

POLYNOMIAL KERNELS FOR PLANAR F-DELETION

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FEDOR V. FOMIN, DANIEL LOKSHTANOV AND SAKET SAURABH

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when F contains connected graphs

Outline

Outline

PROLOGUE

Outline

SKETCHES OF KEY IDEAS

Outline

EPILOGUE

Polynomial Kernels for Planar F -deletion

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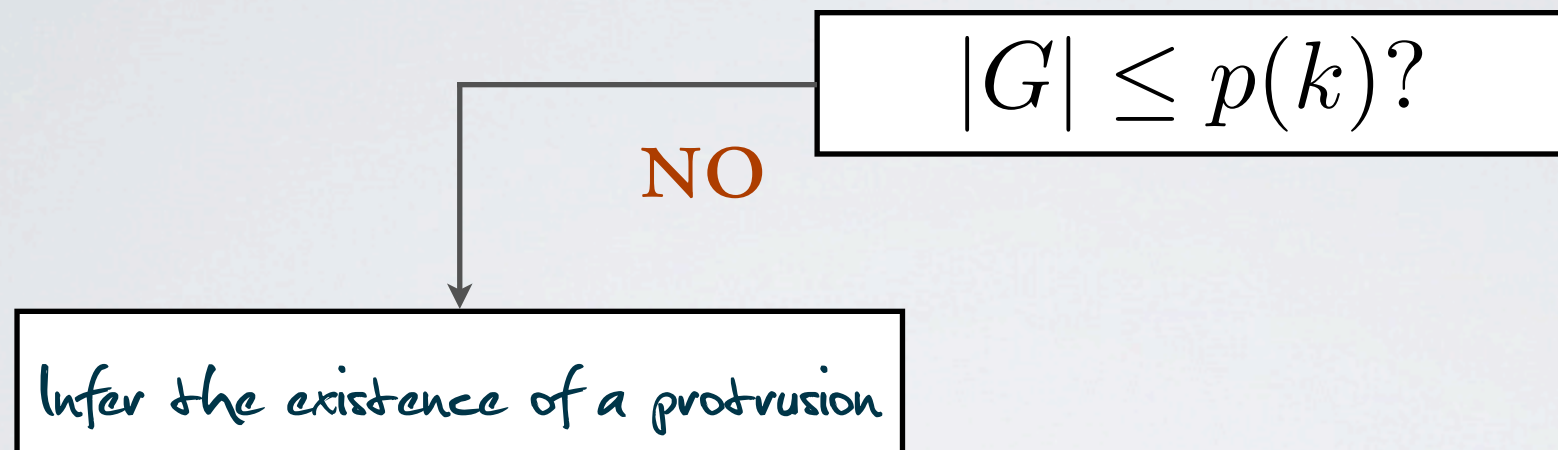
THE STORY SO FAR

