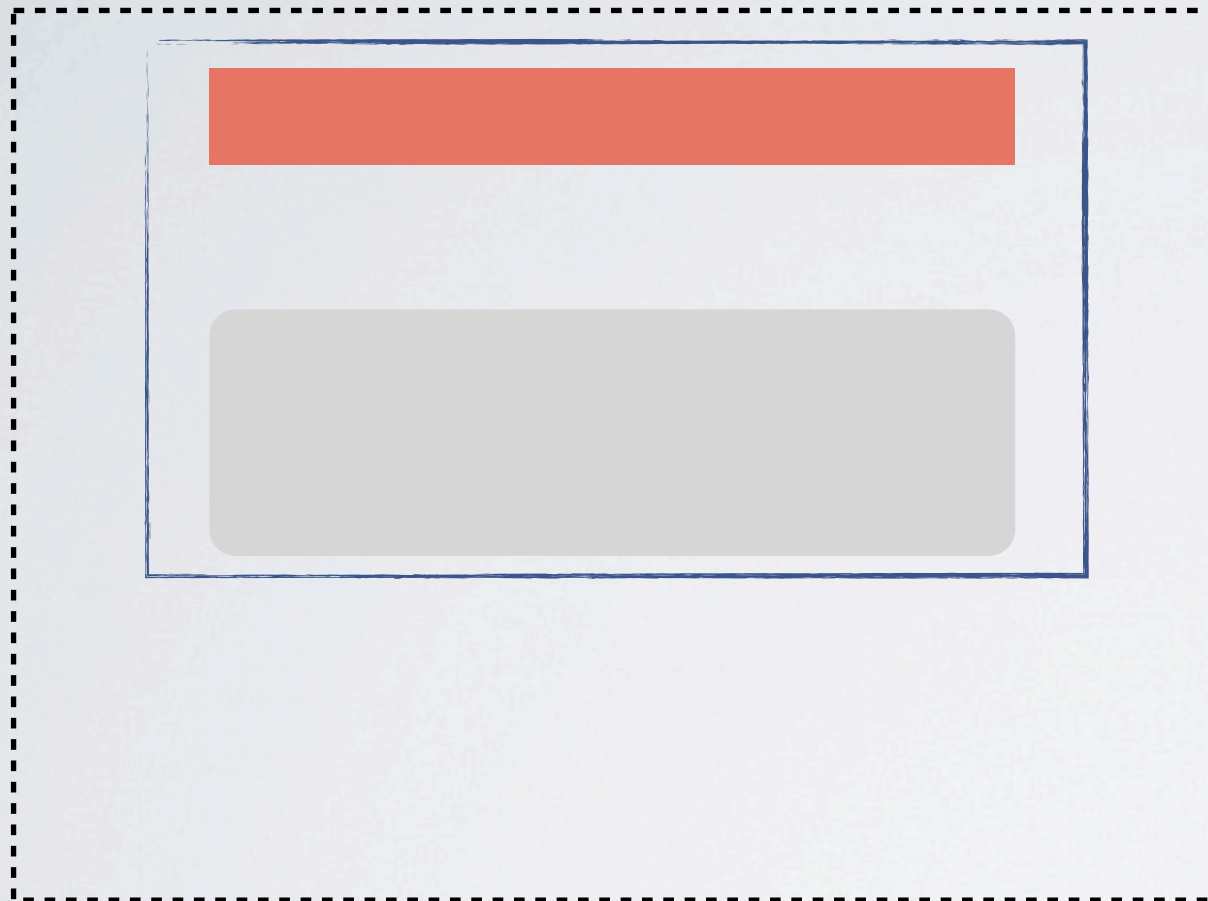
THE 2-CONNECTED STEINER TREE PROBLEM

Try all graphs H for which $H \setminus T$ is a forest.