RESTRICTED CASES OF PLANAR F-DELETION

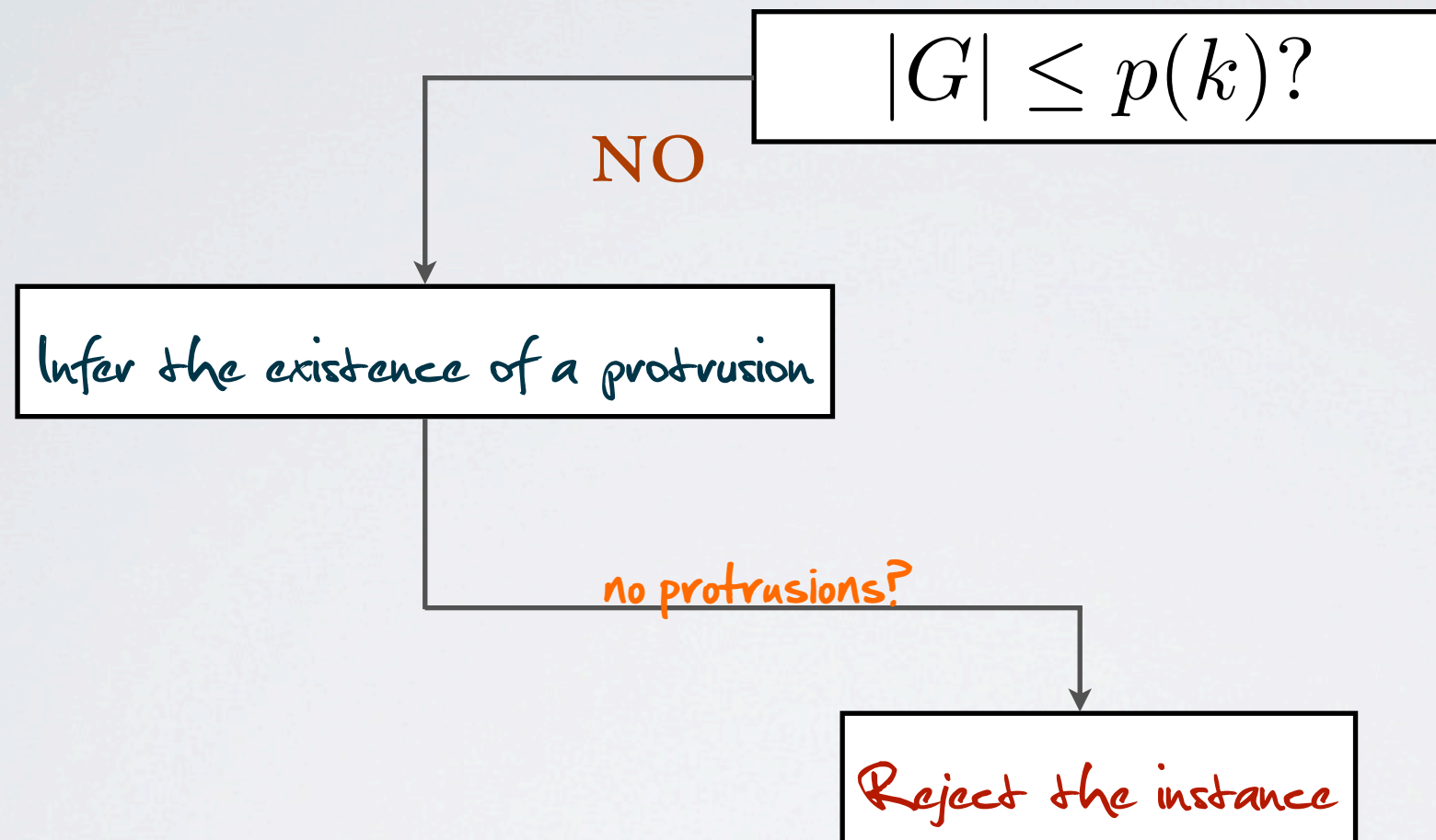
RESTRICTED CASES OF PLANAR F-DELETION

$$|G| \leq p(k)?$$

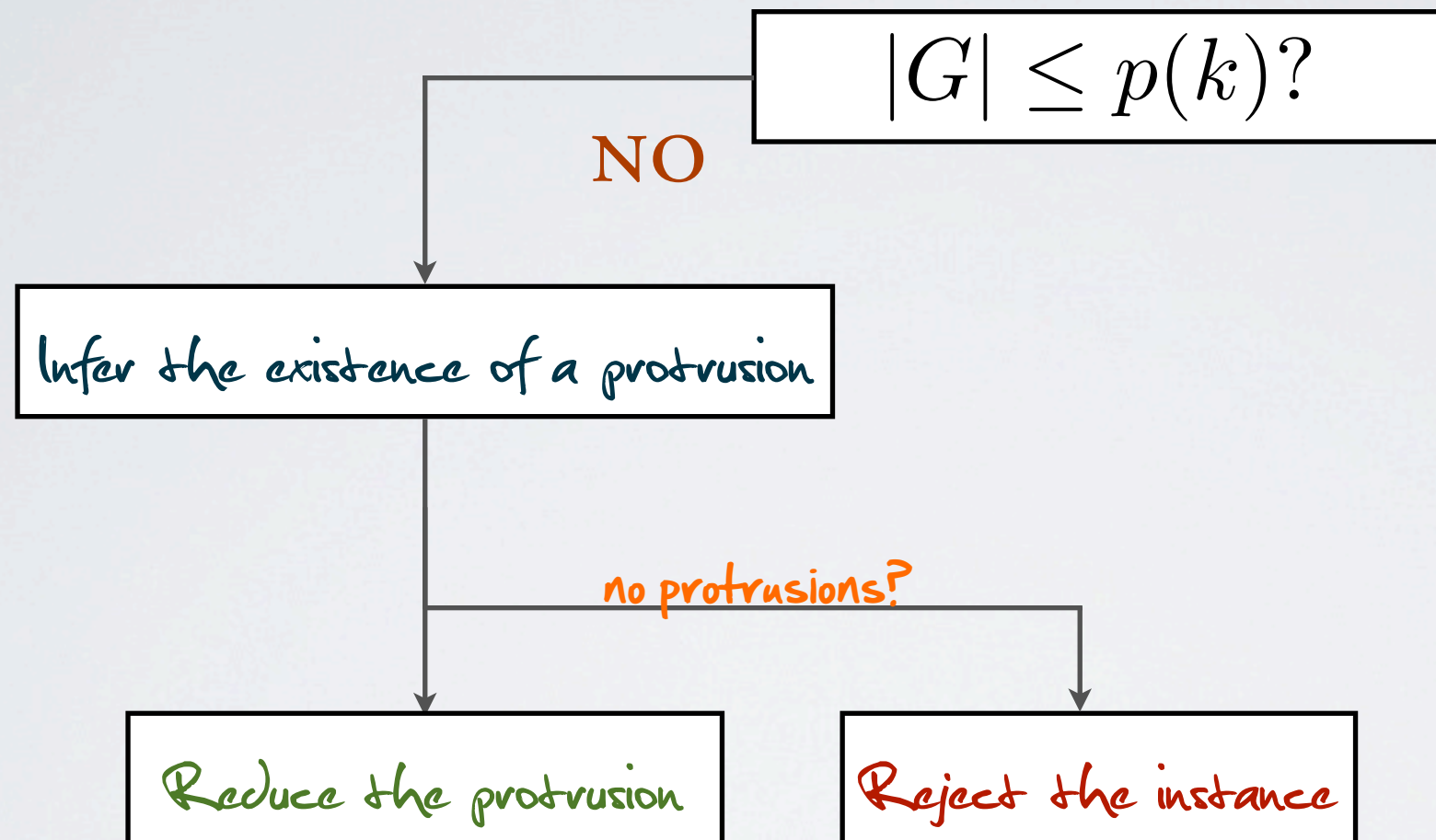
RESTRICTED CASES OF PLANAR F-DELETION



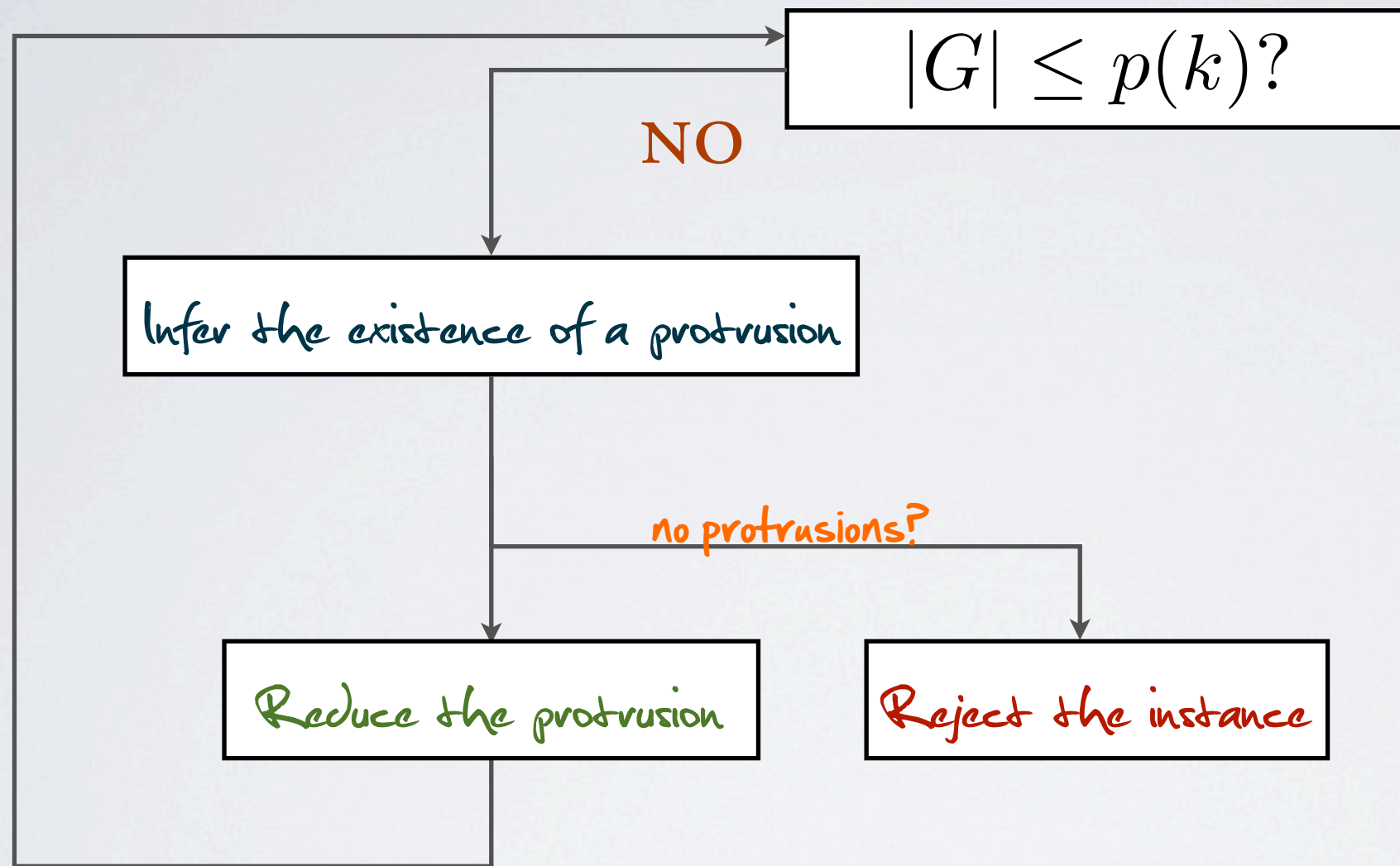
RESTRICTED CASES OF PLANAR F-DELETION



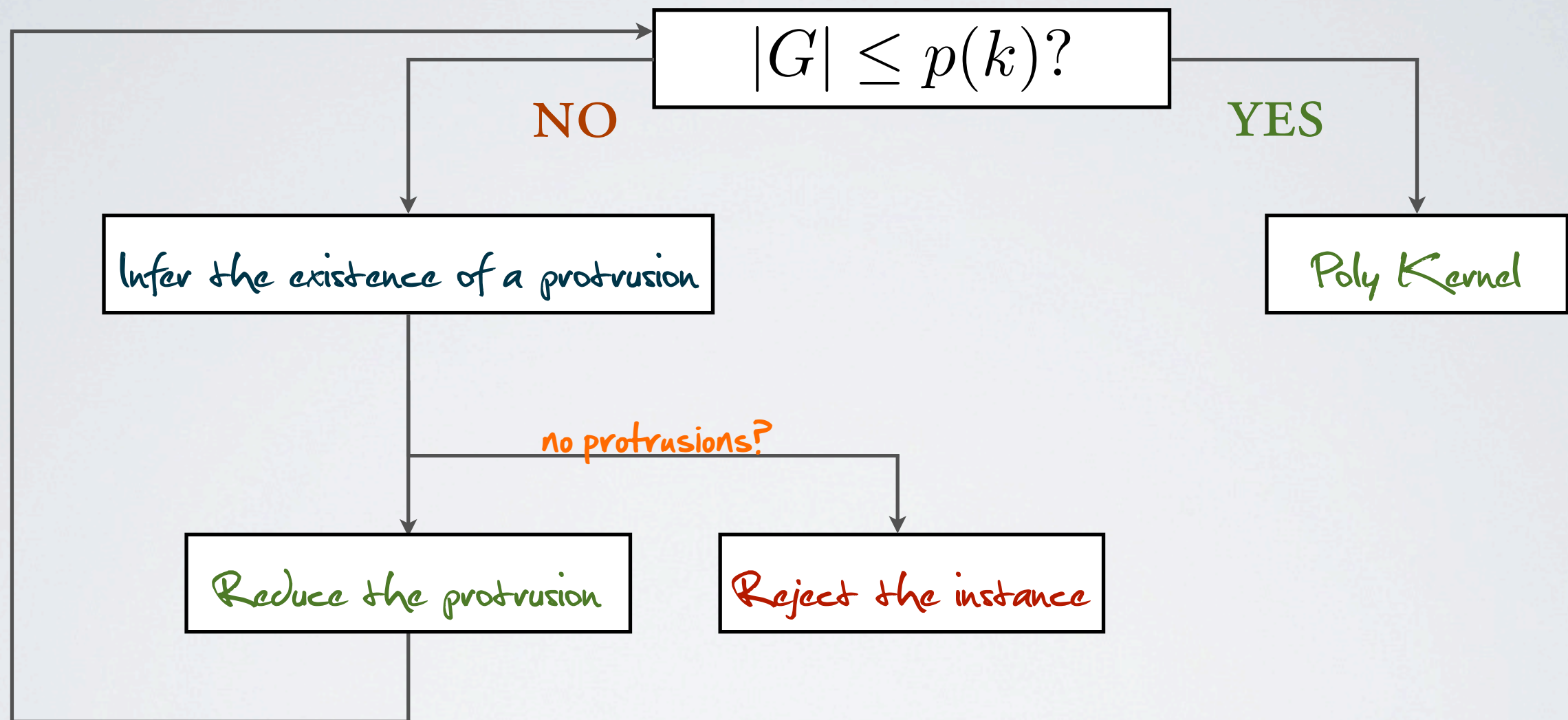
RESTRICTED CASES OF PLANAR F-DELETION



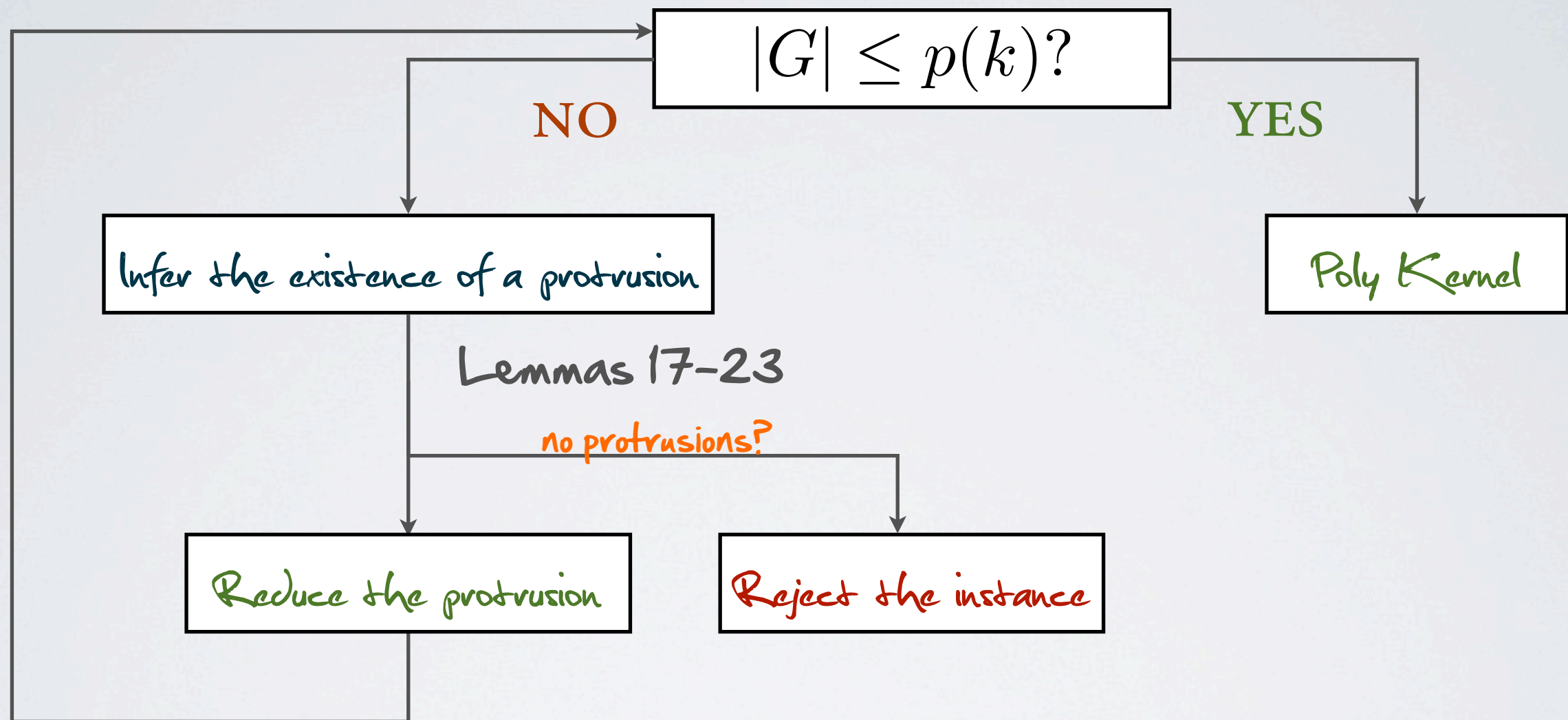
RESTRICTED CASES OF PLANAR F-DELETION



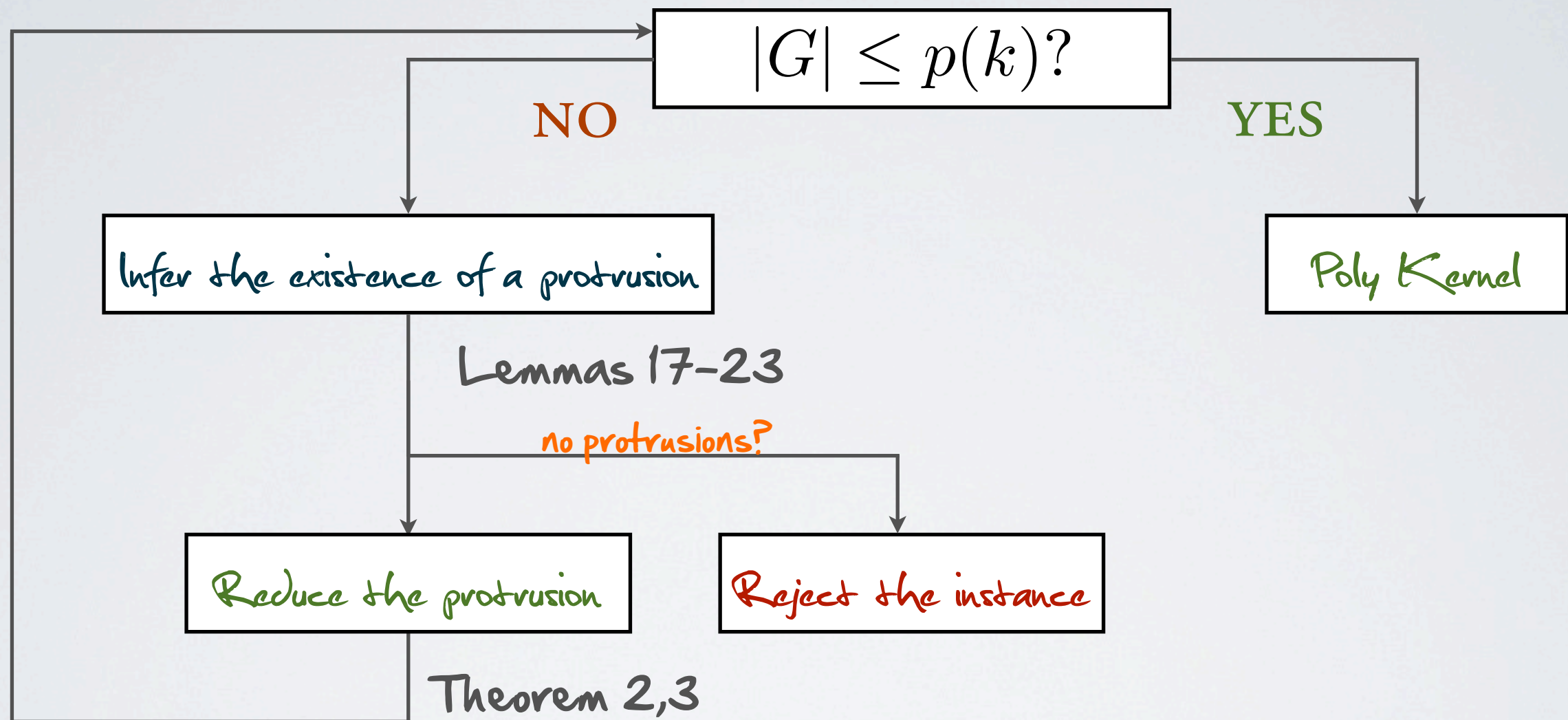
RESTRICTED CASES OF PLANAR F-DELETION



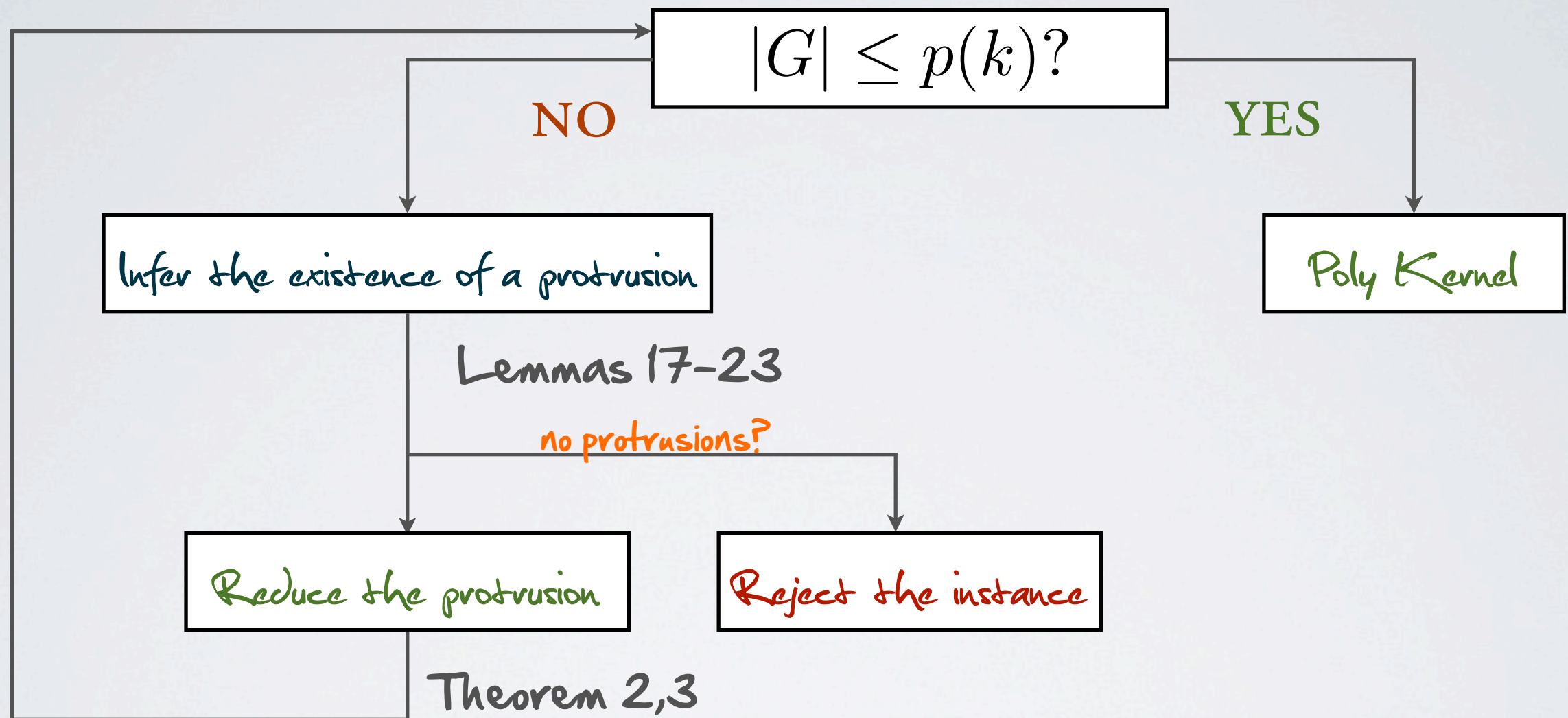
RESTRICTED CASES OF PLANAR F-DELETION



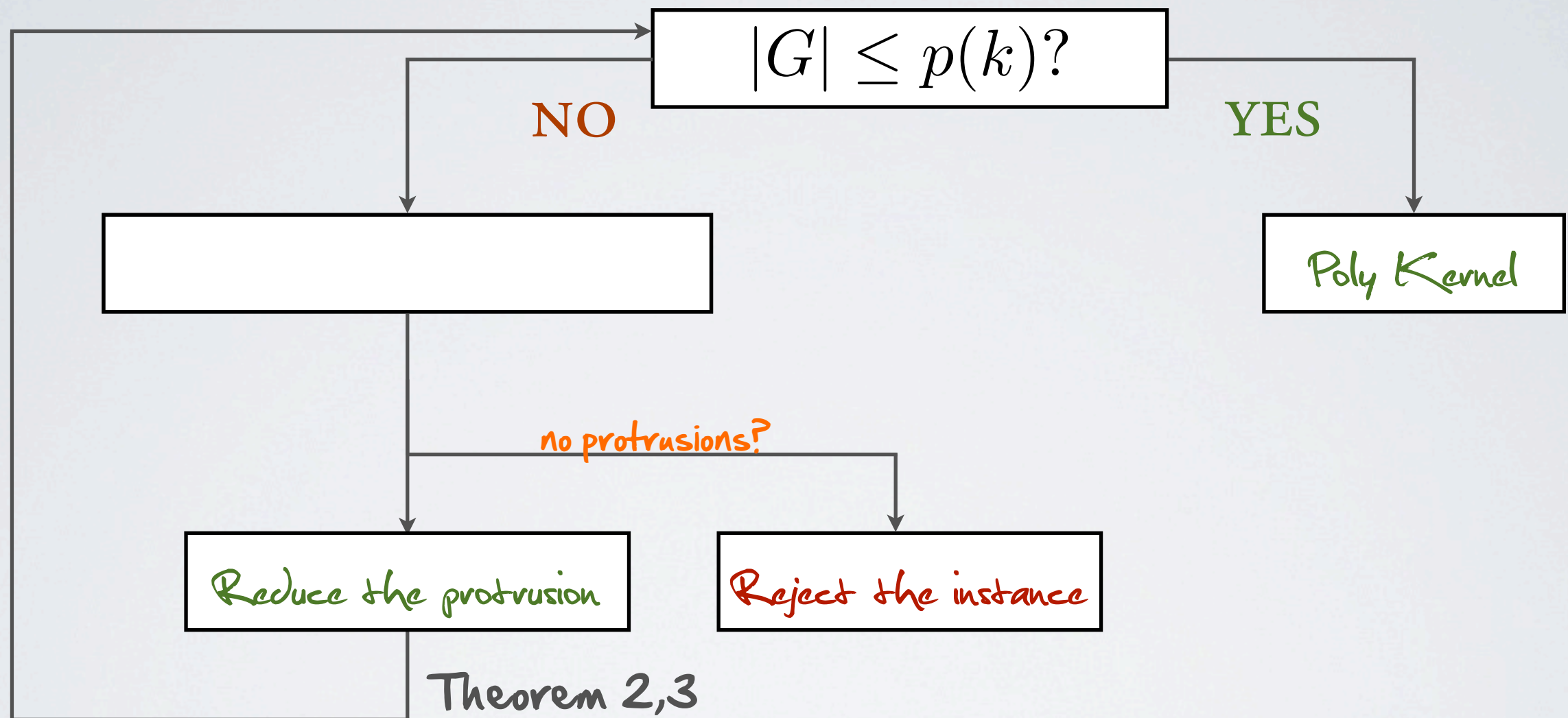
RESTRICTED CASES OF PLANAR F-DELETION



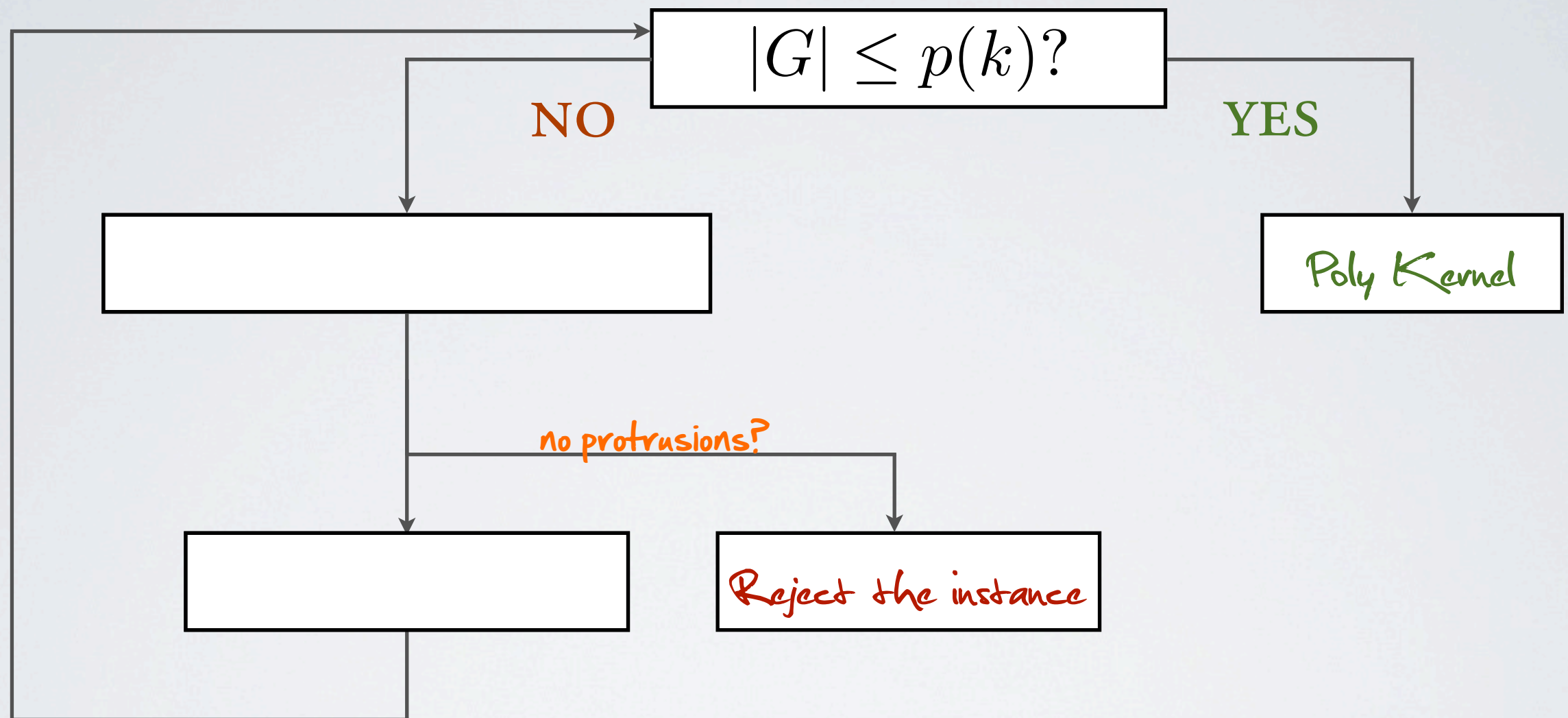
PLANAR F-DELETION



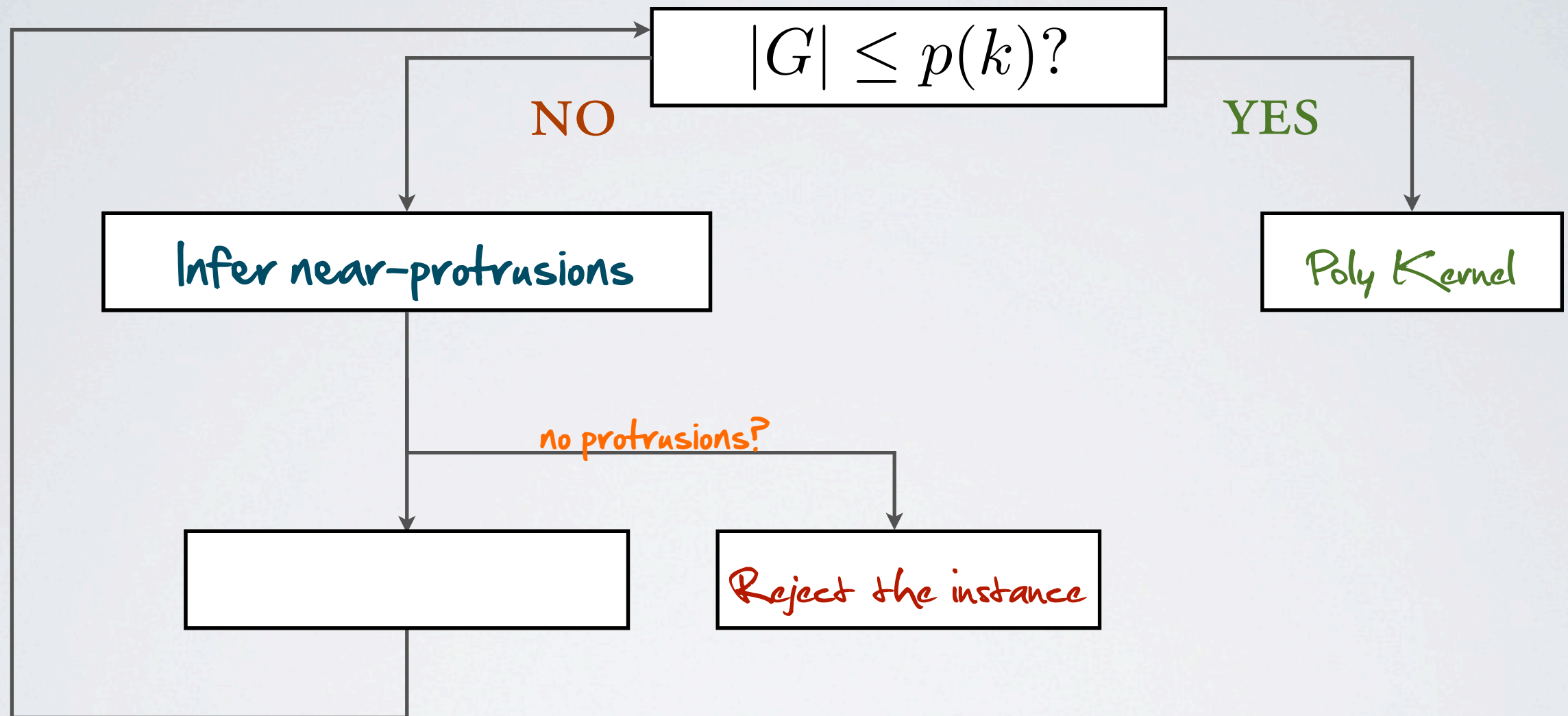
PLANAR F-DELETION



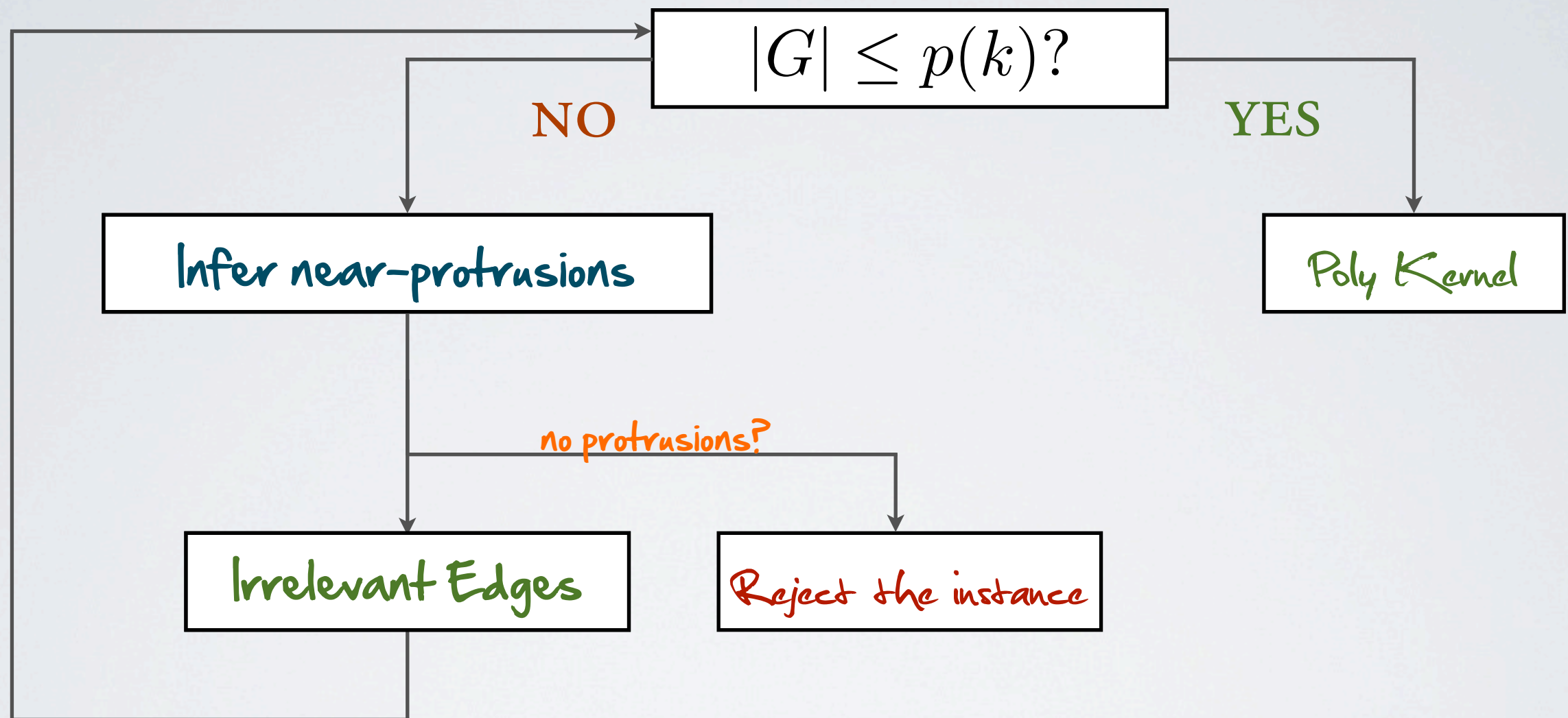
PLANAR F-DELETION