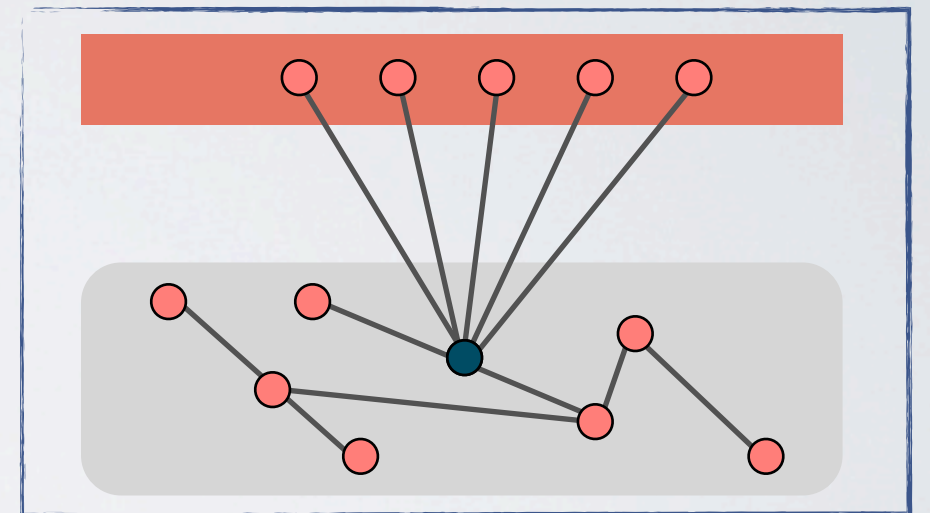
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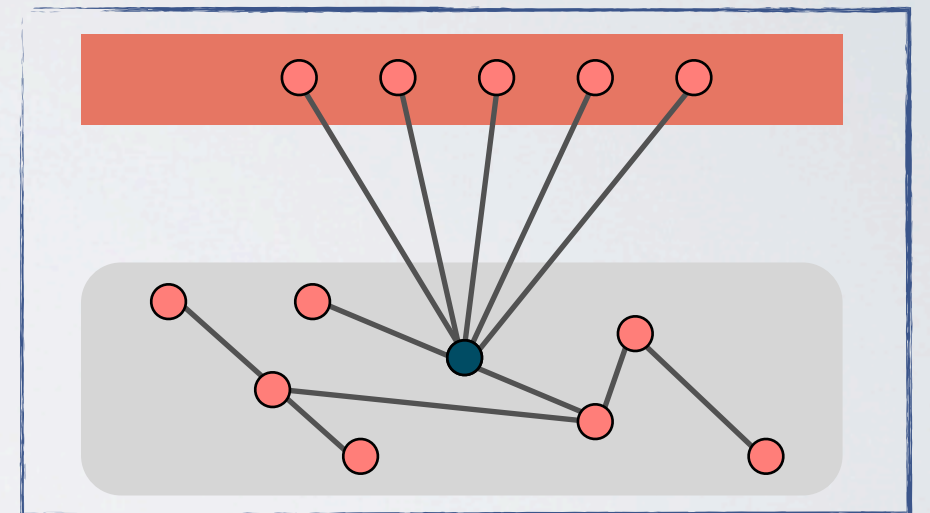
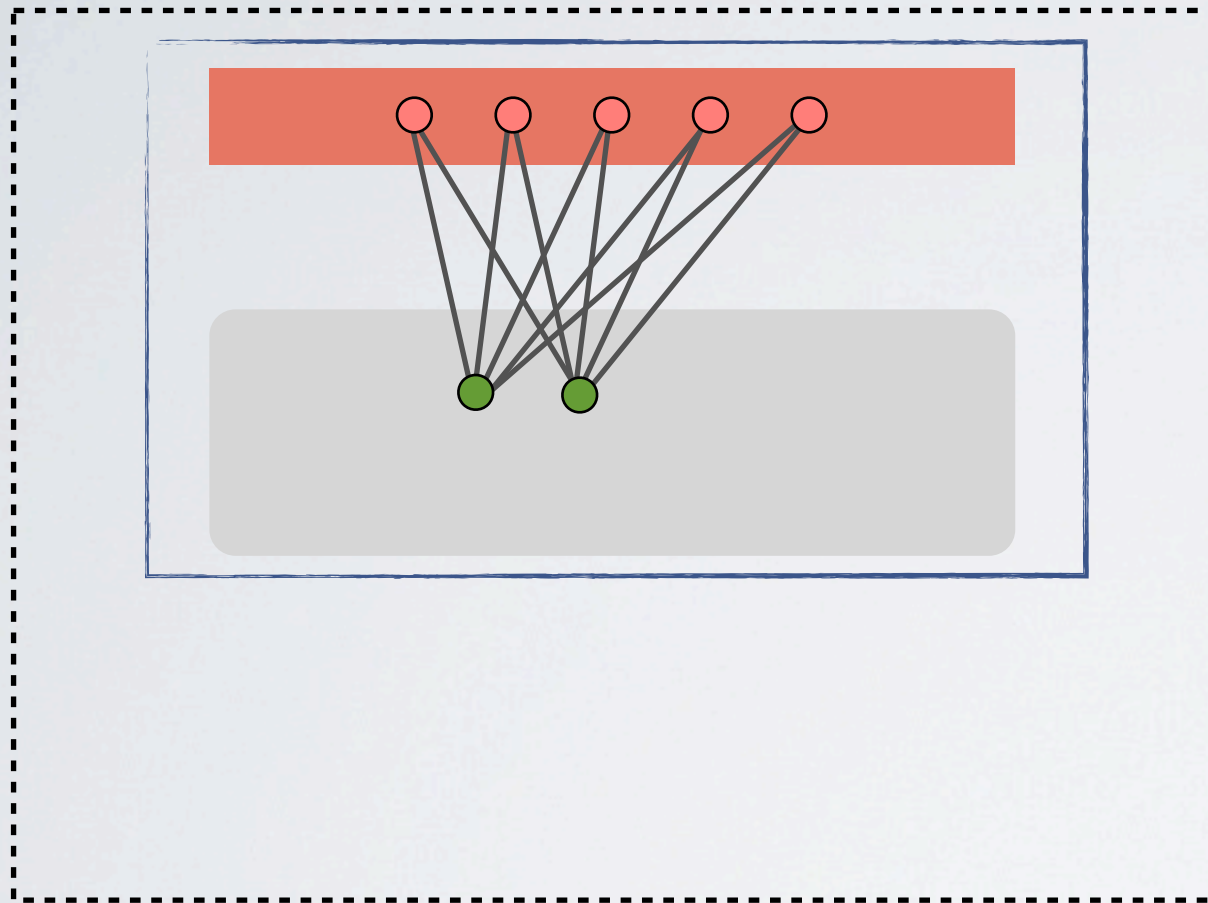
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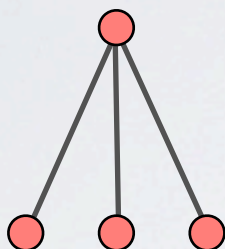
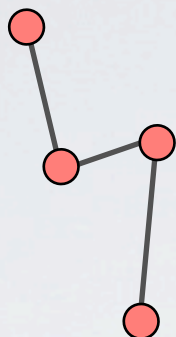
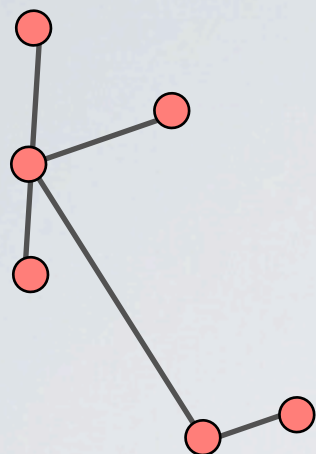
Map H into G such that vertices in H are assigned to their "candidates" in G .