PLANAR F-DELETION



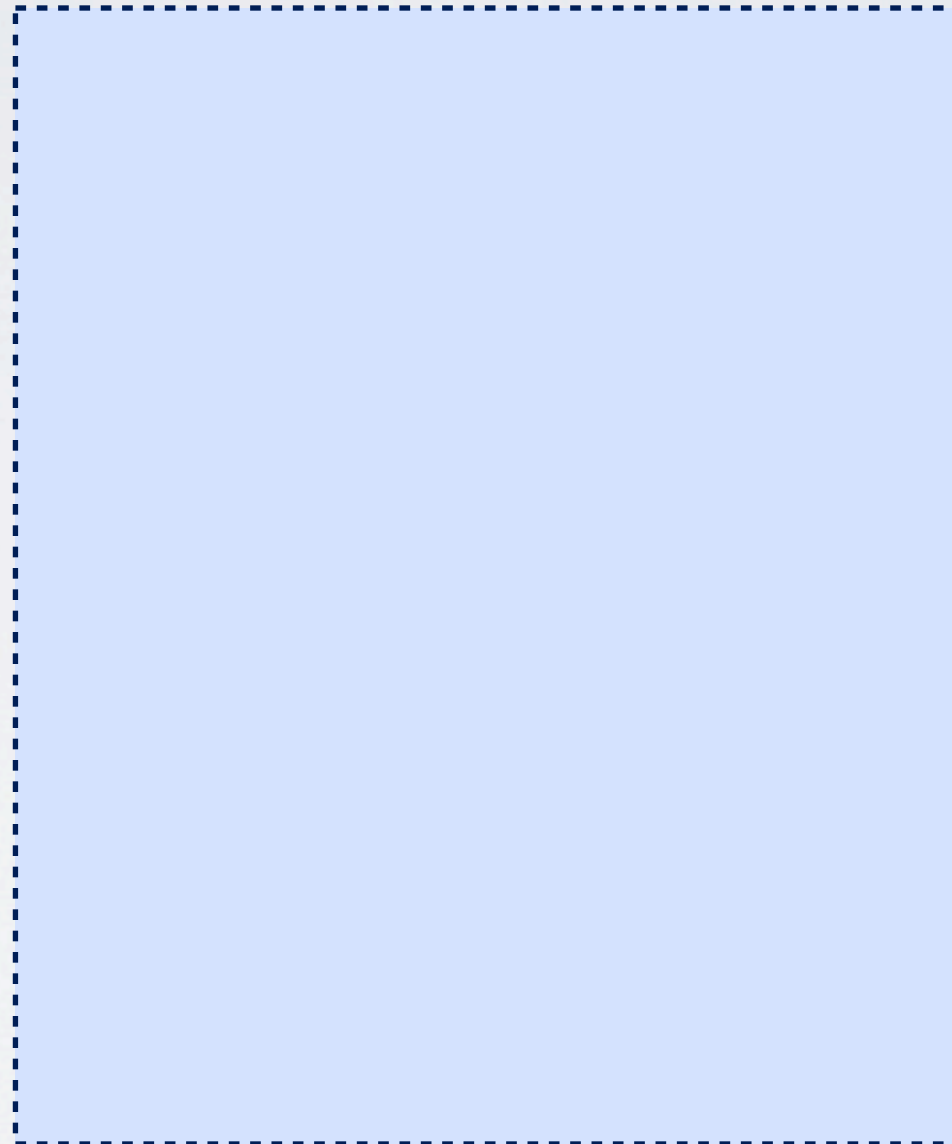
PLANAR F-DELETION



X



$G \setminus X$



X

Approximate F-deletion set

$G \setminus X$

X

Approximate F-deletion set

$G \setminus X$

constant treewidth zone

X

Approximate F-deletion set

$G \setminus X$

constant treewidth zone



X

Approximate F-deletion set



$G \setminus X$



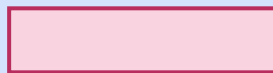
constant treewidth zone

X

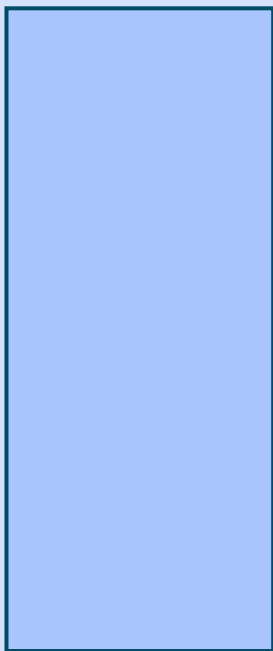
Approximate F-deletion set



$G \setminus X$

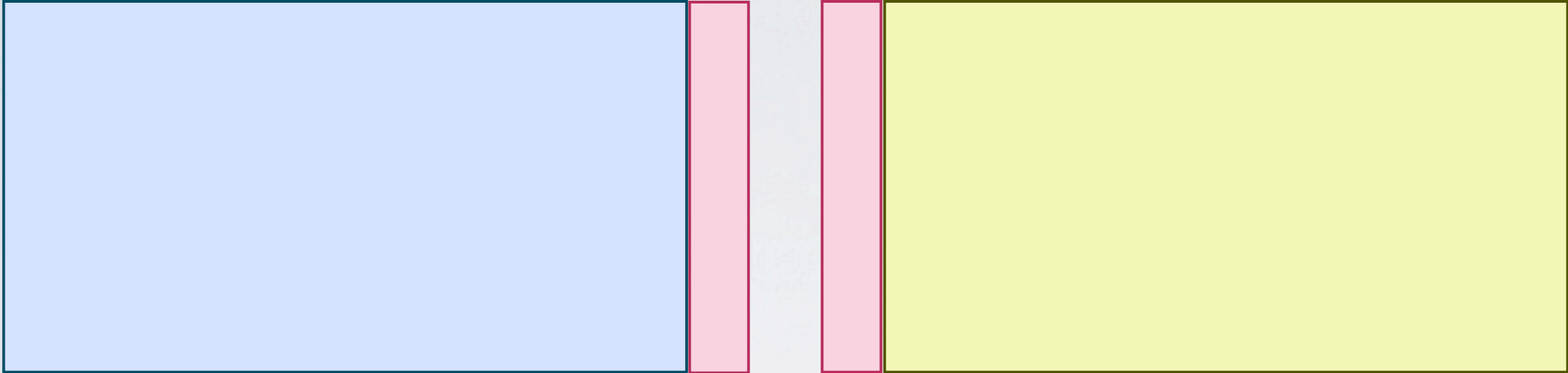


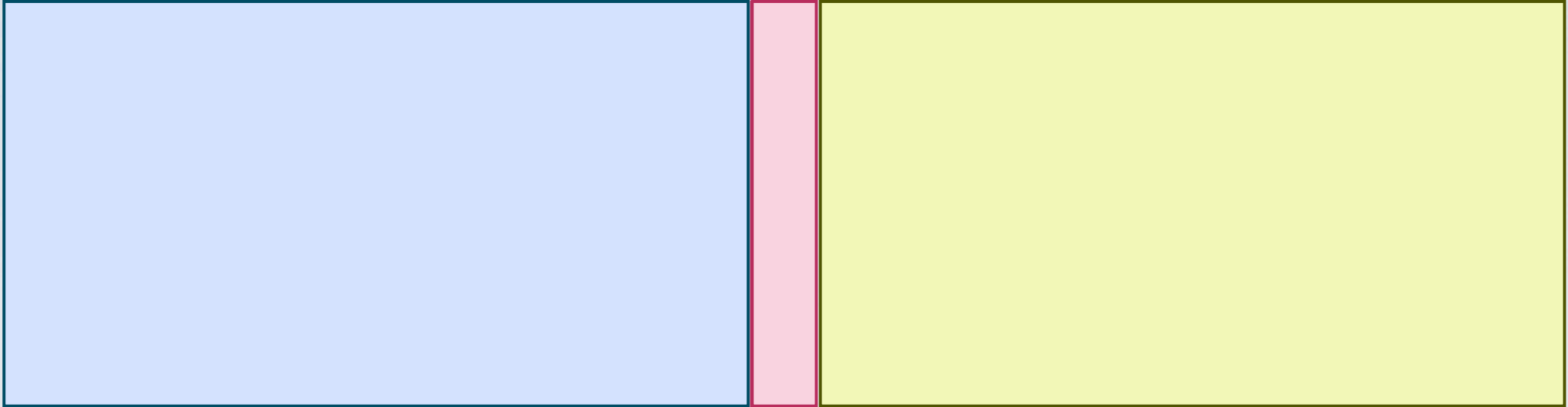
constant boundary...?



constant treewidth zone

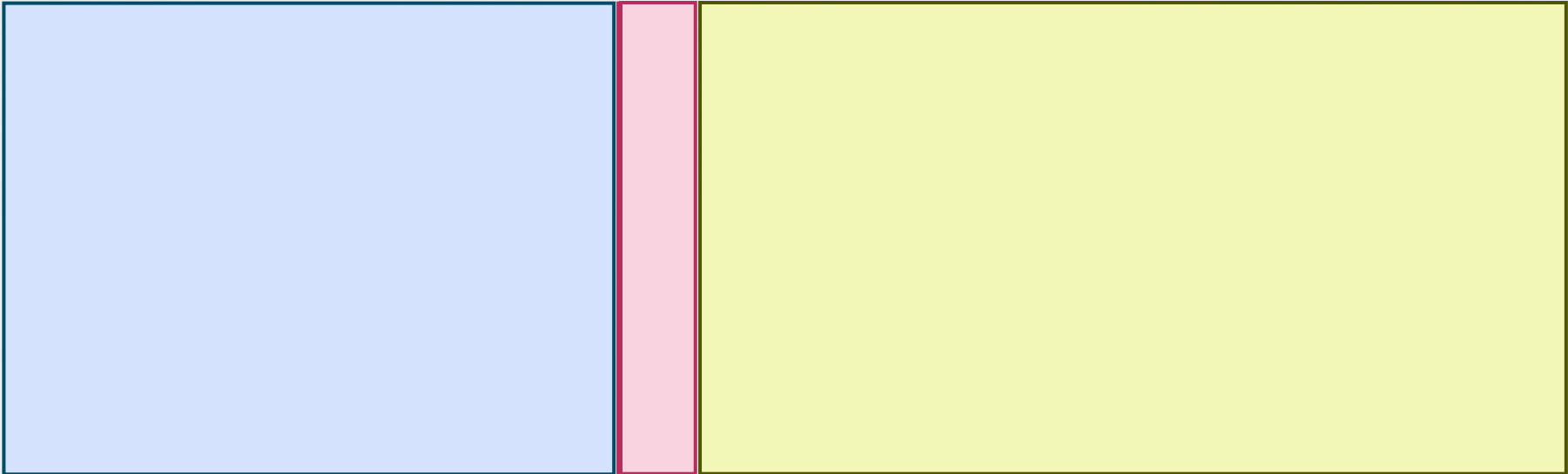
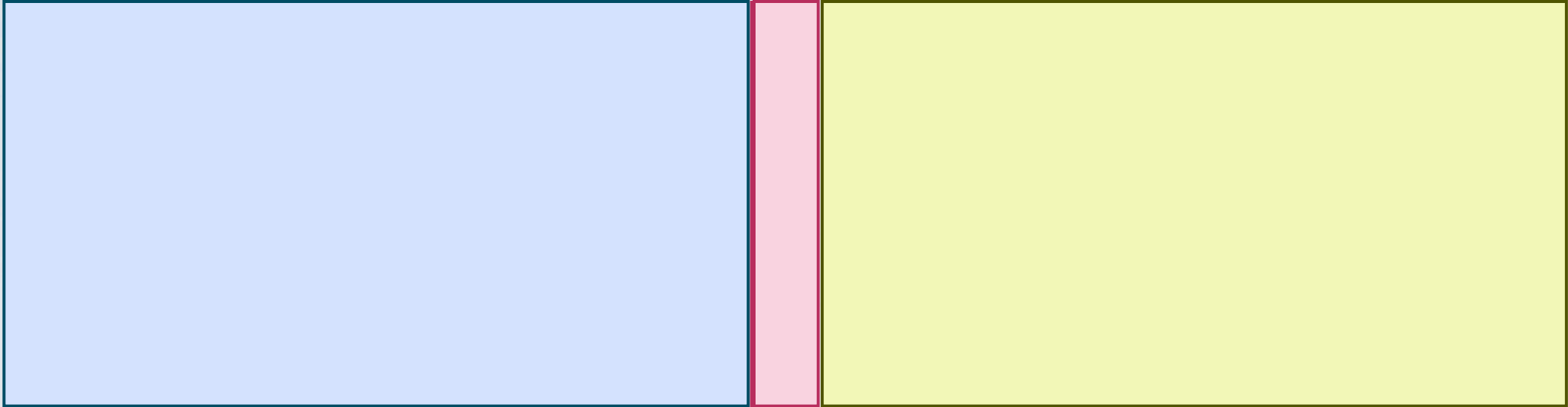


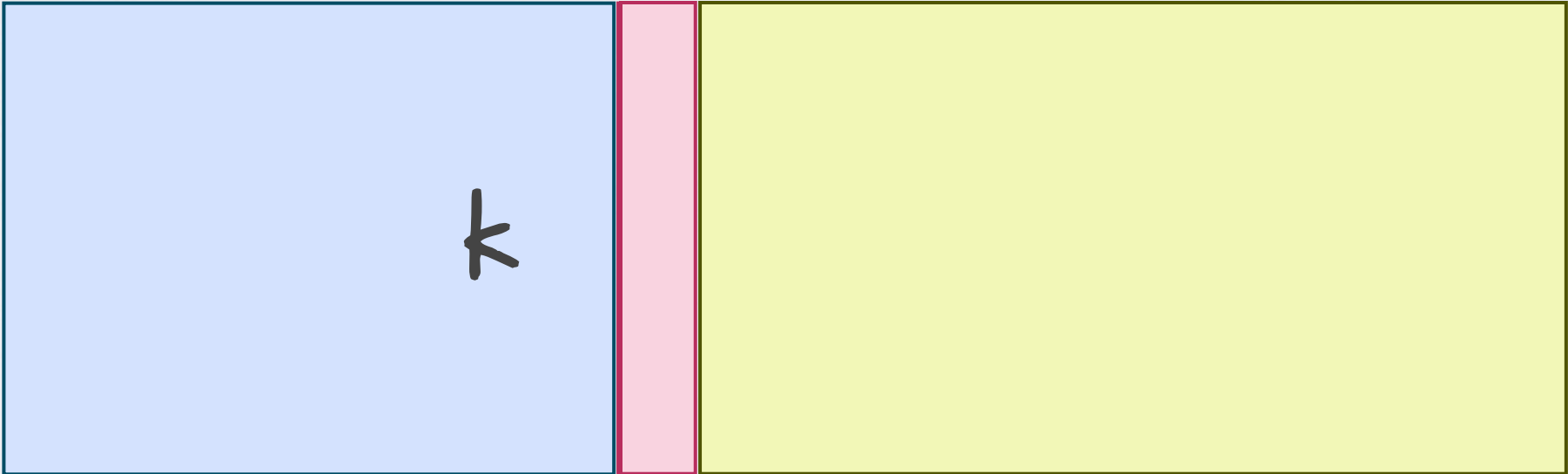
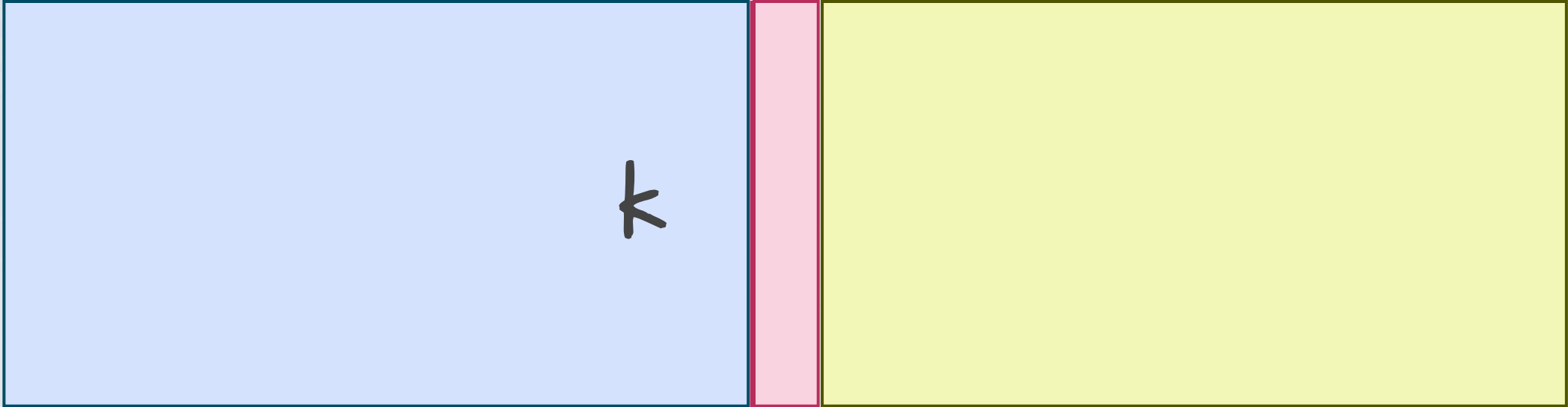


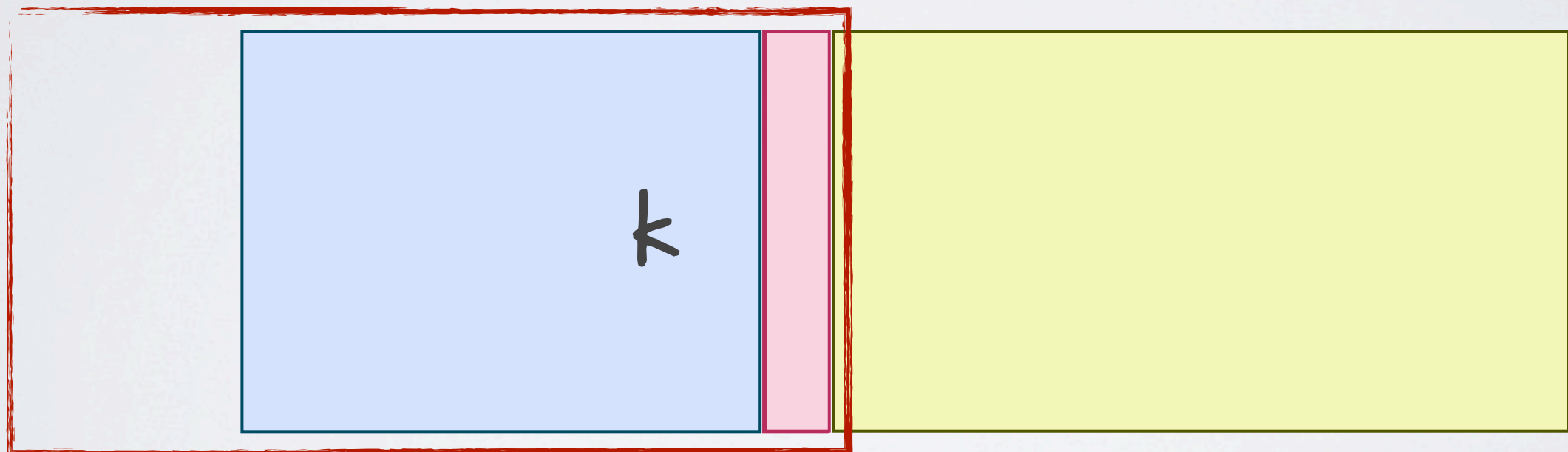
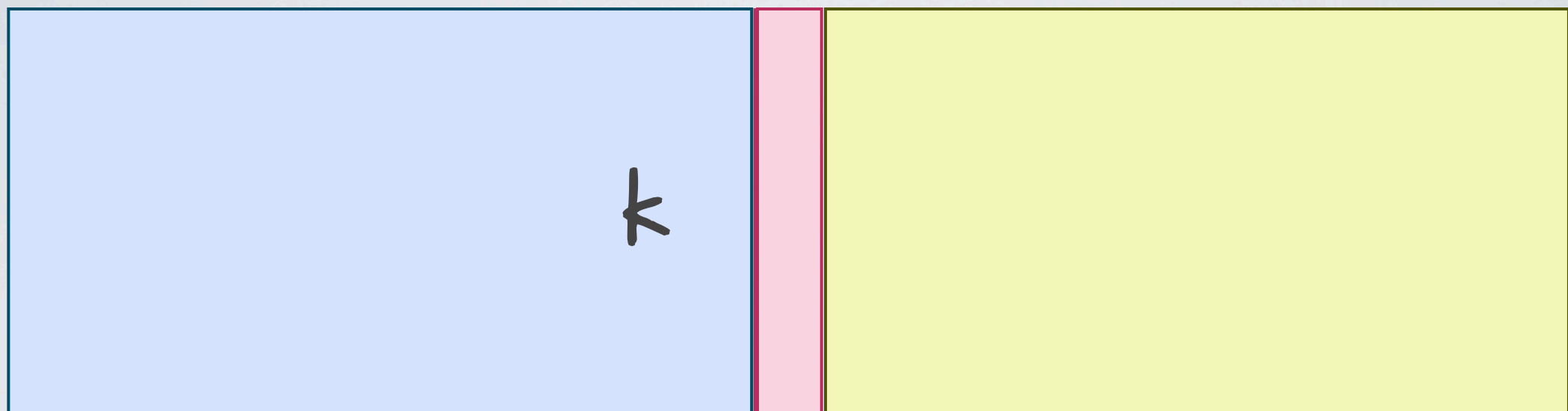


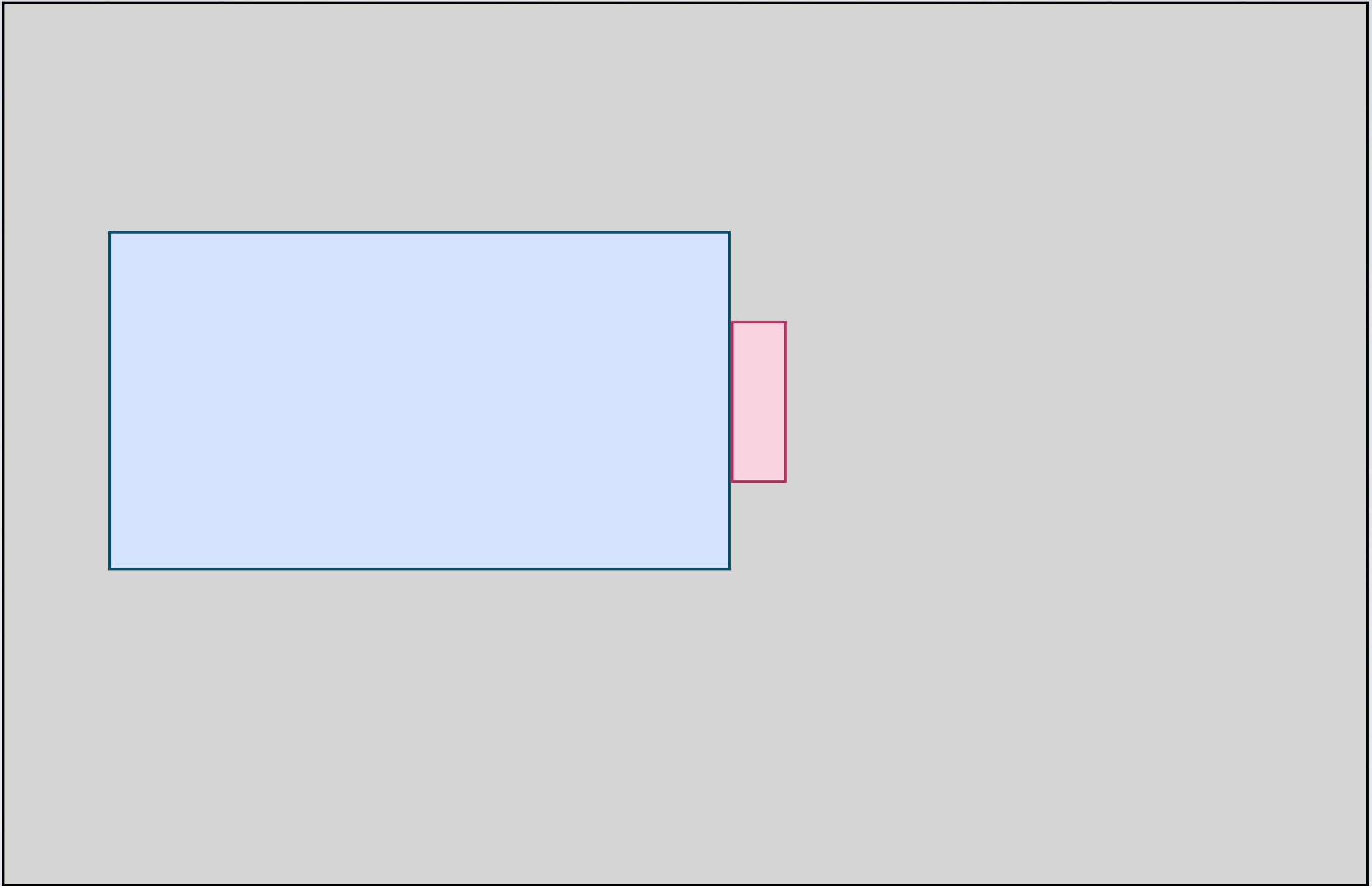


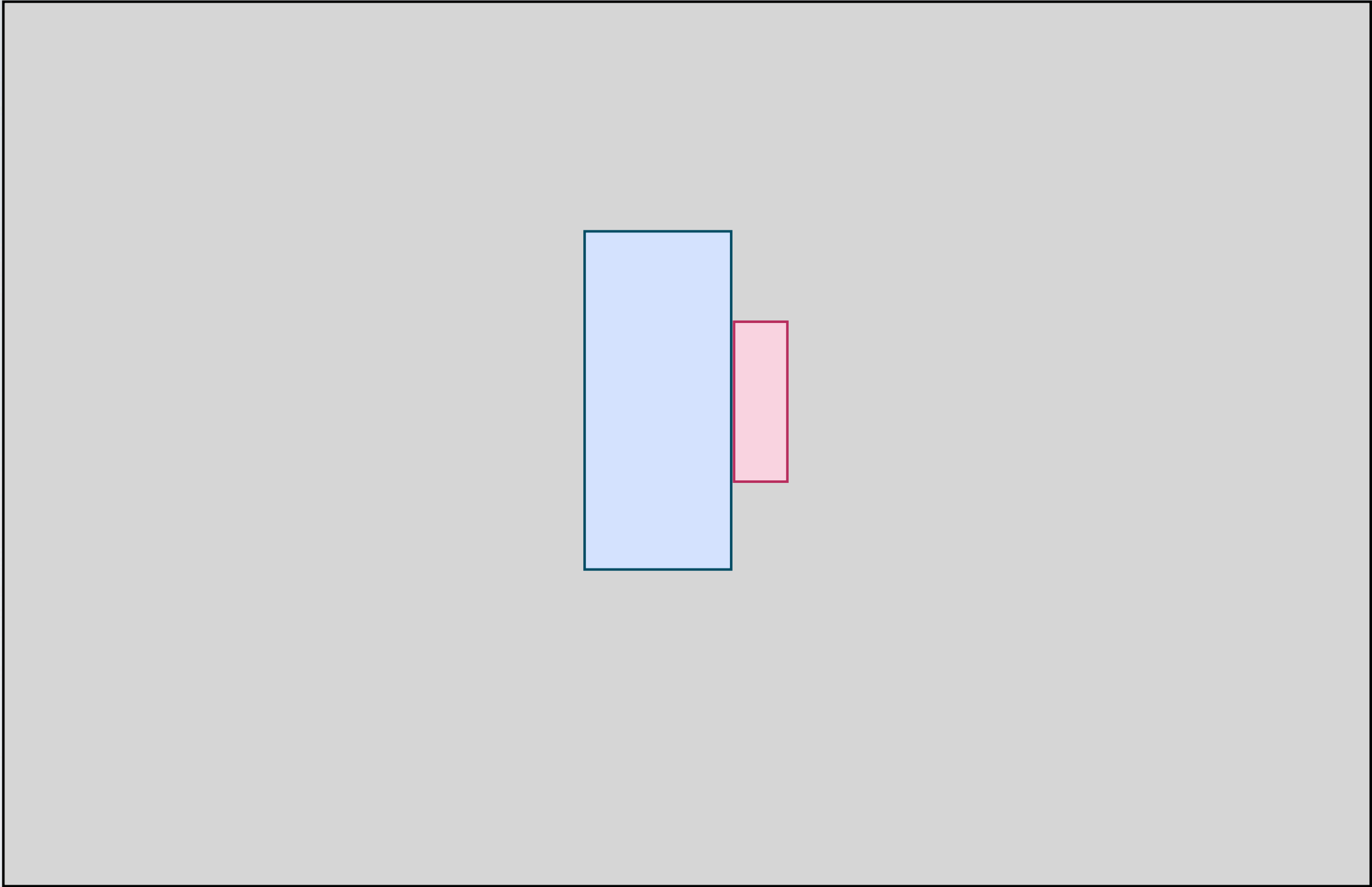




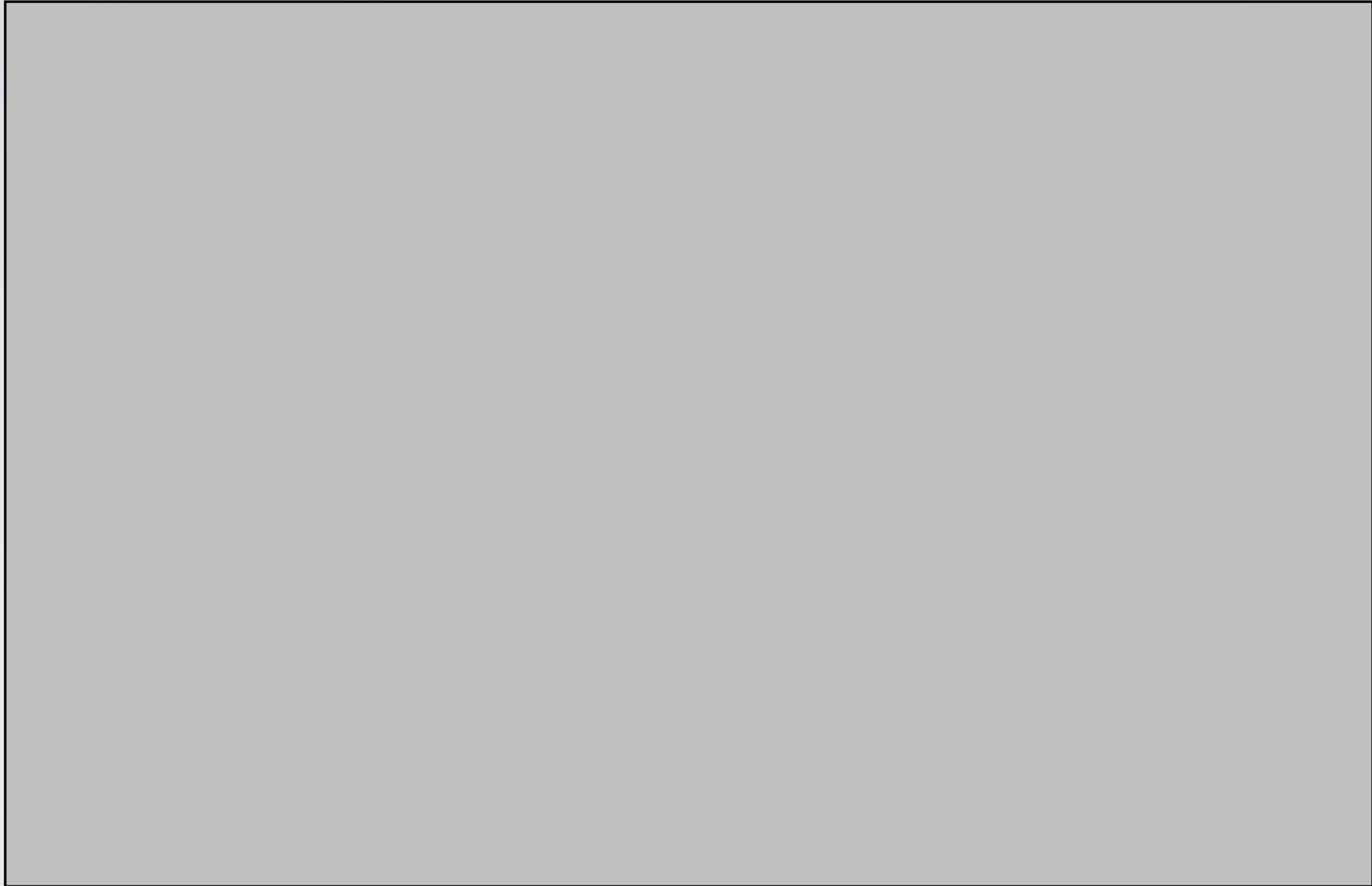




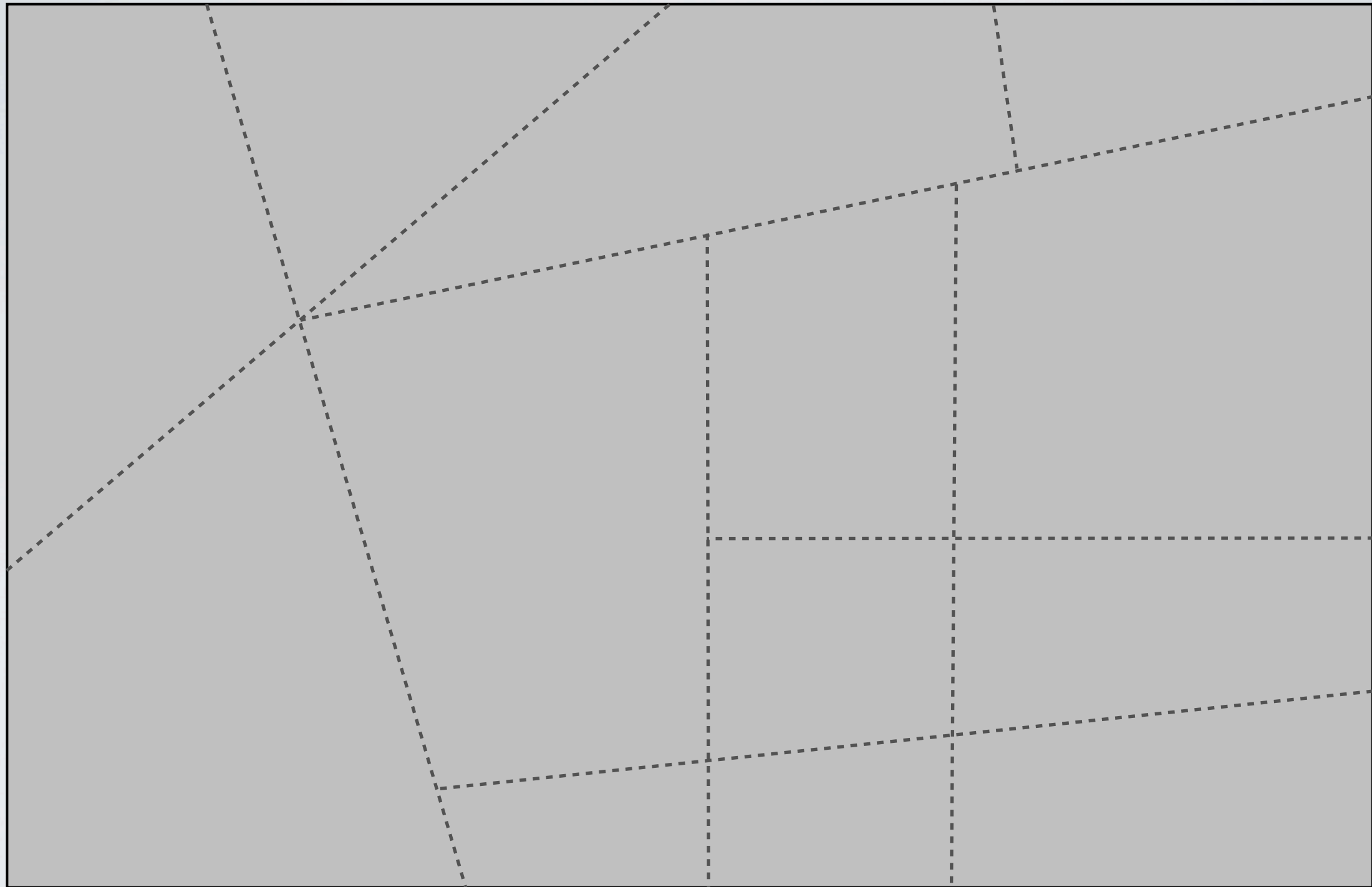




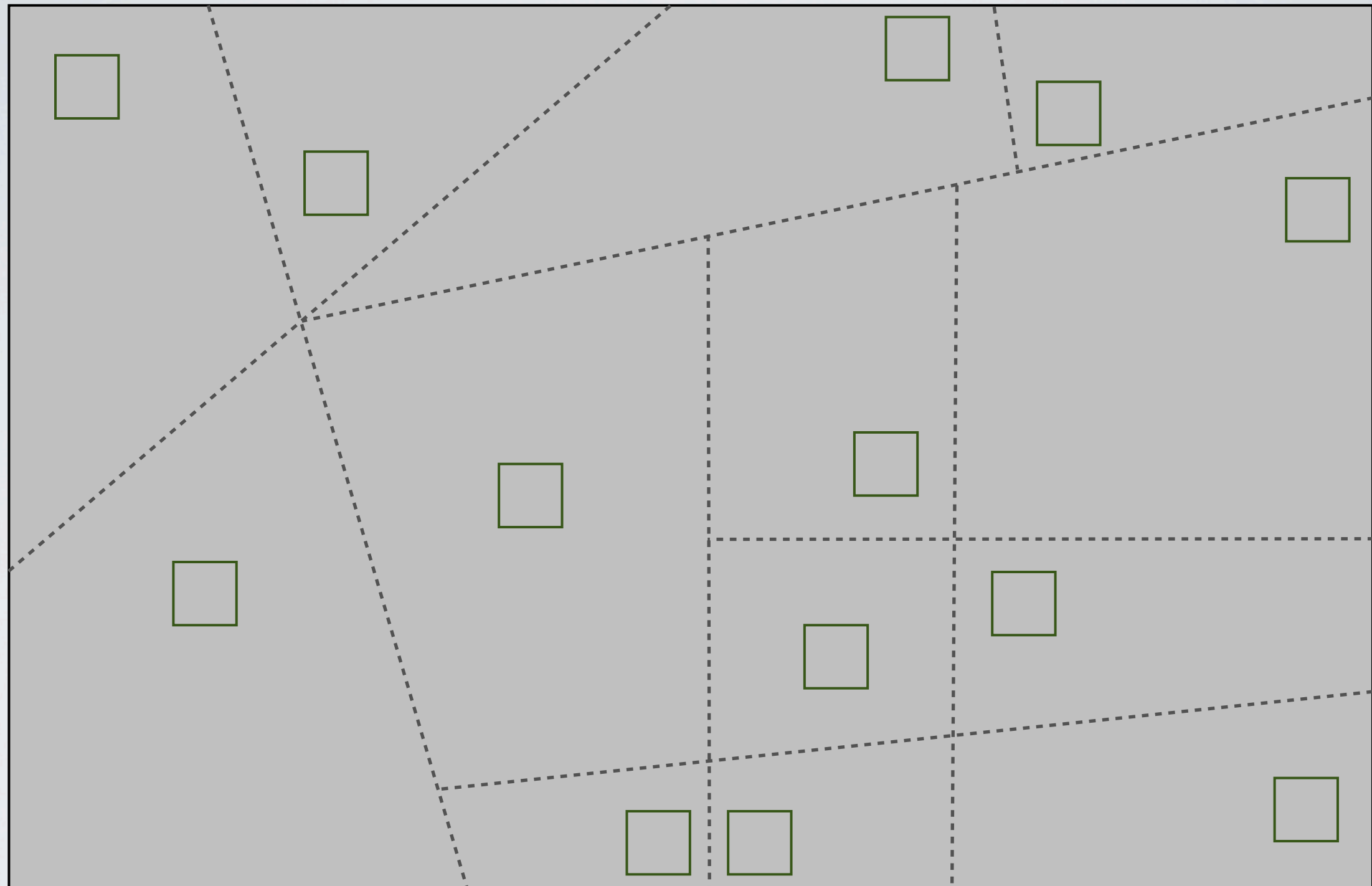
The Space of all t -boundaried graphs



The Space of all t -boundaried graphs



The Space of all t -boundaried graphs



Polynomial Kernels for Planar F -deletion

Polynomial Kernels for Planar F -deletion

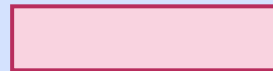
CONTINUED...

X

Approximate F-deletion set



$G \setminus X$



constant boundary...?



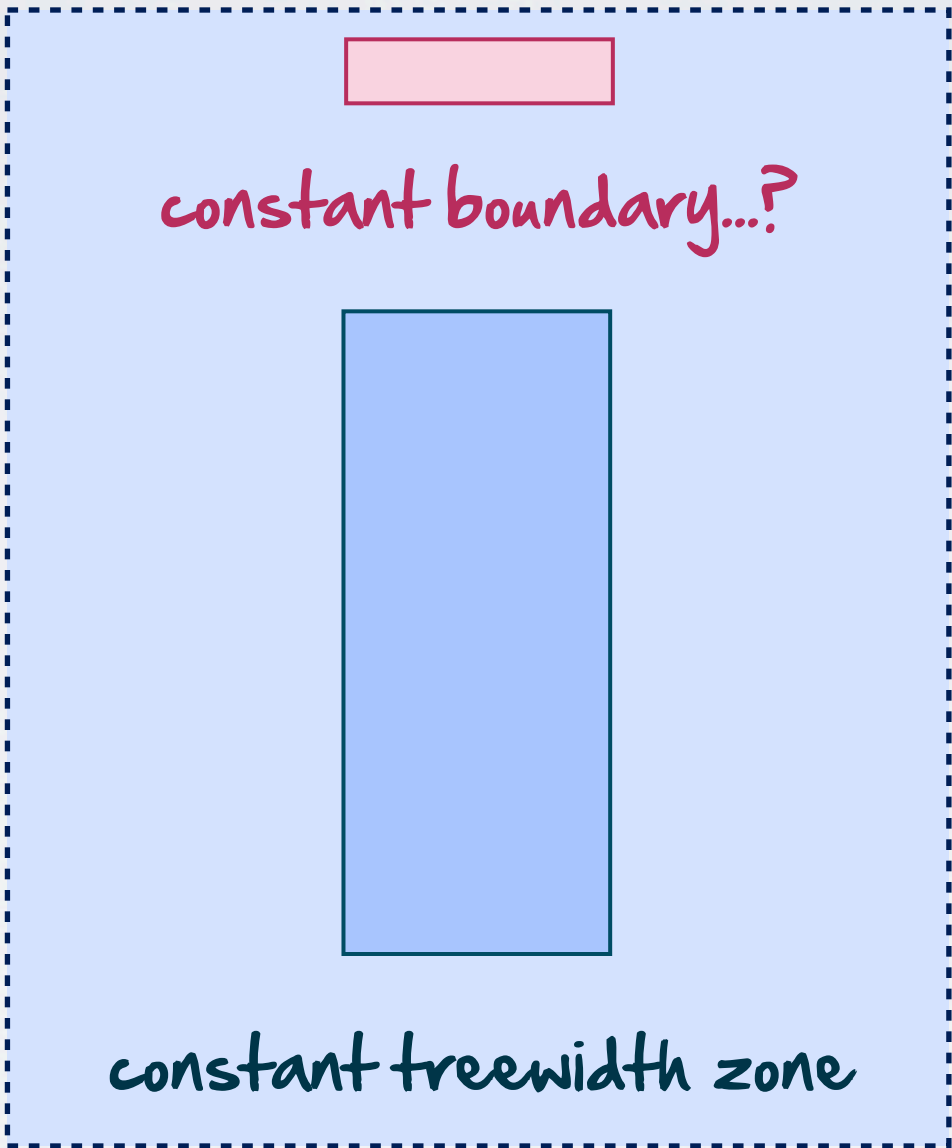
constant treewidth zone



X



$G \setminus X$

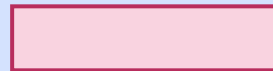


X

Approximate F-deletion set



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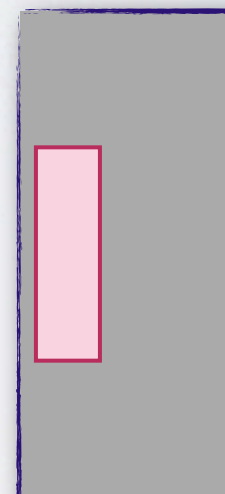


$G \setminus X$

constant boundary...?



constant treewidth zone



For every guess we have a protrusion



But it may not be safe to reduce them!



STRATEGY OUTLINE

SAFETY | **SIZE** | **TIME**

Suppose the protrusion gives us a way of
finding irrelevant edges.

SAFETY | SIZE | TIME

If every guess declares an edge to be irrelevant,
then it is safe to remove it from G .

SAFETY | SIZE | TIME

If every guess declares an edge to be **irrelevant**,
then it is safe to remove it from G .

SAFETY | **SIZE** | **TIME**

If there are **more than $\text{poly}(k)$ edges** incident on a single vertex, then one of them is not relevant to any guess.

Deleting an edge never increases OPT.

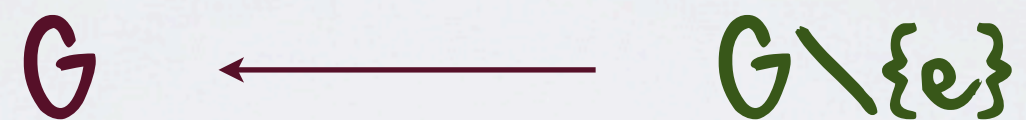
Deleting an edge never increases OPT.

G

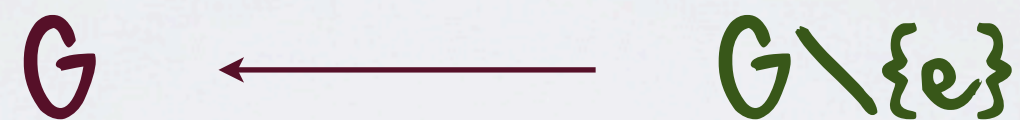
Deleting an edge never increases OPT.

$$G \longrightarrow G \setminus \{e\}$$

Deleting an edge never increases OPT.



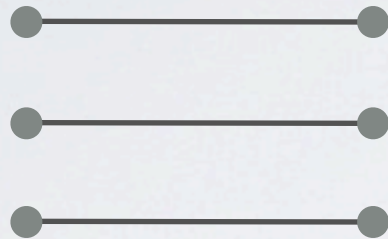
Deleting an edge never increases OPT.



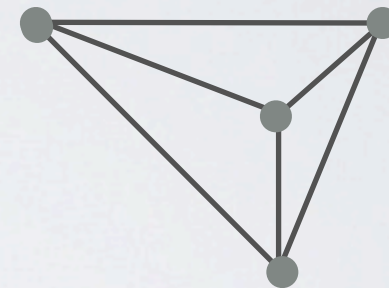
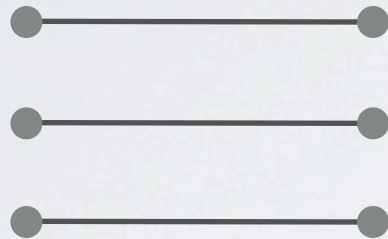
Determine **why** G is a no-instance,
and don't interfere with it.