THE 2-CONNECTED STEINER TREE PROBLEM

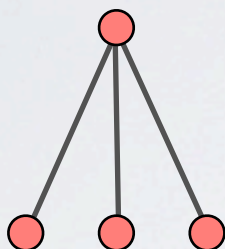
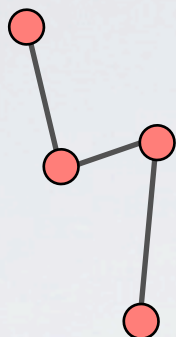
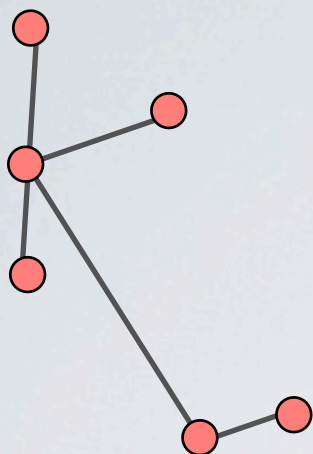
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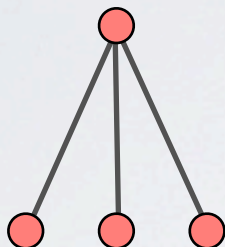
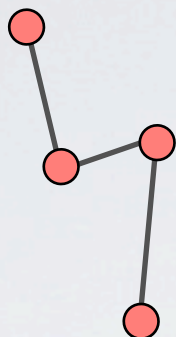
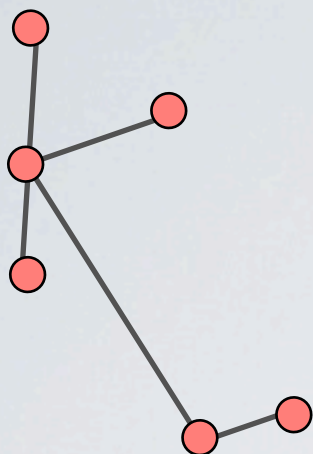
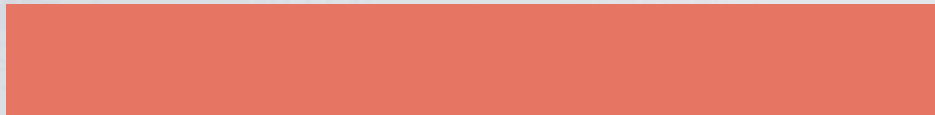
H



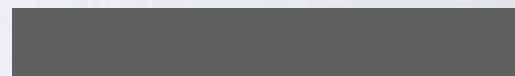
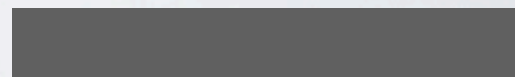
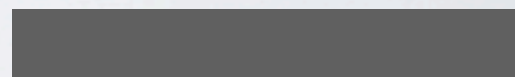
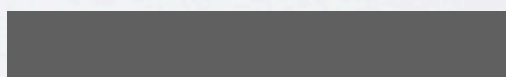
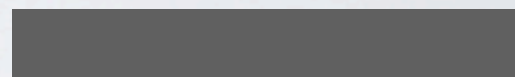
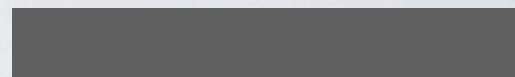
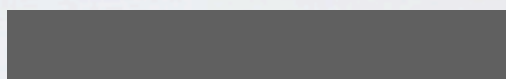
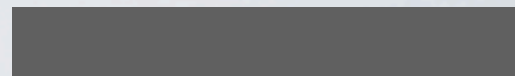
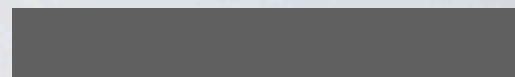
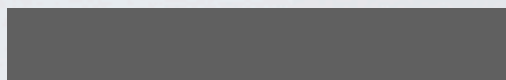
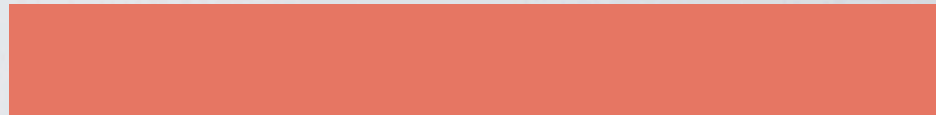