G does not have a Vertex Cover of size 2

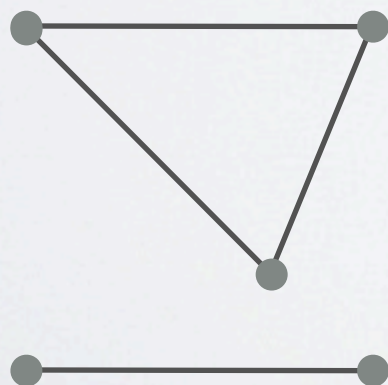
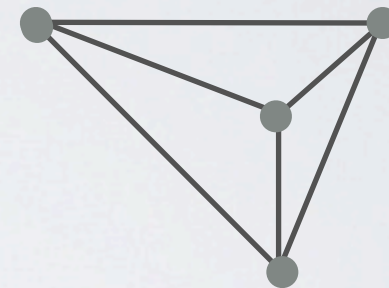
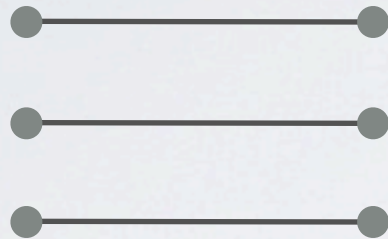
G does not have a Vertex Cover of size 2



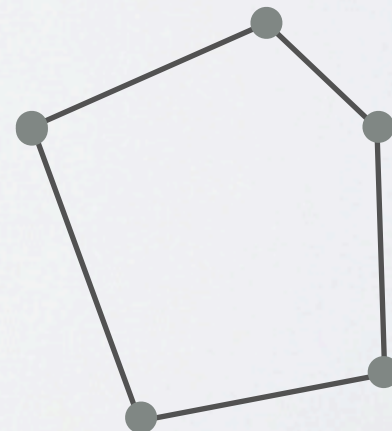
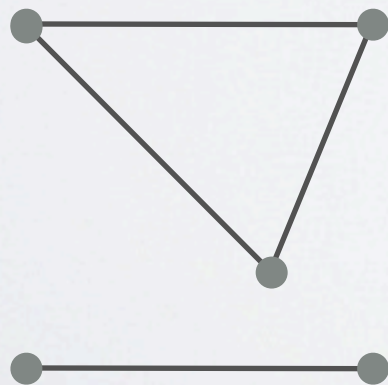
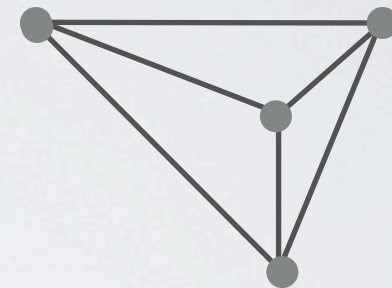
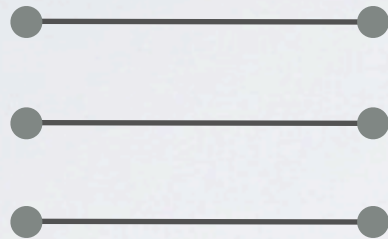
G does not have a Vertex Cover of size 2



G does not have a Vertex Cover of size 2



G does not have a Vertex Cover of size 2



G does not have a Vertex Cover of size 2

G does not have a F -deletion Set of size 2

G does not have a F -deletion Set of size k

G does not have a F -deletion Set of size k

G does not belong to $[F\text{Del}]_k$

G does not have a F -deletion Set of size k

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But $[F\text{Del}]_k$ is closed under minors, and hence has a finite obstruction set S .

G does not have a F -deletion Set of size k

G does not belong to $[F\text{Del}]_k$

But $[F\text{Del}]_k$ is closed under minors, and hence has a finite obstruction set S .

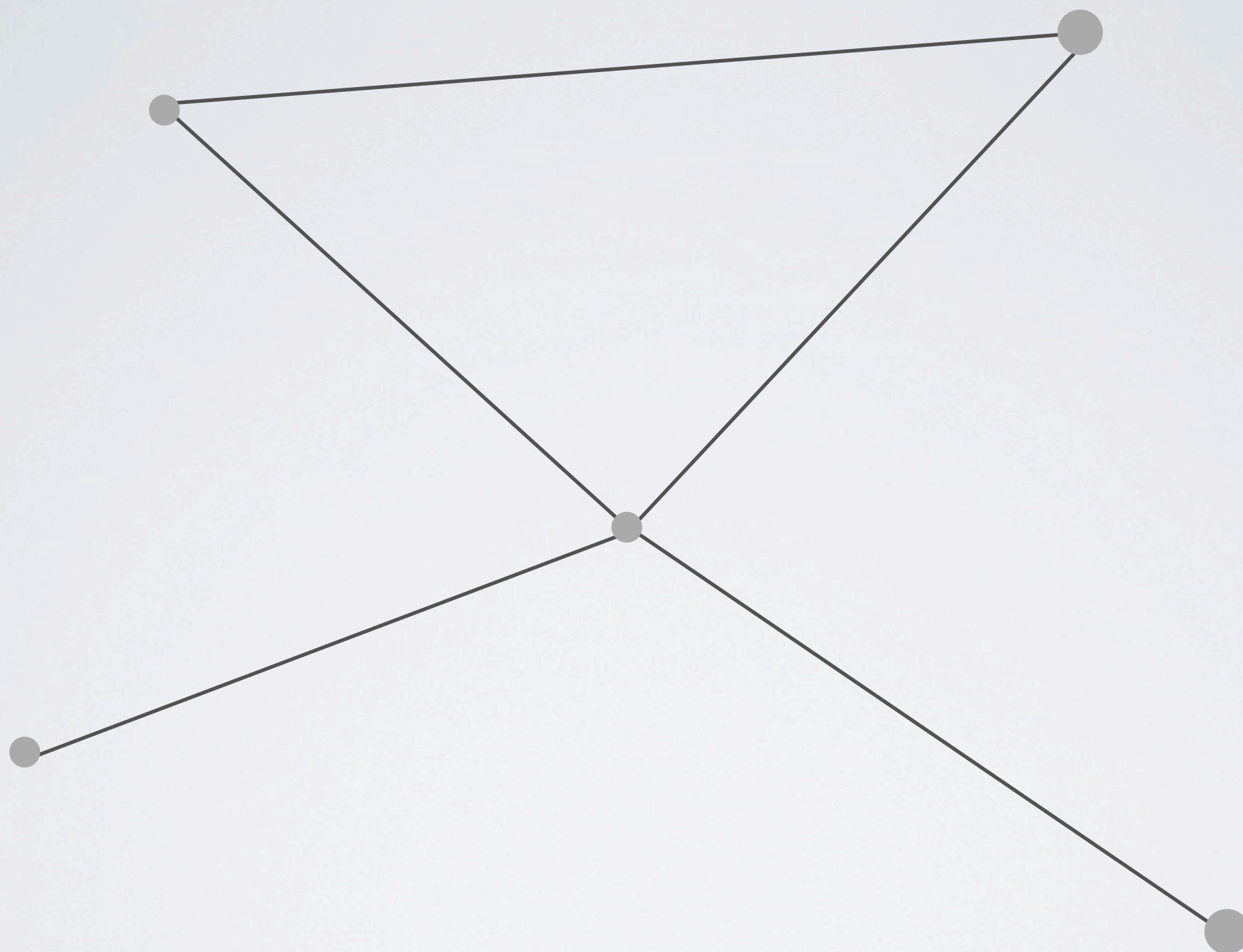
S "witnesses" the fact that G is a NO instance.

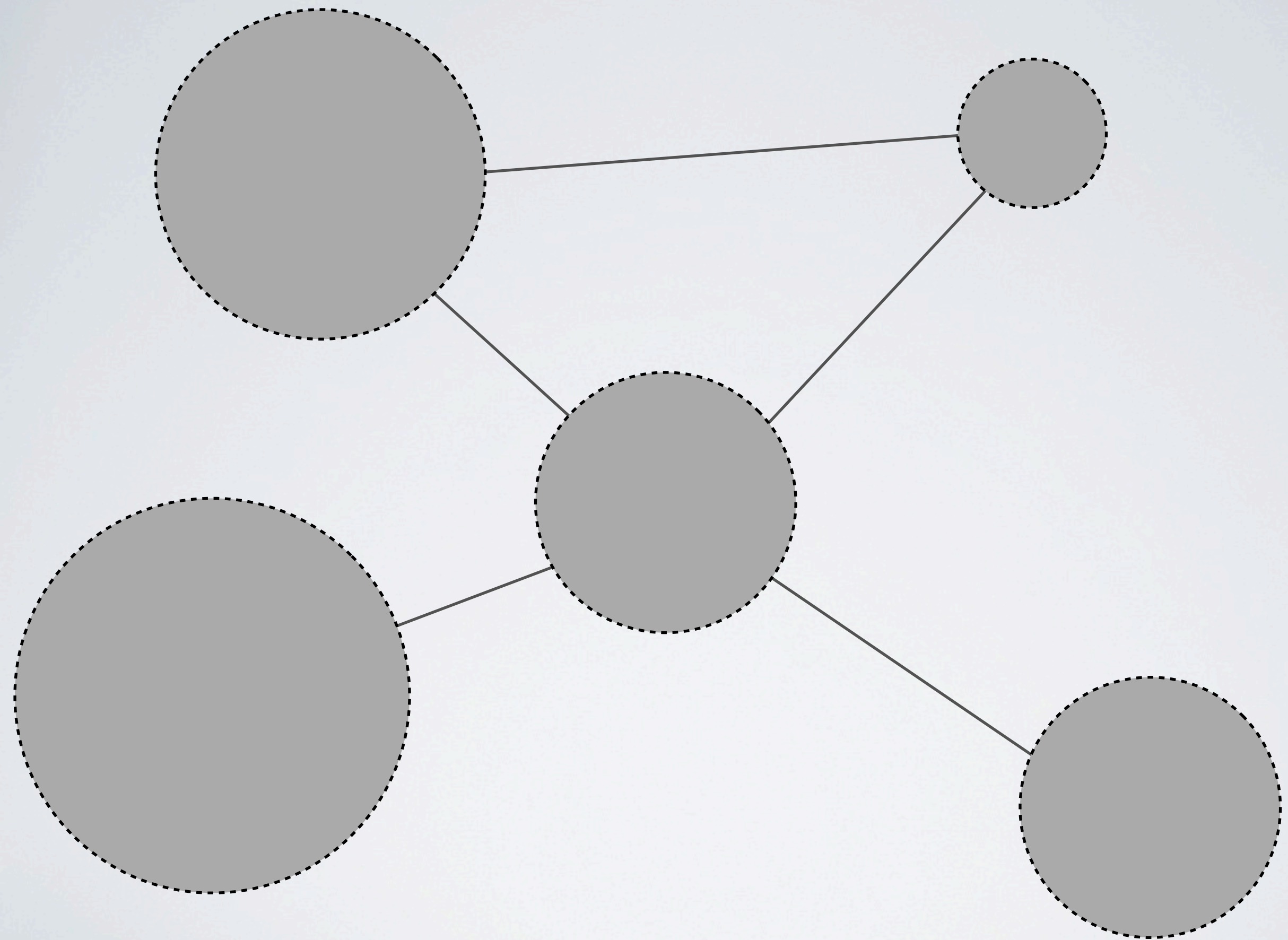
Edges not involved in copies of S are... irrelevant!

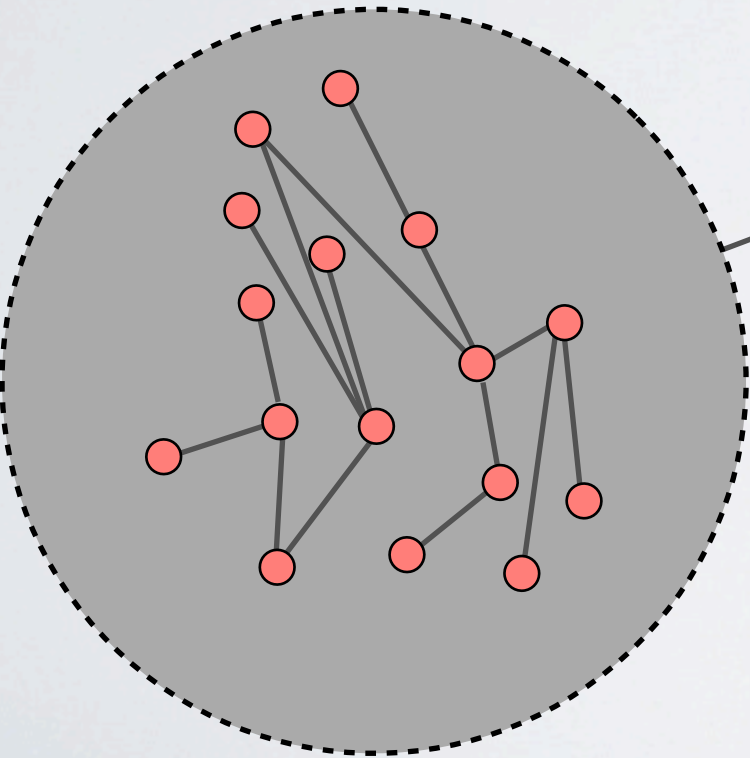
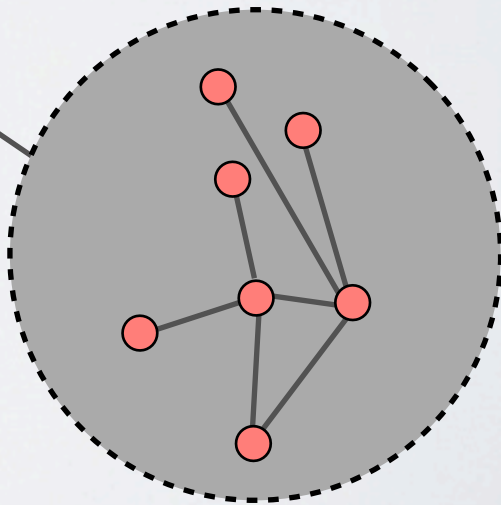
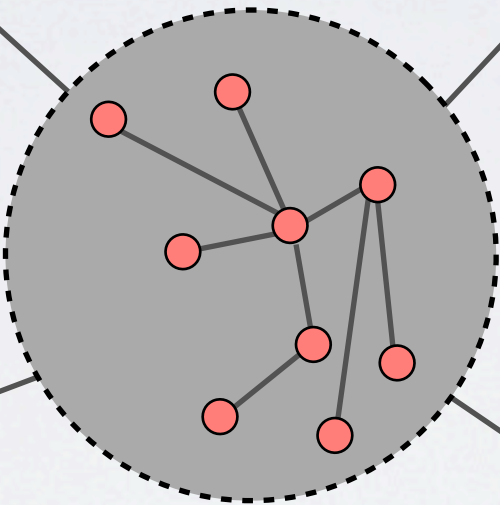
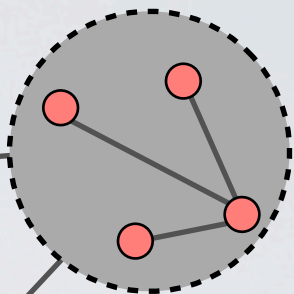
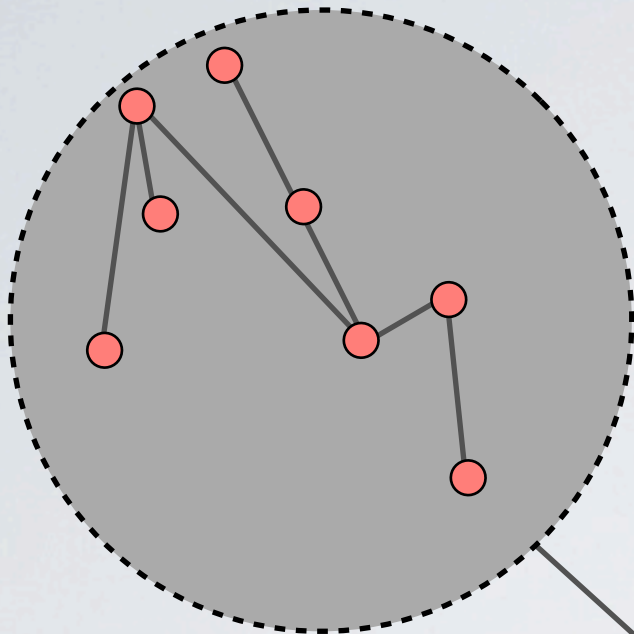
We don't know the obstruction sets.

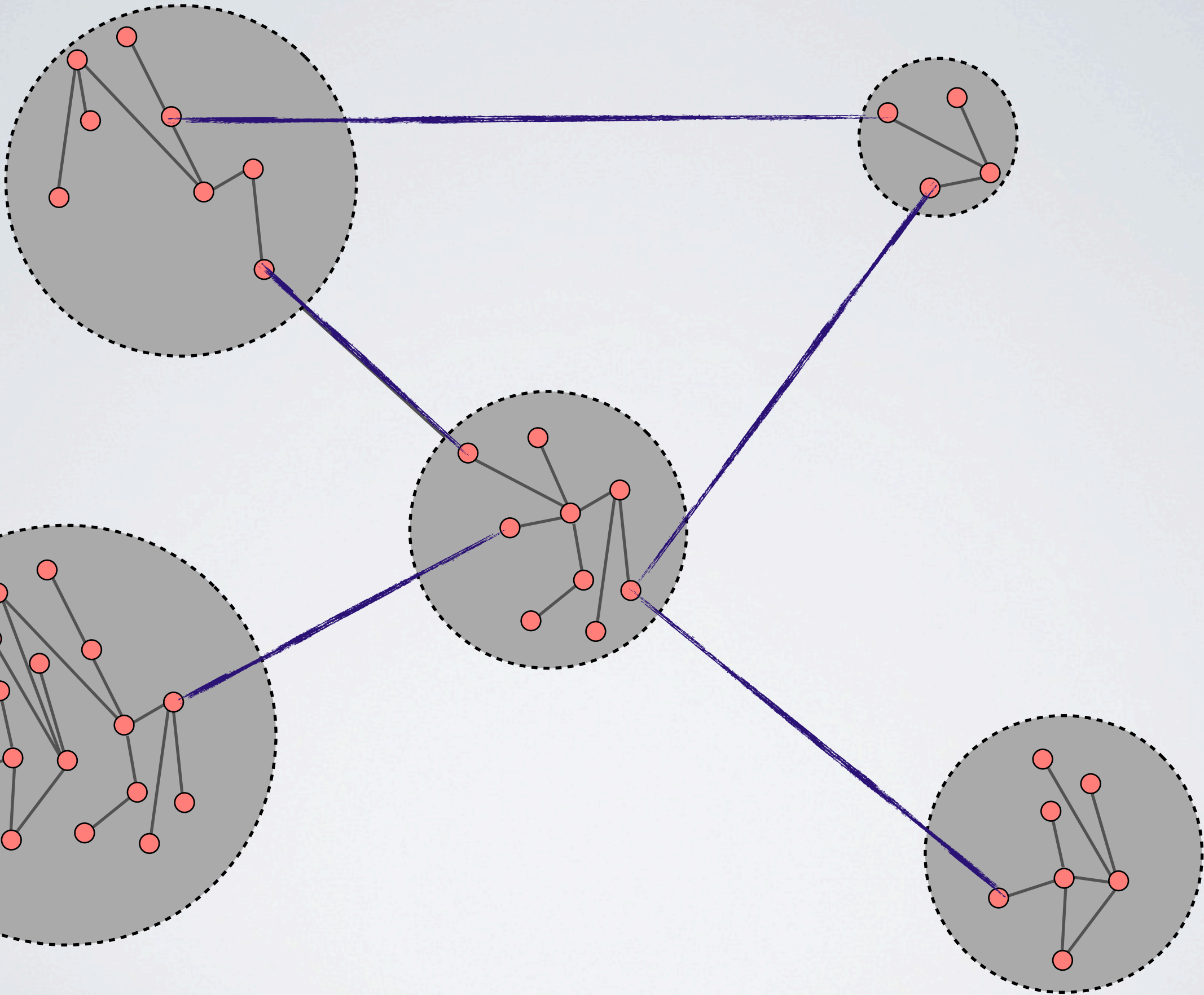
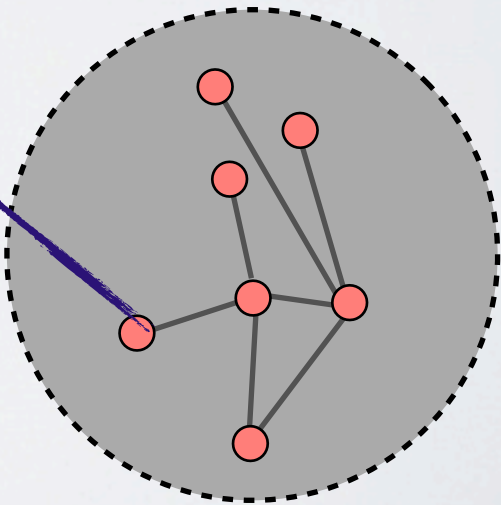
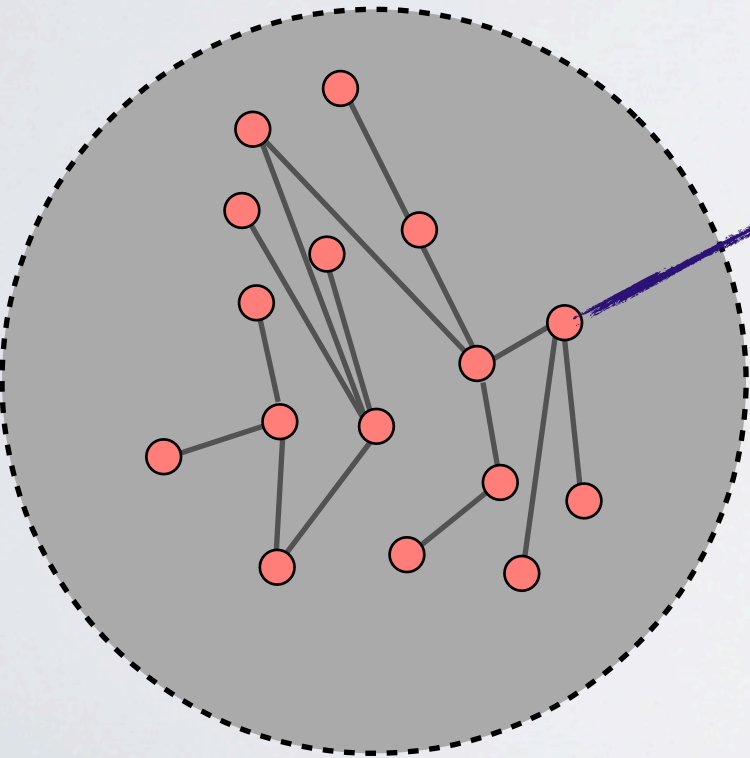
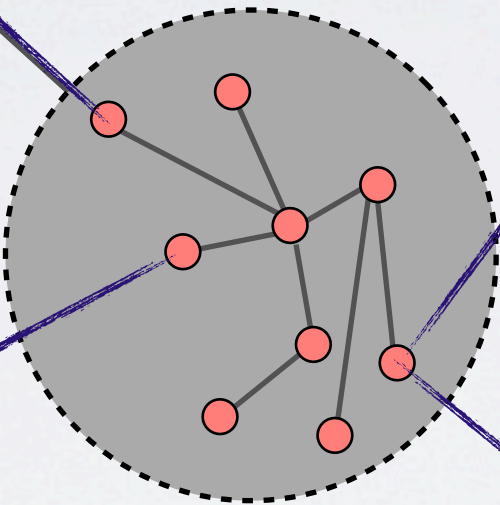
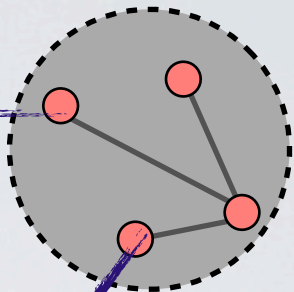
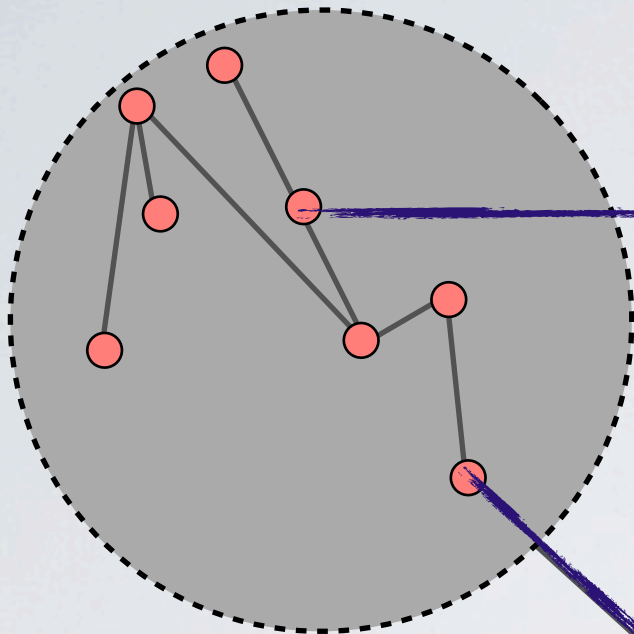
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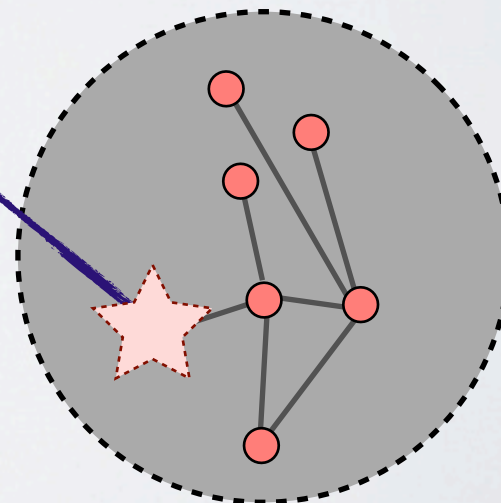
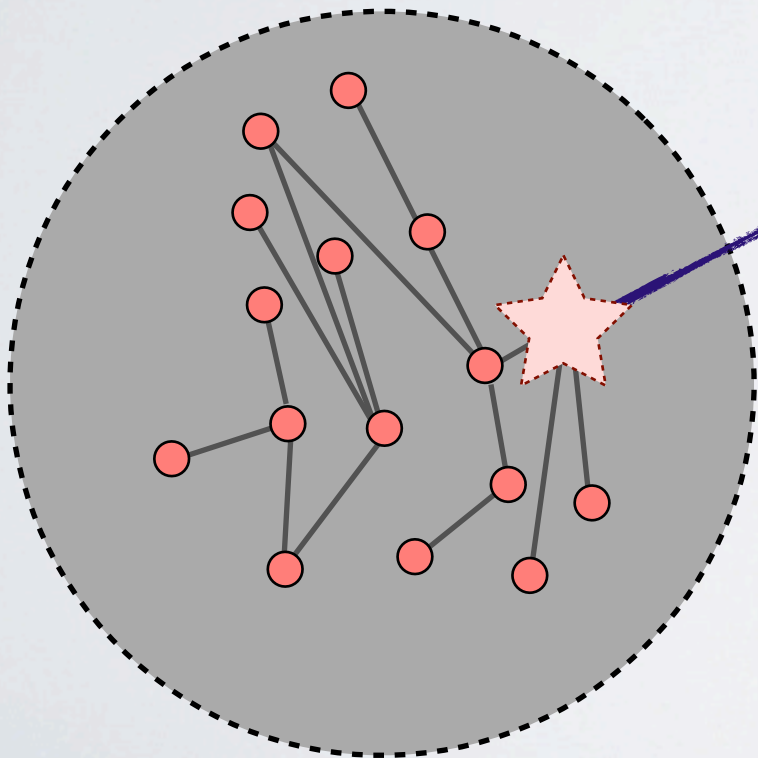
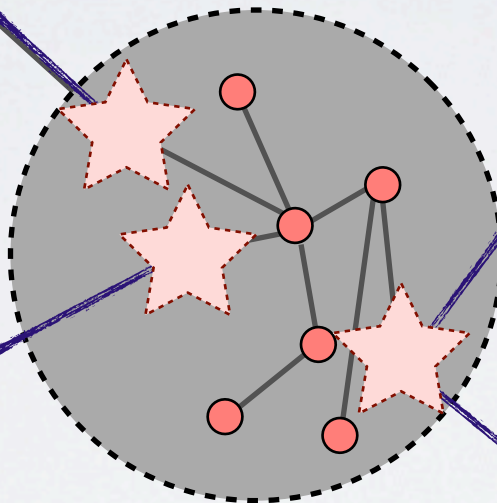
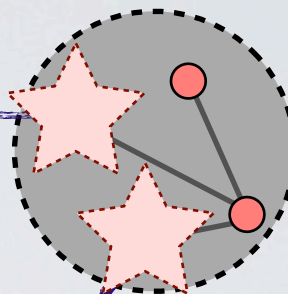
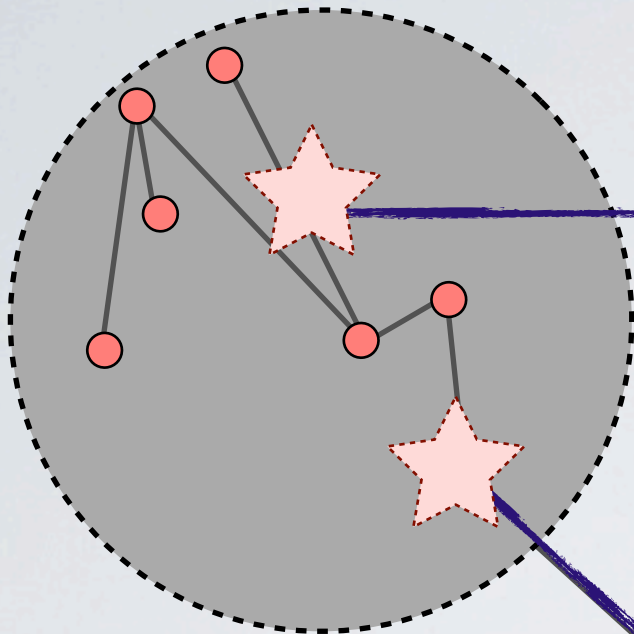
Even if we did, minor models of graphs in S could be arbitrarily large.