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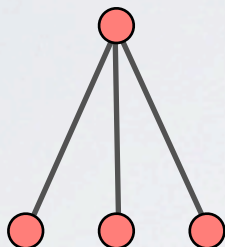
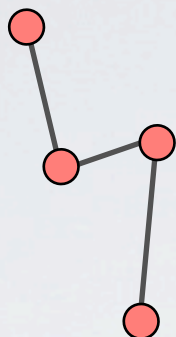
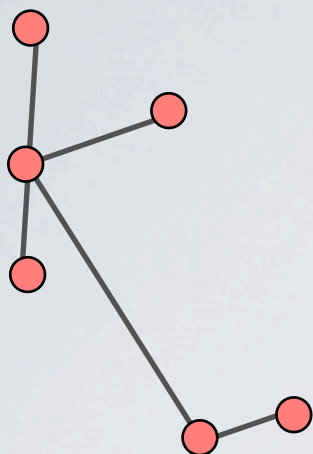
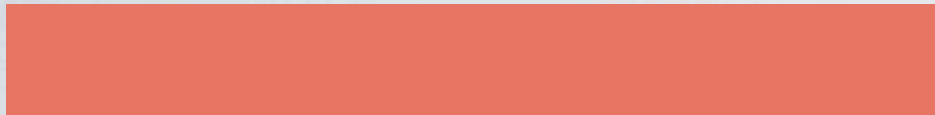
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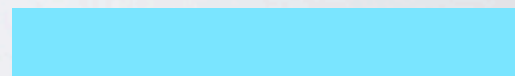
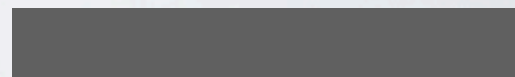
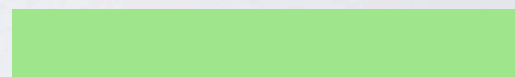
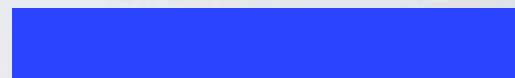