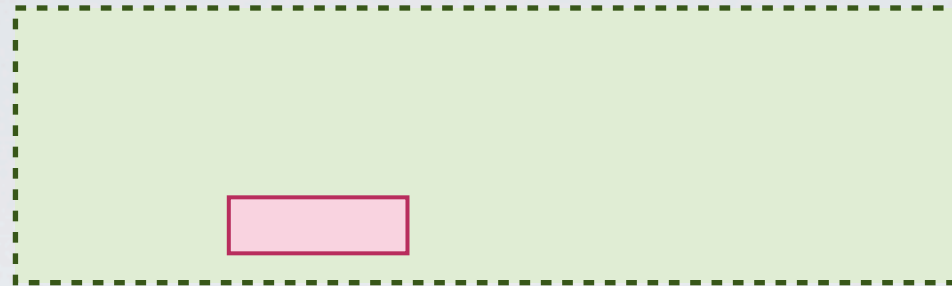




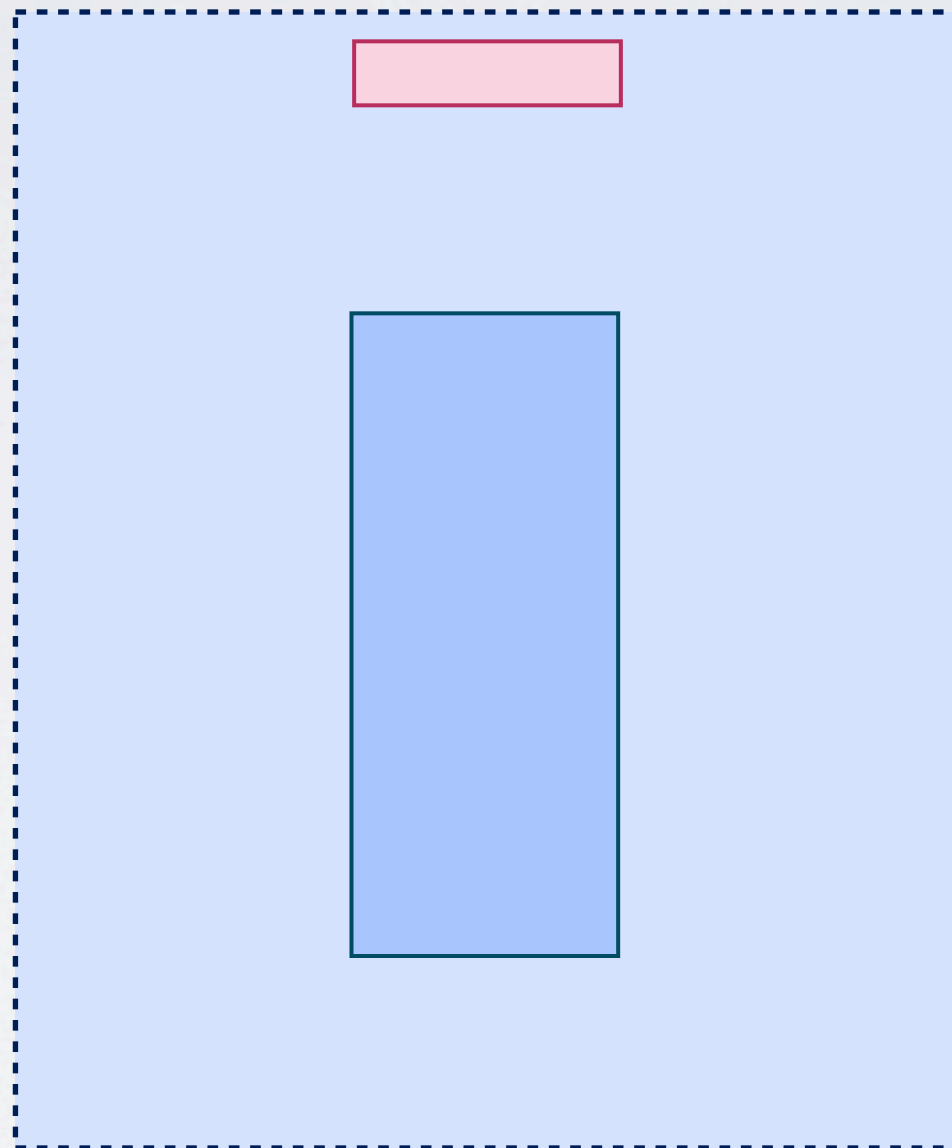


Approximate F-deletion set

X



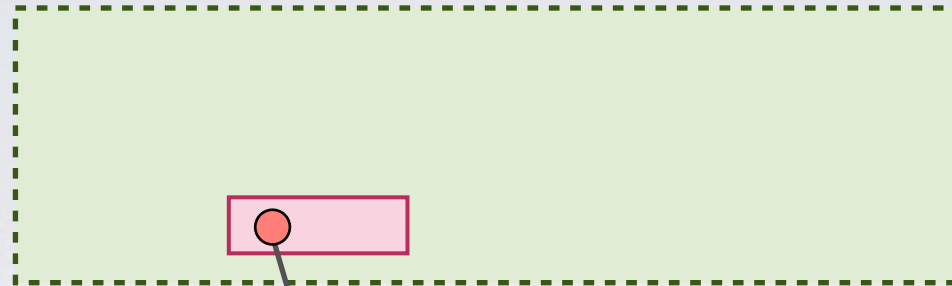
$G \setminus X$



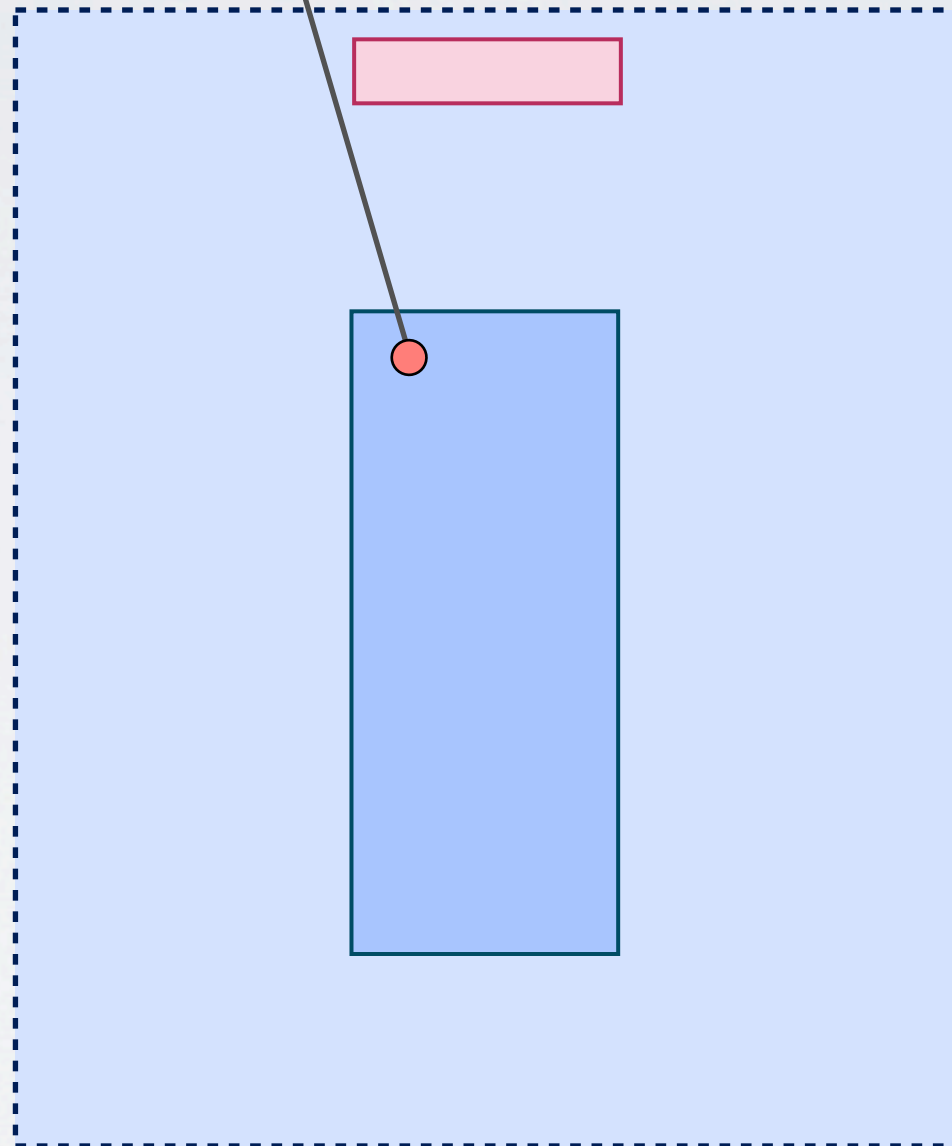
constant treewidth zone

Approximate F-deletion set

X



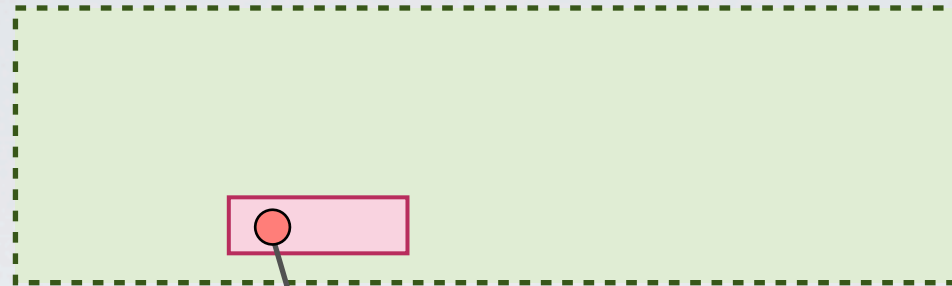
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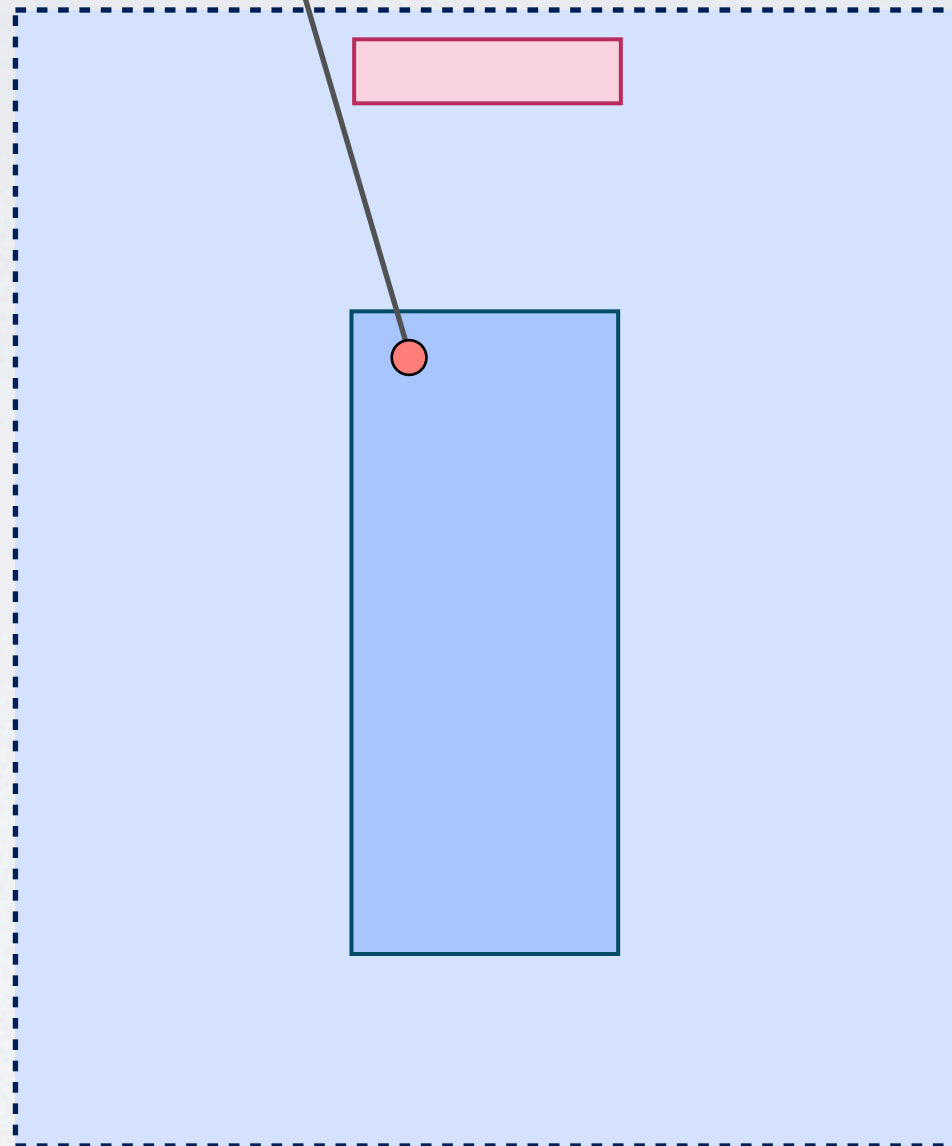
constant treewidth zone

Approximate F-deletion set

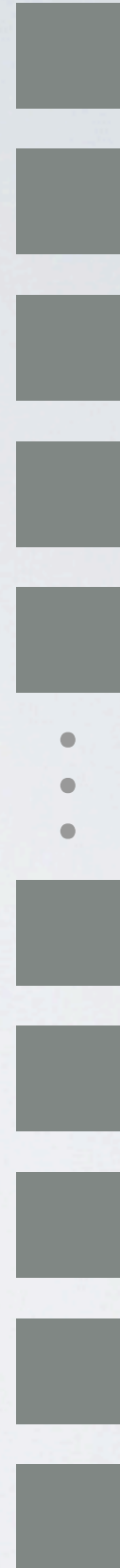
X



$G \setminus X$

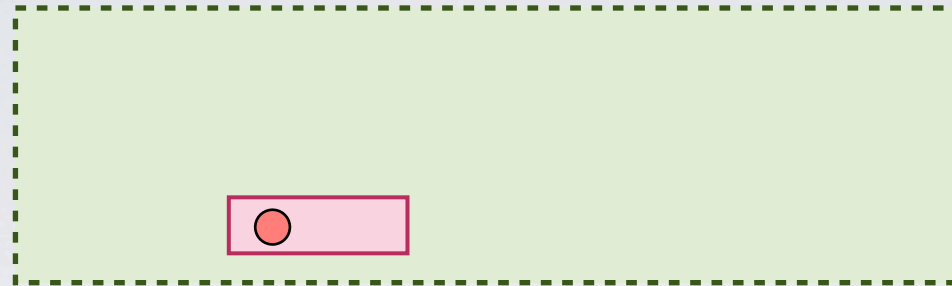


constant treewidth zone

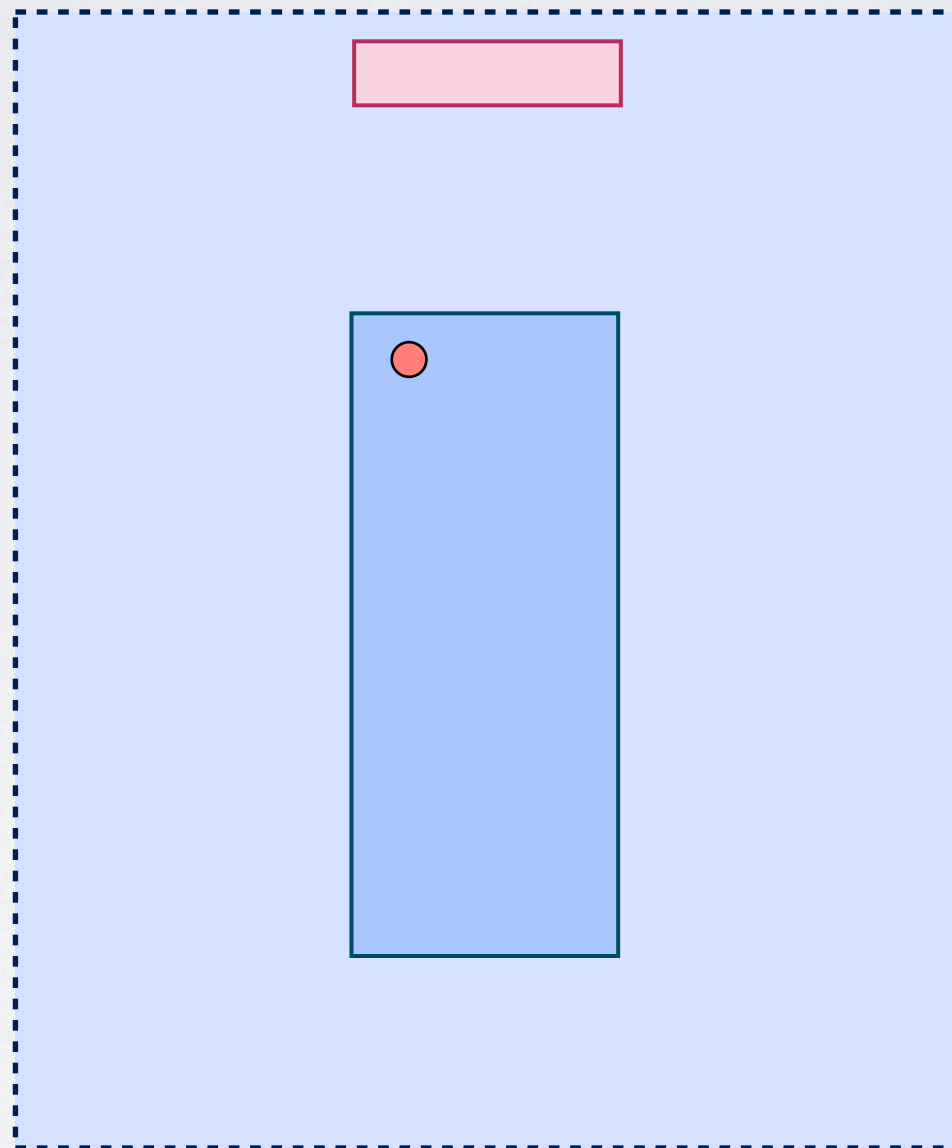


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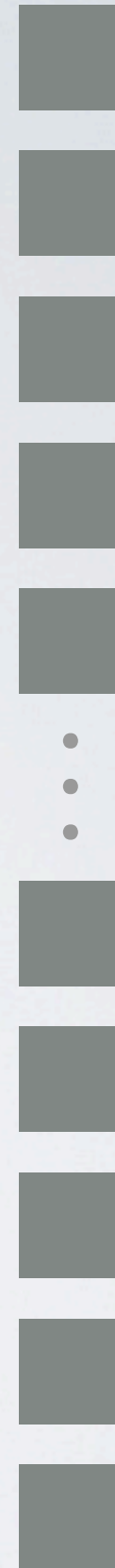
X