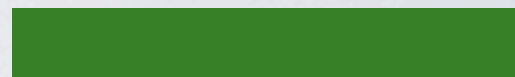
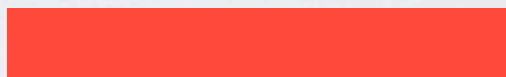
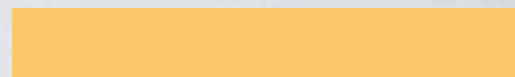
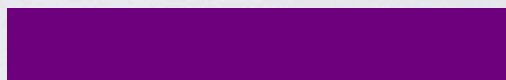
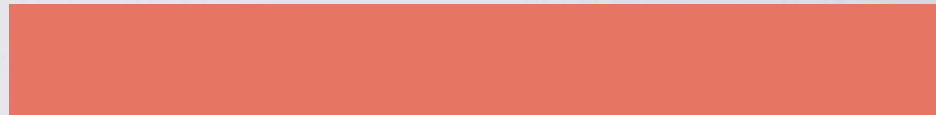
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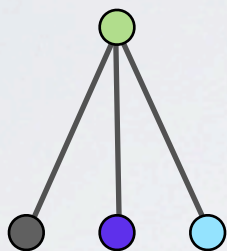
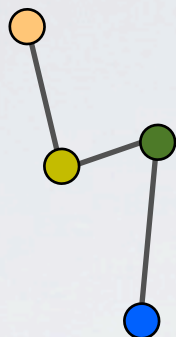
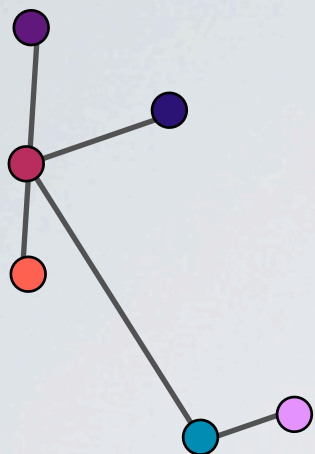
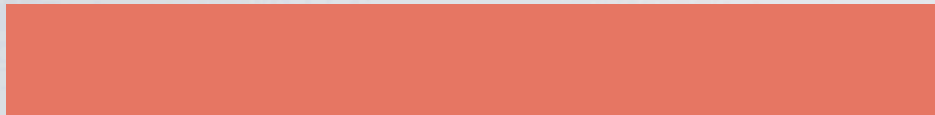
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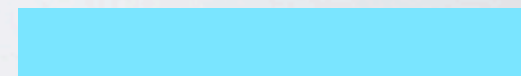
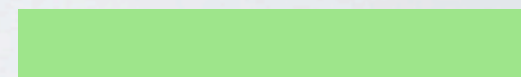
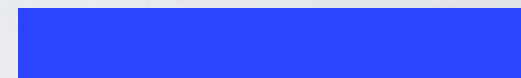
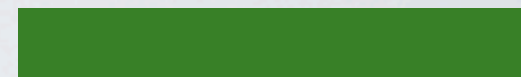
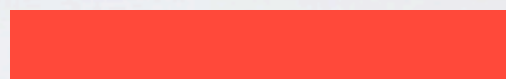
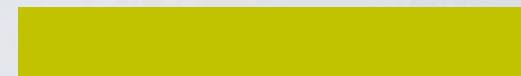
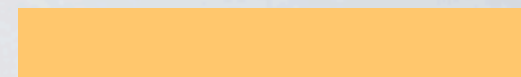
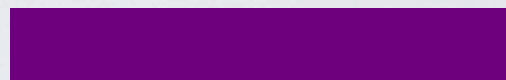
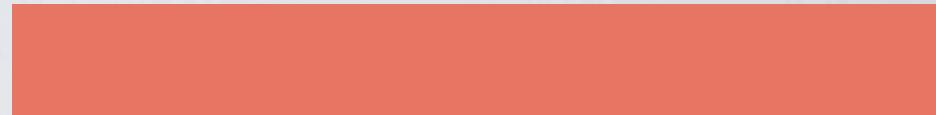