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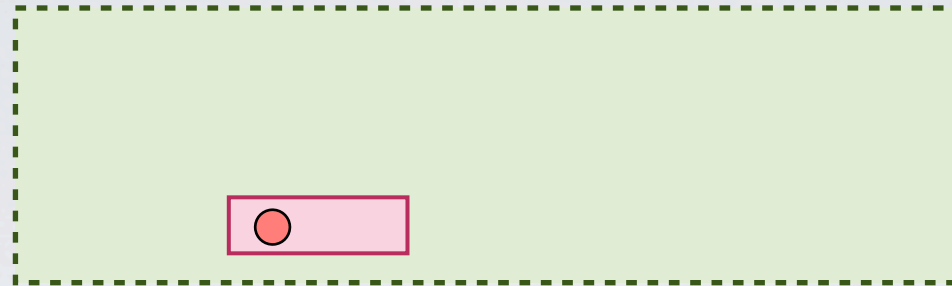


constant treewidth zone

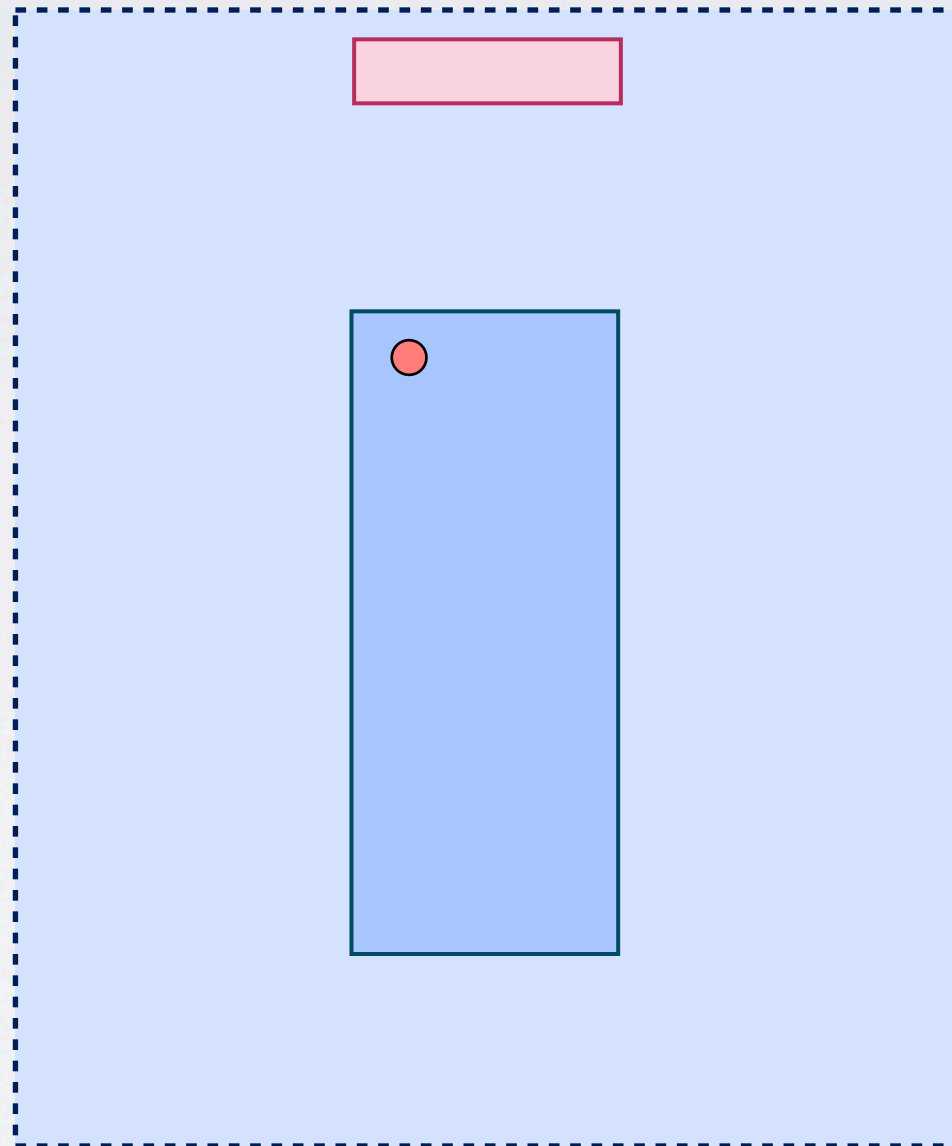


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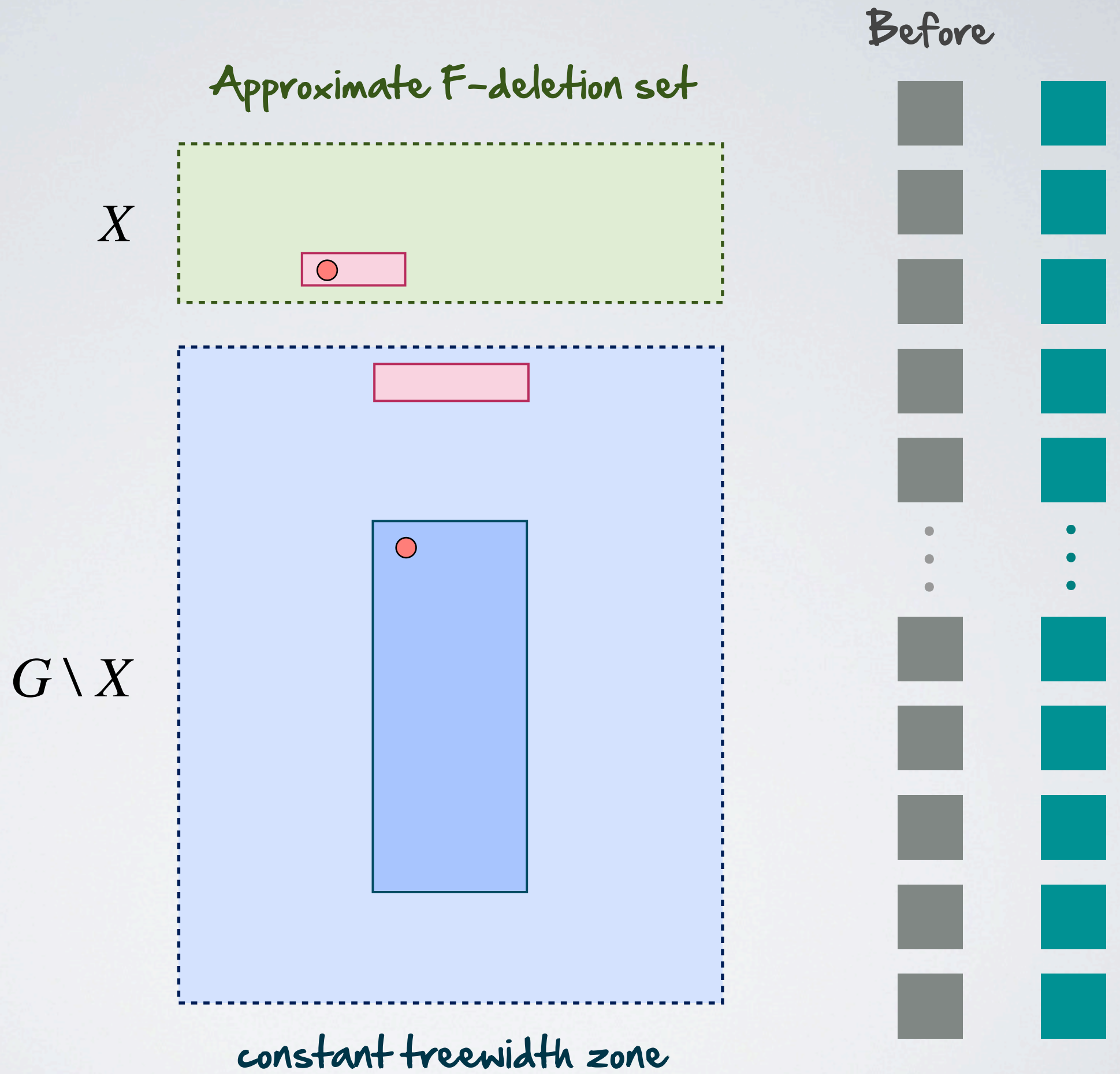


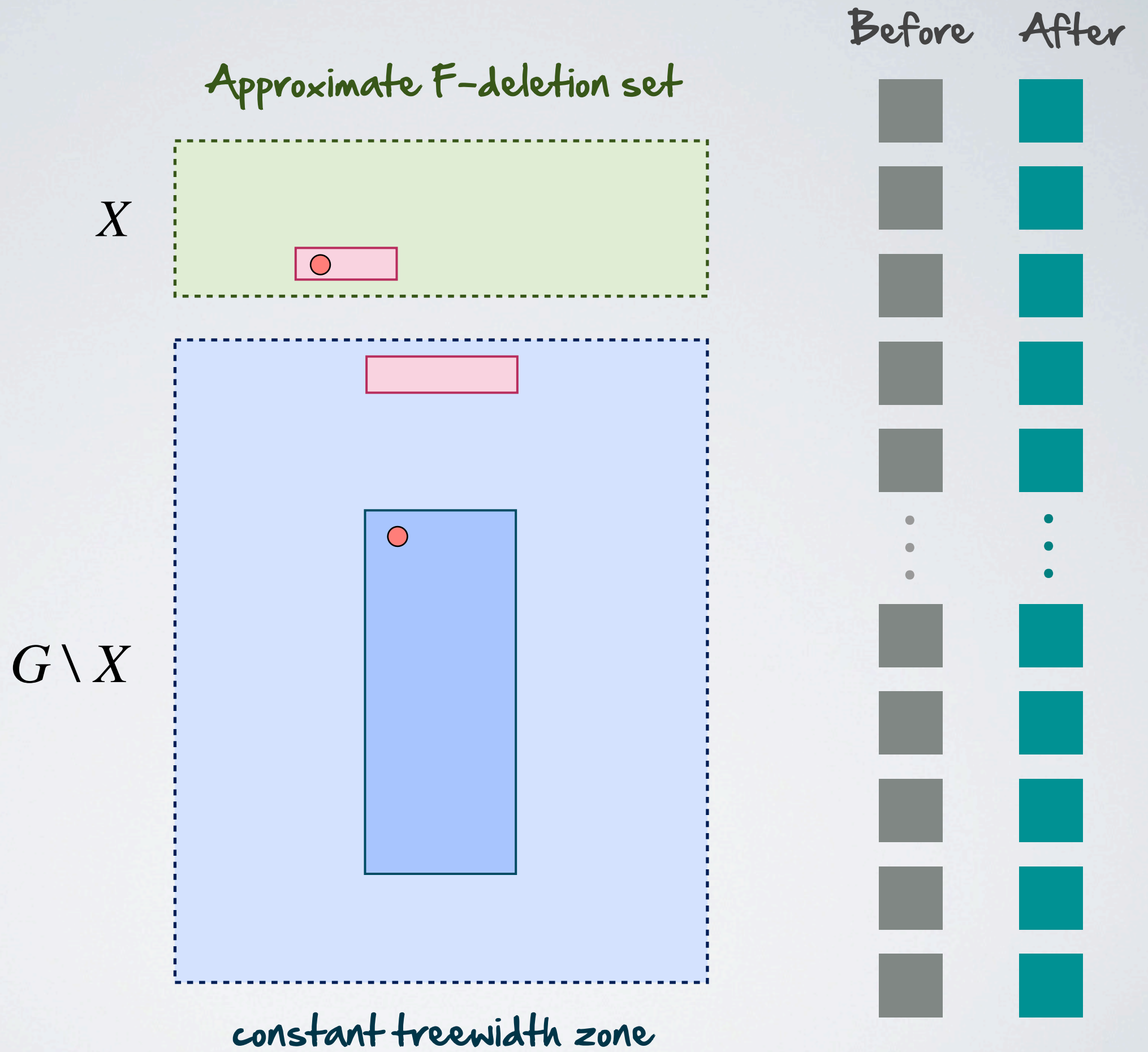
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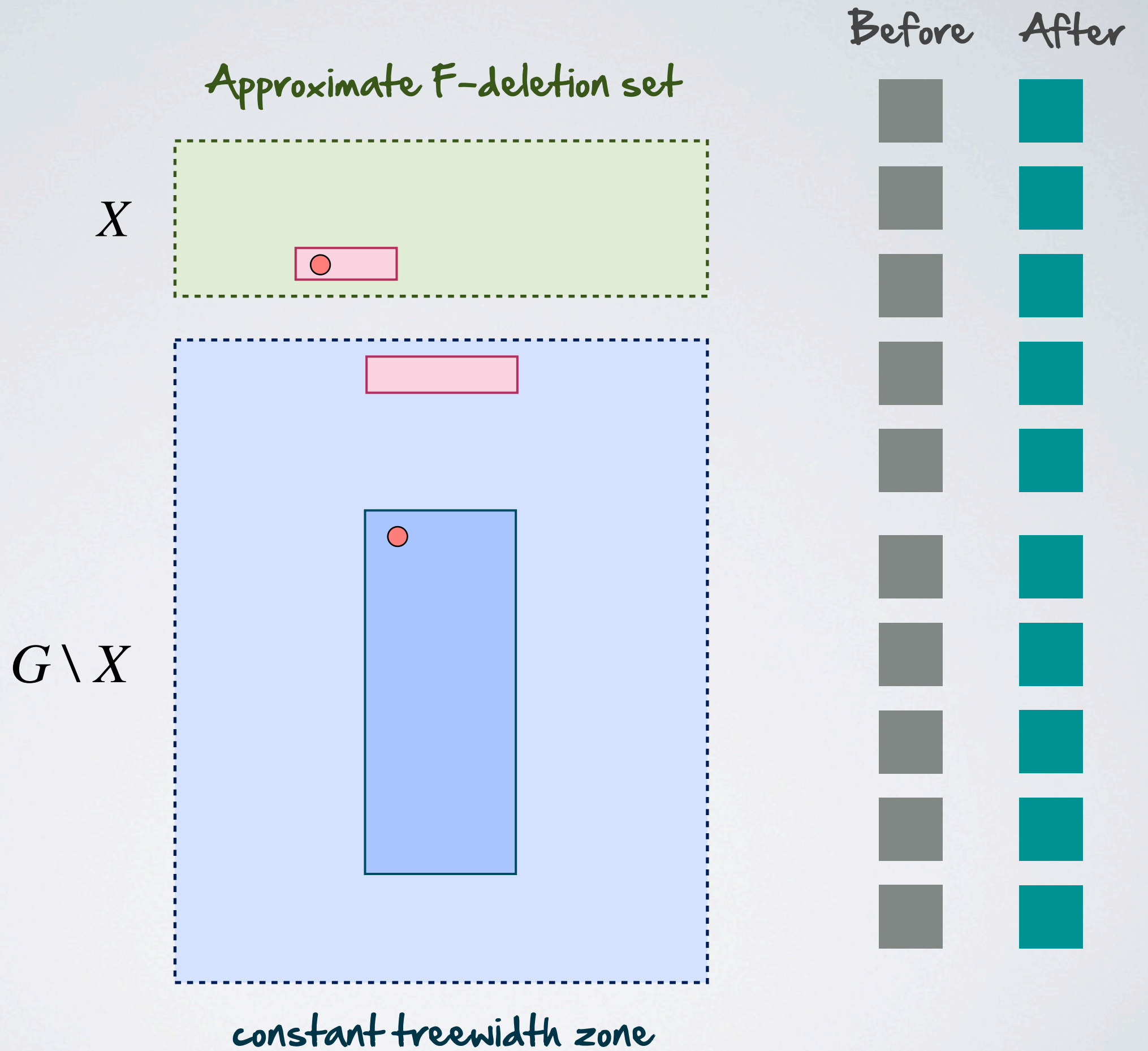


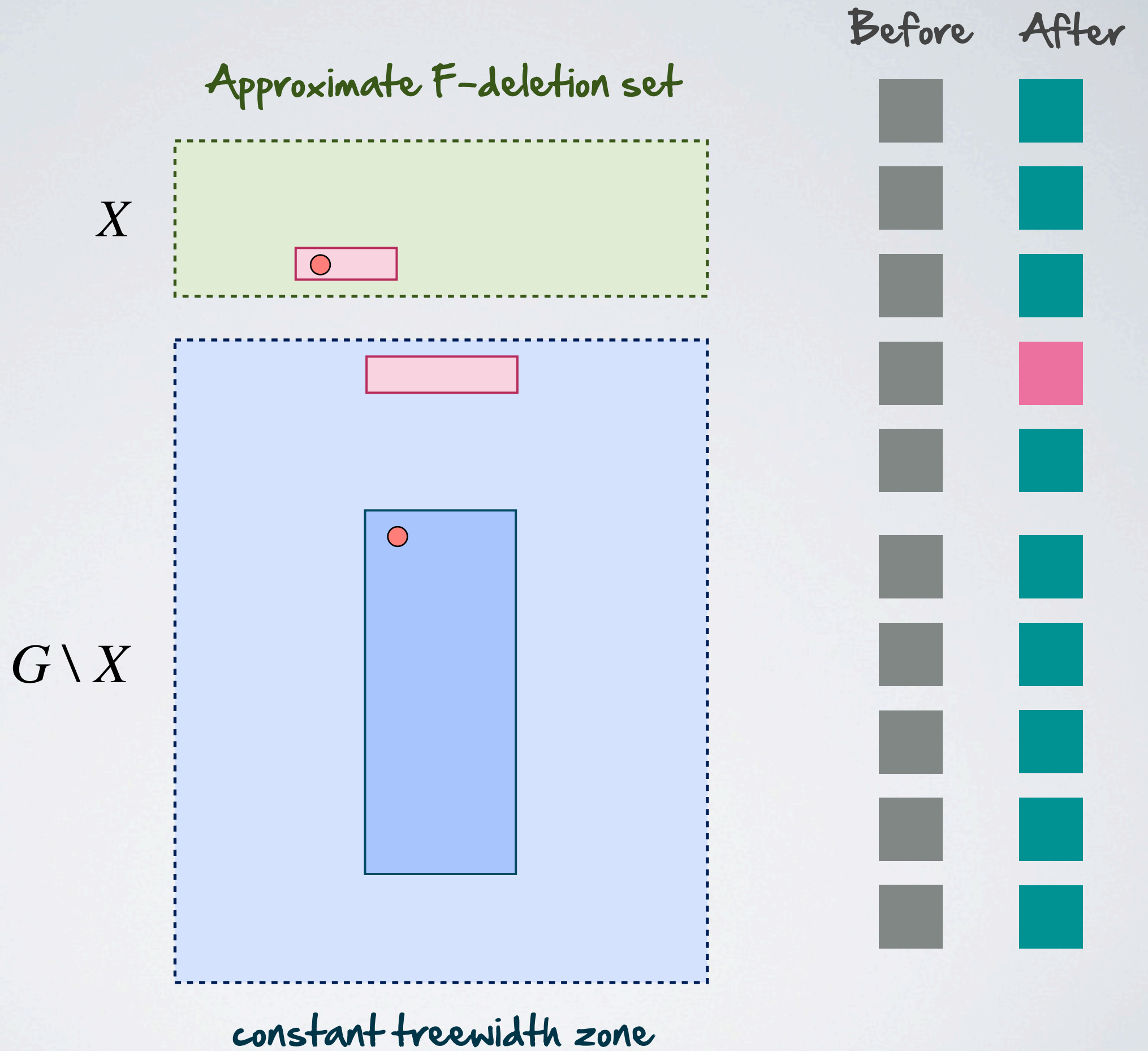
constant treewidth zone











Before

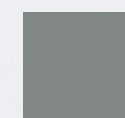
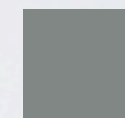
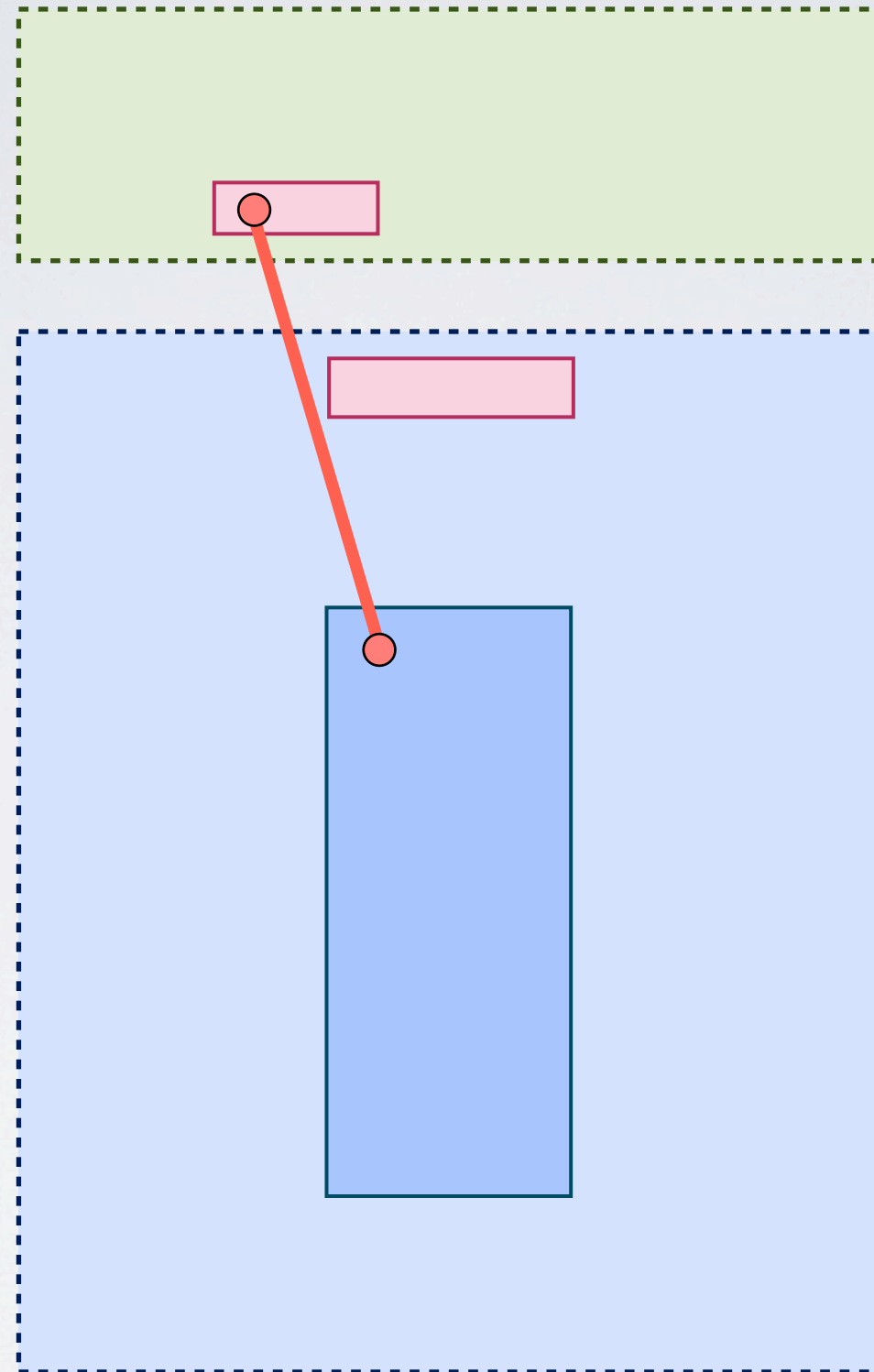
After