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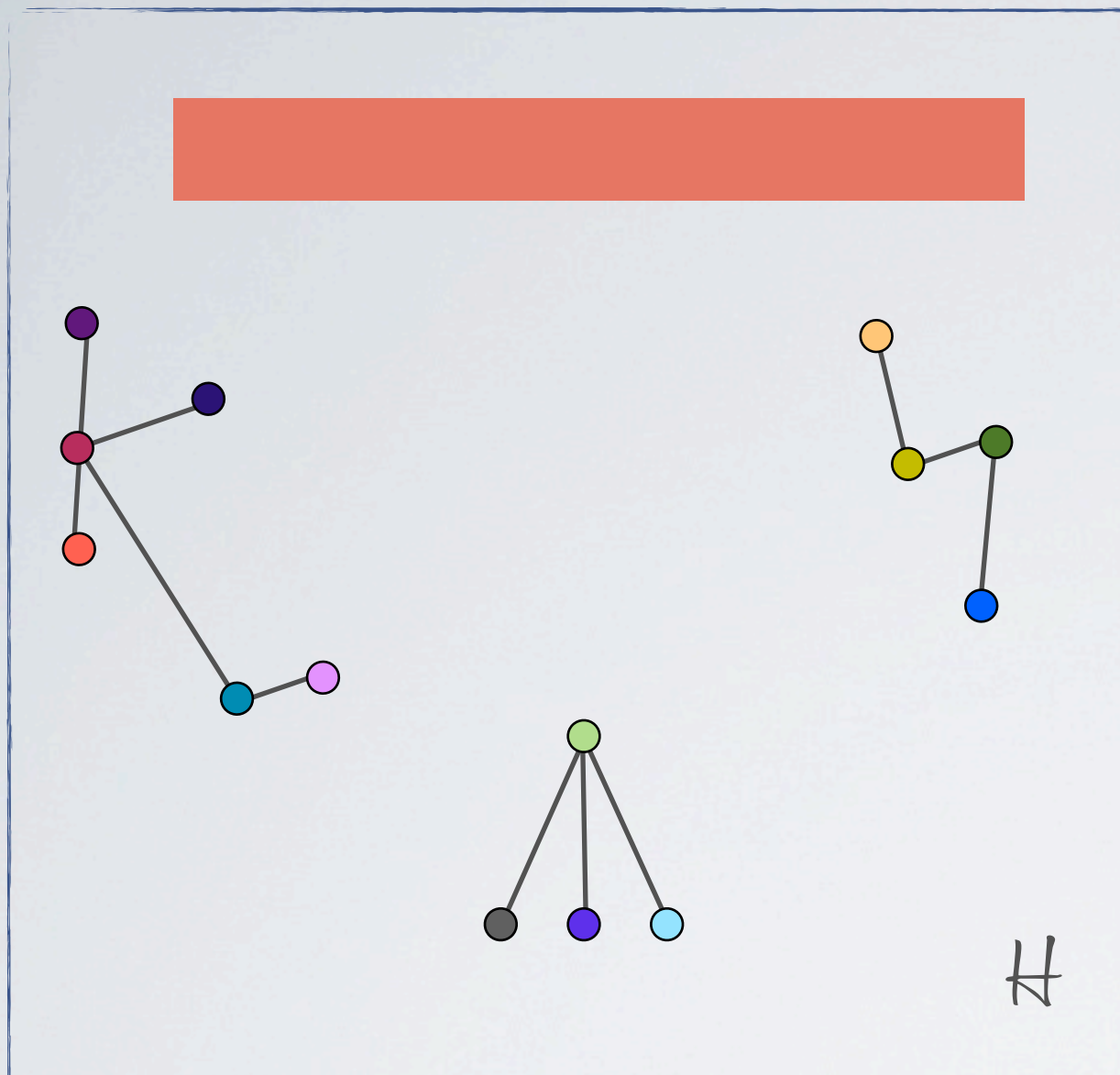
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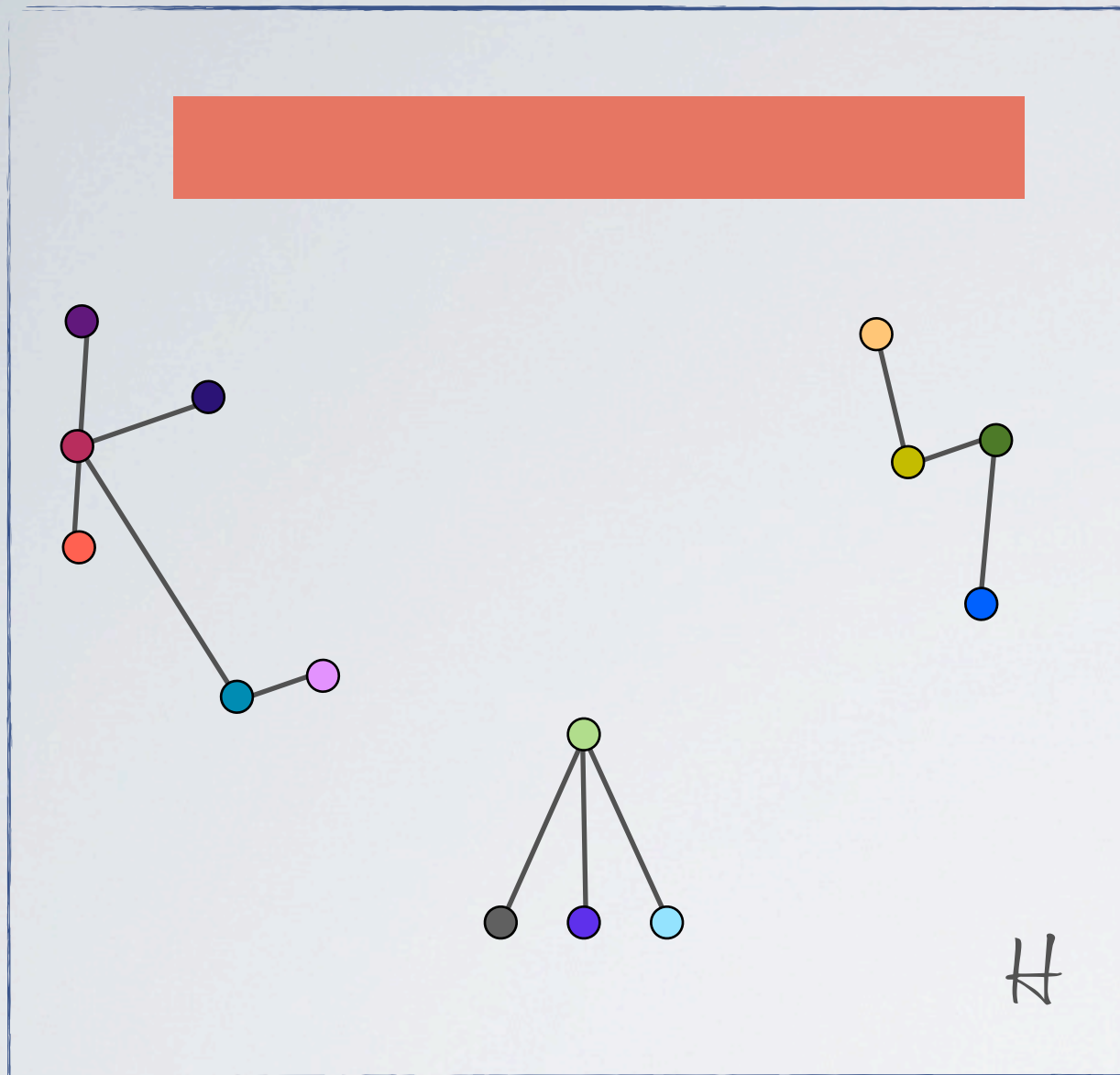


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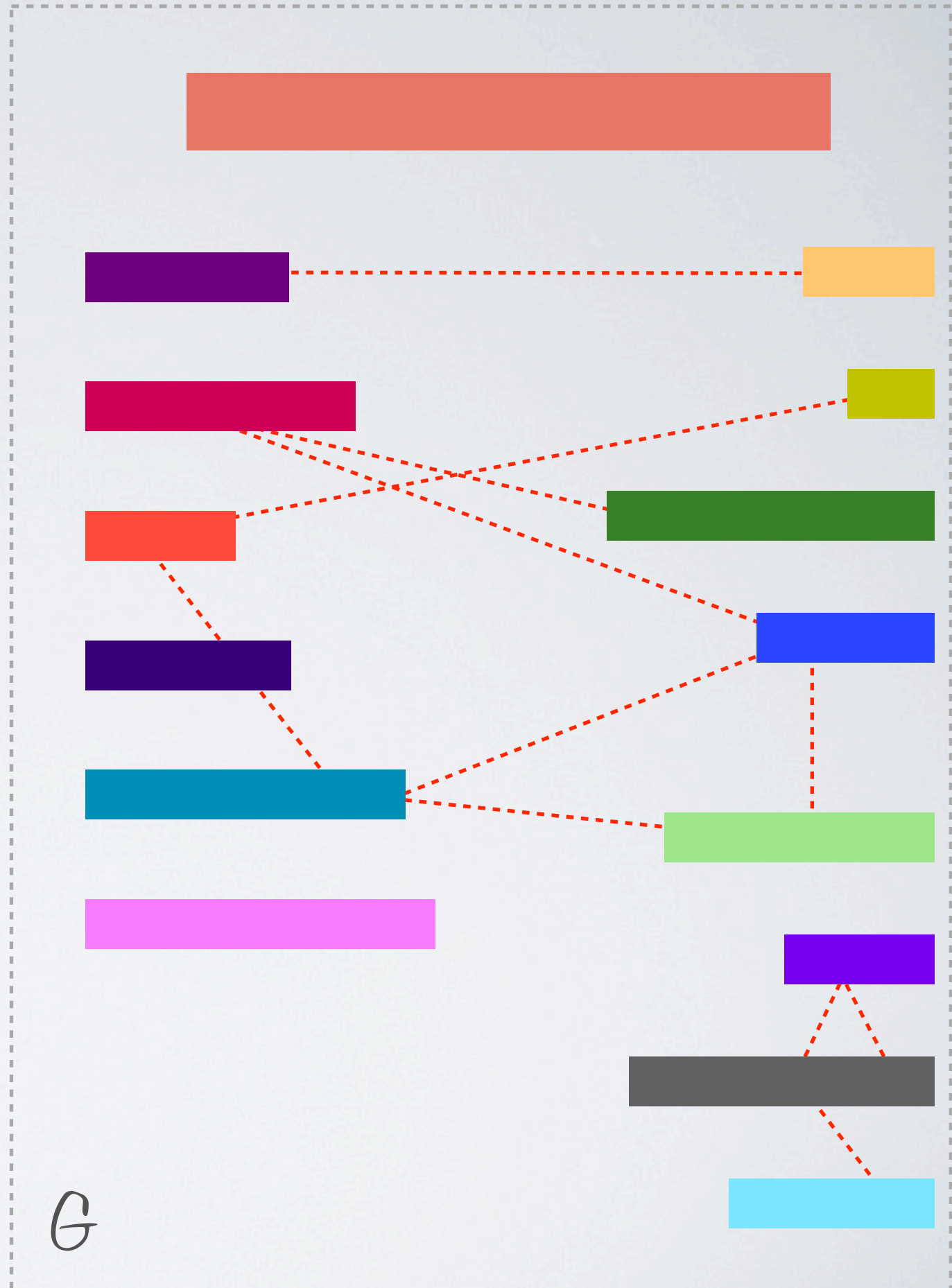


Prune the color classes to get rid of non-candidates.

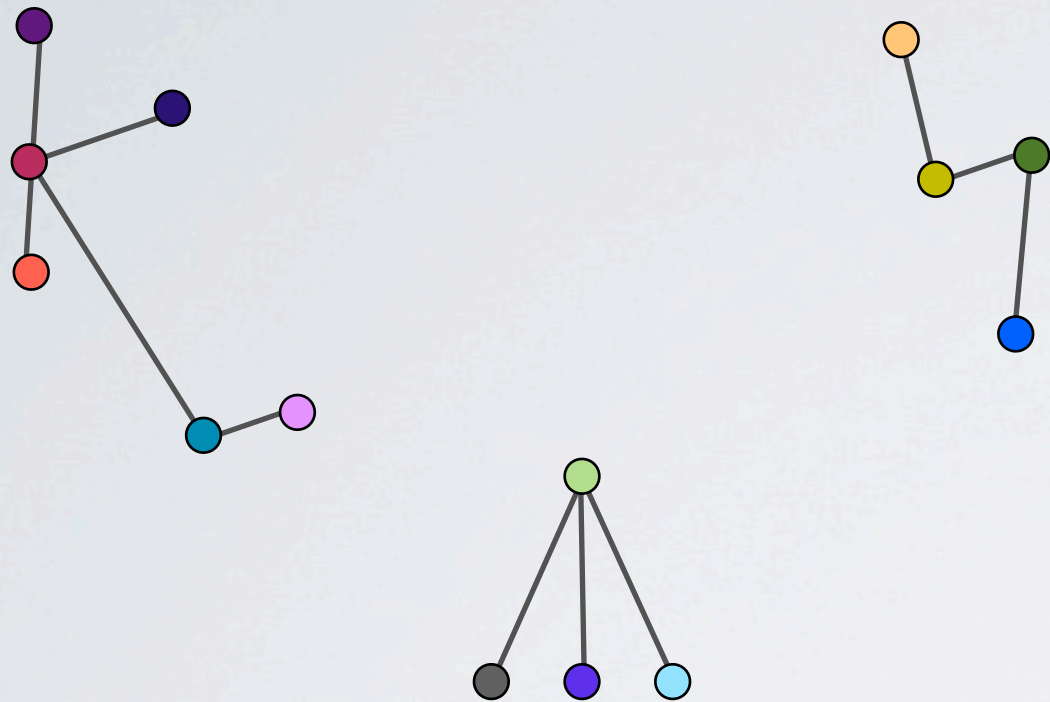




Delete irrelevant edges.



Perform
a breadth-first
search.



H

G