Approximate F-deletion set

X

$$G \setminus X$$

constant treewidth zone



SAFETY | **SIZE** | **TIME**

Avoiding some obstruction to membership in FDel_k

SAFETY | SIZE | TIME

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Large degree implies the existence of at least one irrelevant edge

SAFETY | SIZE | TIME

Avoiding some obstruction to membership in FDe_k

Large degree implies the existence of at least one irrelevant edge

Use near-protrusions, cost vectors, finite index, CMSO expressibility.

SAFETY | SIZE | TIME

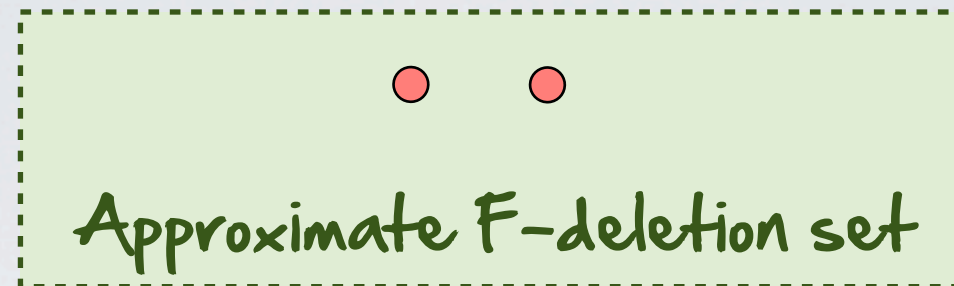
X

Approximate F-deletion set

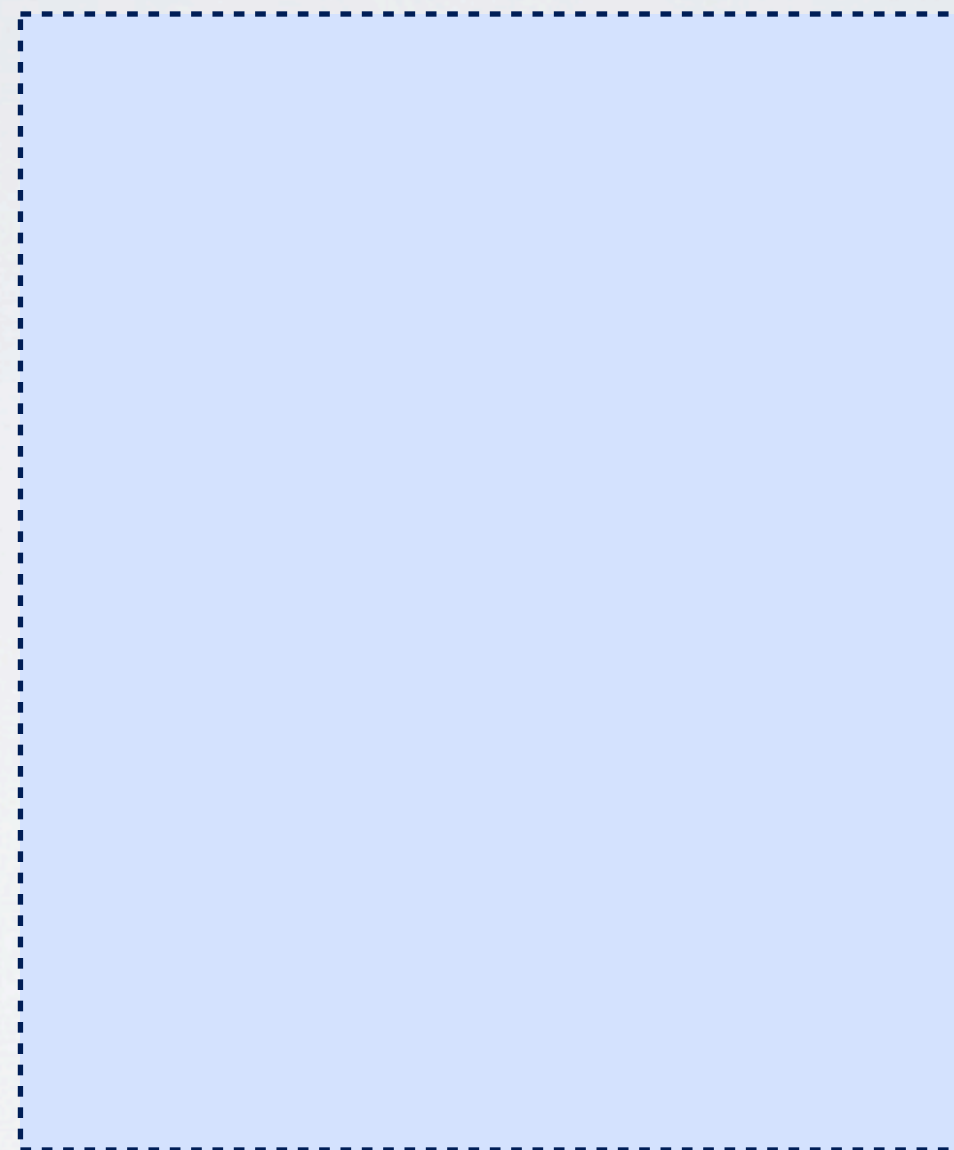
$G \setminus X$

constant treewidth zone: $tw=c$

X

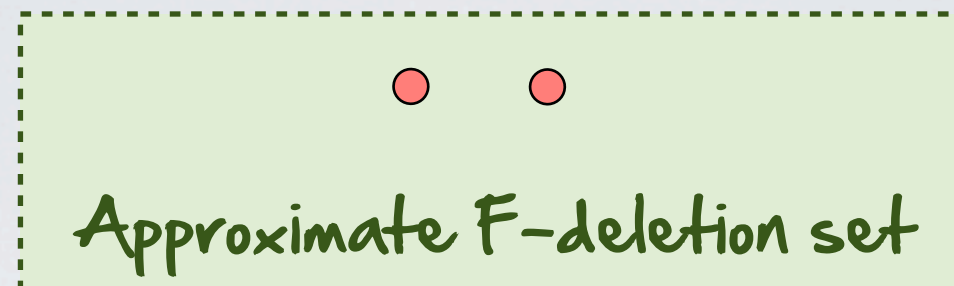


$G \setminus X$

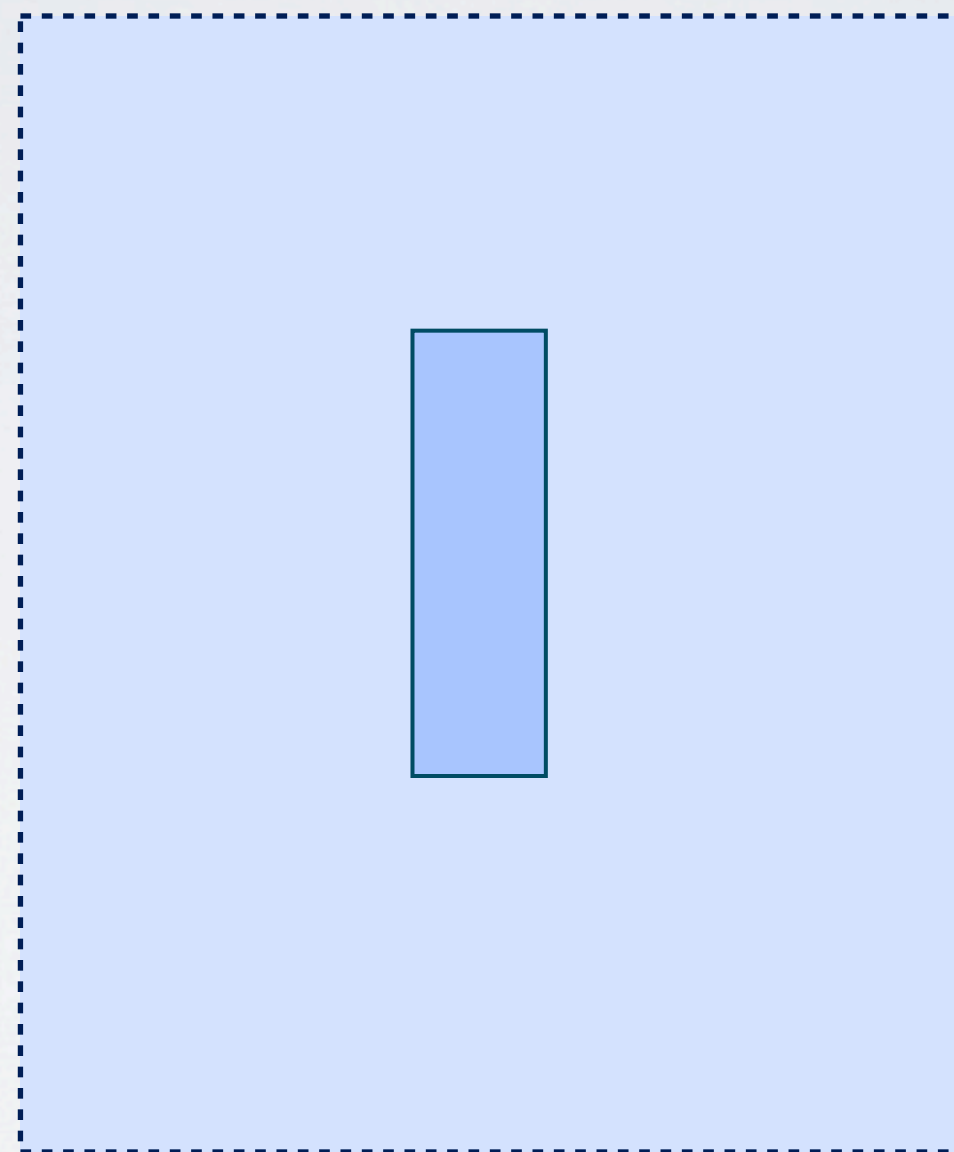


constant treewidth zone: $tw=c$

X



$G \setminus X$



constant treewidth zone: $tw=c$

Case 1

There is a $(k+c+1)$ -sized
 (u,v) -separator.

X

Approximate F -deletion set

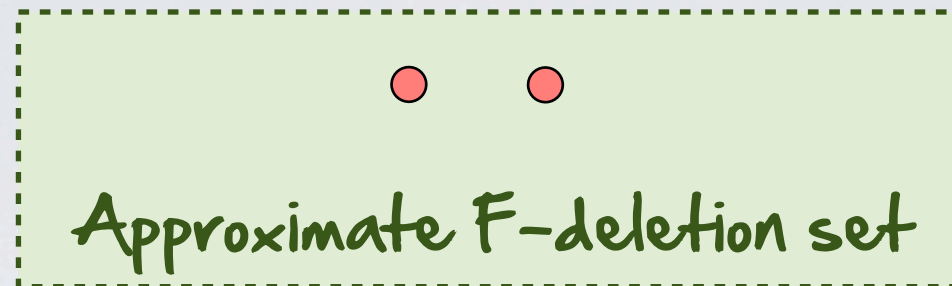
$G \setminus X$

Case 1

There is a $(k+c+1)$ -sized
 (u,v) -separator.

constant treewidth zone: $tw=c$

X



$G \setminus X$

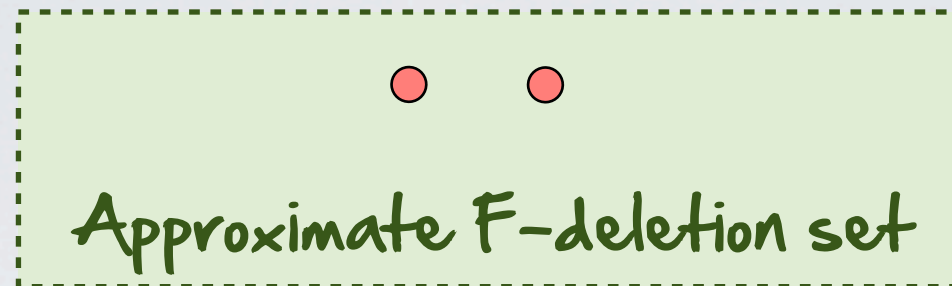
Case 1

There is a $(k+c+1)$ -sized
 (u,v) -separator.

bounded by $\text{poly}(k)$

constant treewidth zone: $tw=c$

X



$G \setminus X$

Case 2

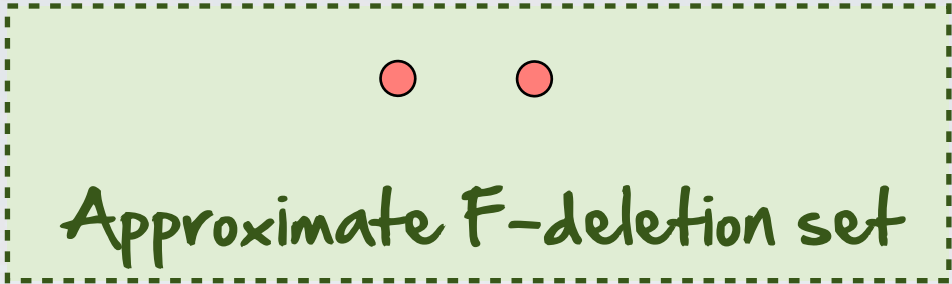
There is a $(k+c+1)$ flow
between u and v .

bounded by $\text{poly}(k)$

constant treewidth zone: $tw=c$



X



$G \setminus X$



bounded by $\text{poly}(k)$

constant treewidth zone: $tw=c$

Case 2

There is a $(k+c+1)$ flow
between u and v .



$G \setminus S$

Case 2

There is a $(k+c+1)$ flow
between u and v .

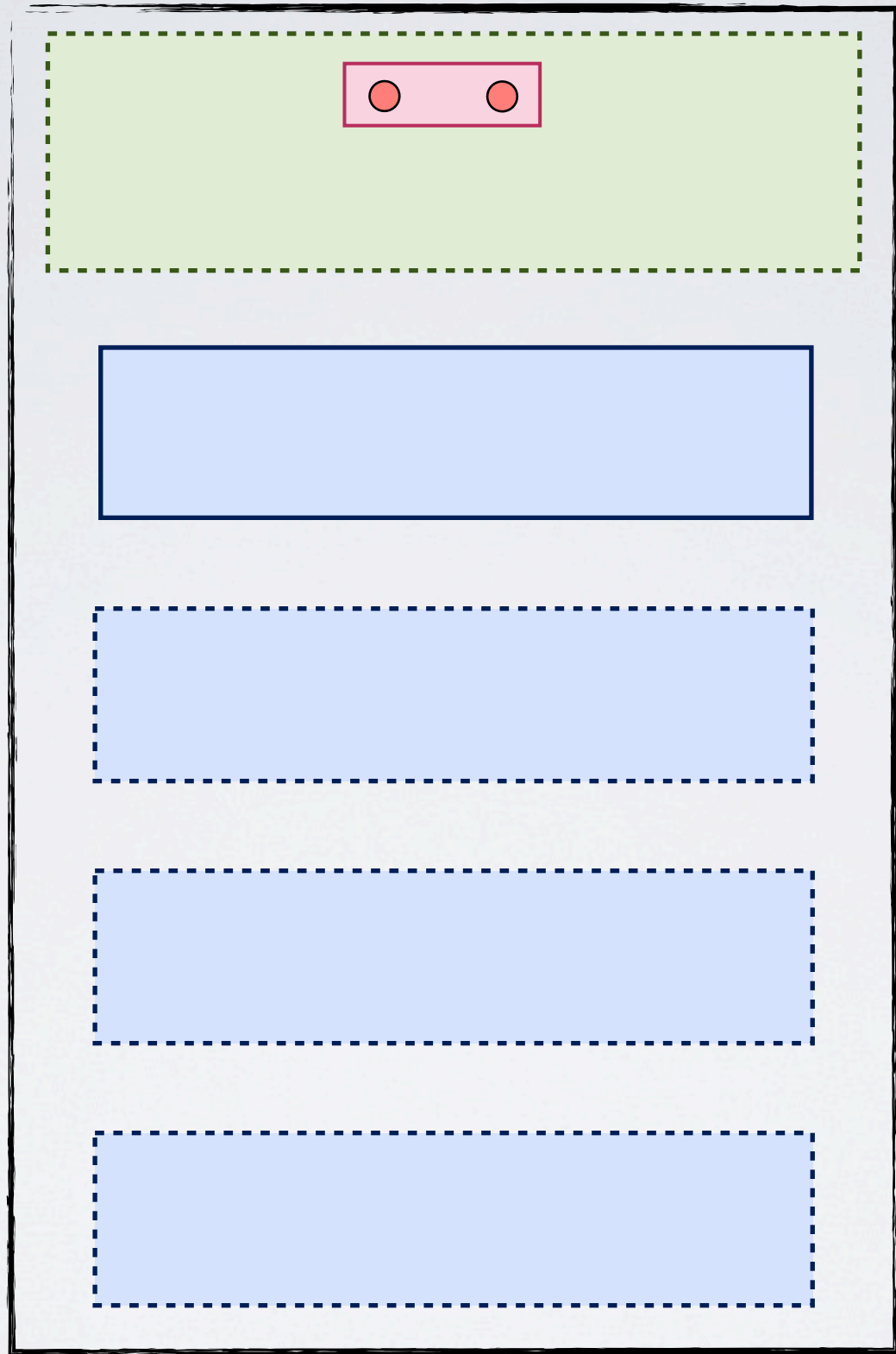
bounded by $\text{poly}(k)$

constant treewidth zone: $tw=c$



$G \setminus S$

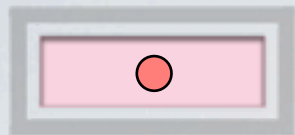
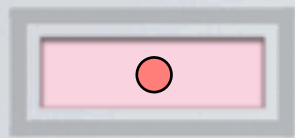
bounded by $\text{poly}(k)$



constant treewidth zone: $tw=c$

Case 2

There is a $(k+c+1)$ flow
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$G \setminus S$

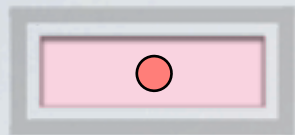
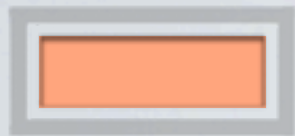
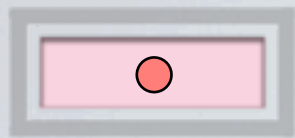


constant treewidth zone: $tw=c$

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$G \setminus S$

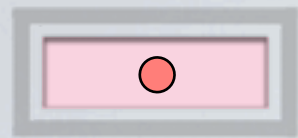


constant treewidth zone: $tw=c$

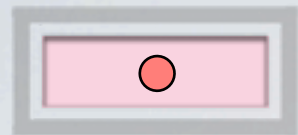
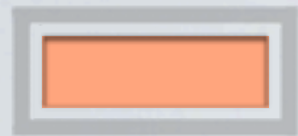
Case 2

There is a $(k+c+1)$ flow
between u and v .

bounded by $\text{poly}(k)$



a separator of size $(c+1)+k$



$G \setminus S$

bounded by $\text{poly}(k)$



constant treewidth zone: $tw=c$

Case 2

There is a $(k+c+1)$ flow
between u and v .

Polynomial Kernels for Planar $\mathcal{F}$ -deletion

Polynomial Kernels for Planar F -deletion

THE CENSORED DETAILS

The presence of disconnected graphs in F opens up a can of worms, various details need substantial tweaking.

The finite obstructions are implied by WQO of t -bounded graphs of bounded treewidth with special minor operations.

Consequences of the kernel.
Obstructions to $[F\text{Del}]_k$ are polynomially
bounded in k .

Polynomial Kernels for Planar $\mathcal{F}$ -deletion

Polynomial Kernels for F -deletion

Polynomial Kernels for F -deletion
To Be CONTINUED...

Daniel will answer your questions!



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