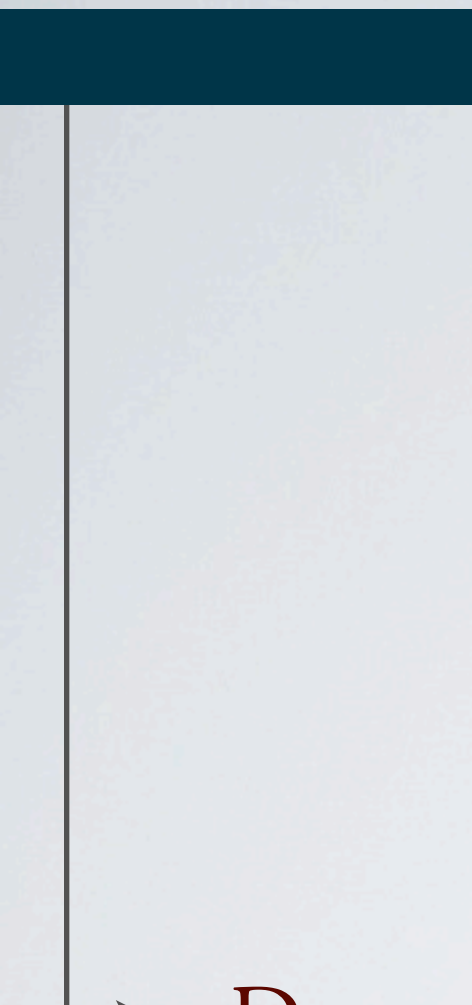
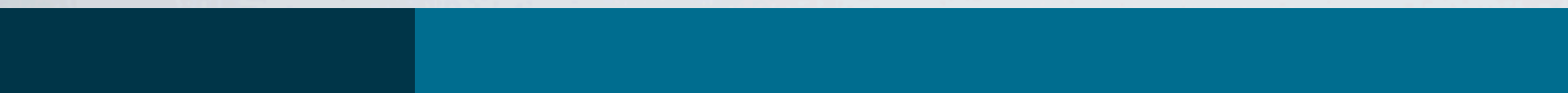


KERNELS FOR F-DELETION



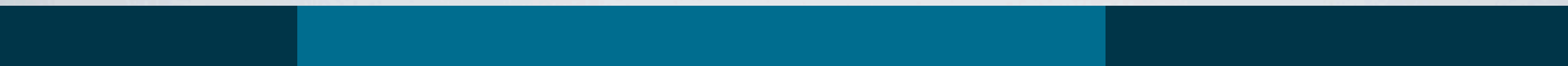
DEFINITIONS

KERNELS & THE F-DELETION PROBLEM



APPROXIMATION
GETTING CLOSE TO
AN OPTIMAL F-HITTING SET

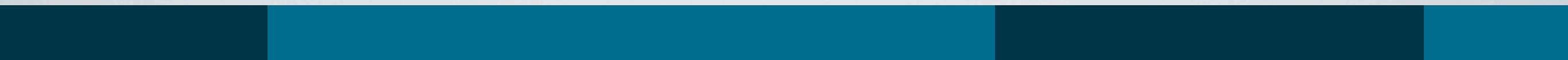
DEFINITIONS
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APPROXIMATION
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DEFINITIONS
KERNELS & THE
F-DELETION PROBLEM

OTHER RESULTS
AN OVERVIEW

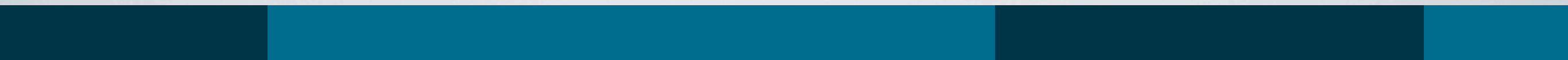


→ DEFINITIONS
KERNELS & THE
F-DELETION PROBLEM

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GETTING CLOSE TO
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REMARKS

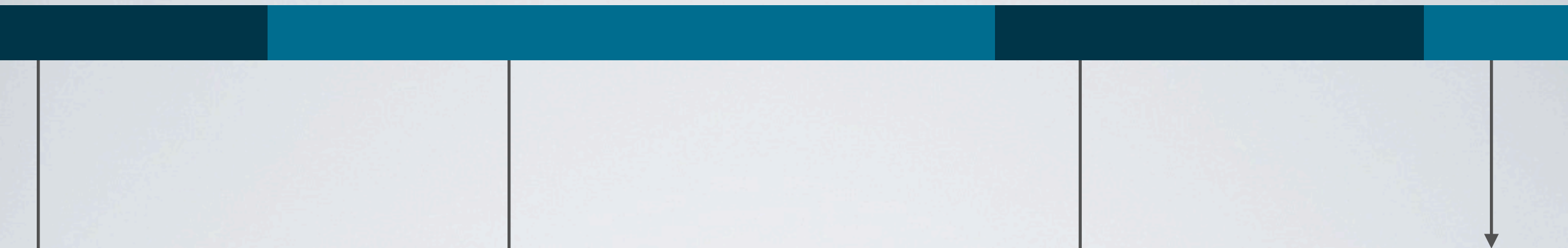


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KERNELIZATION

A kernelization procedure

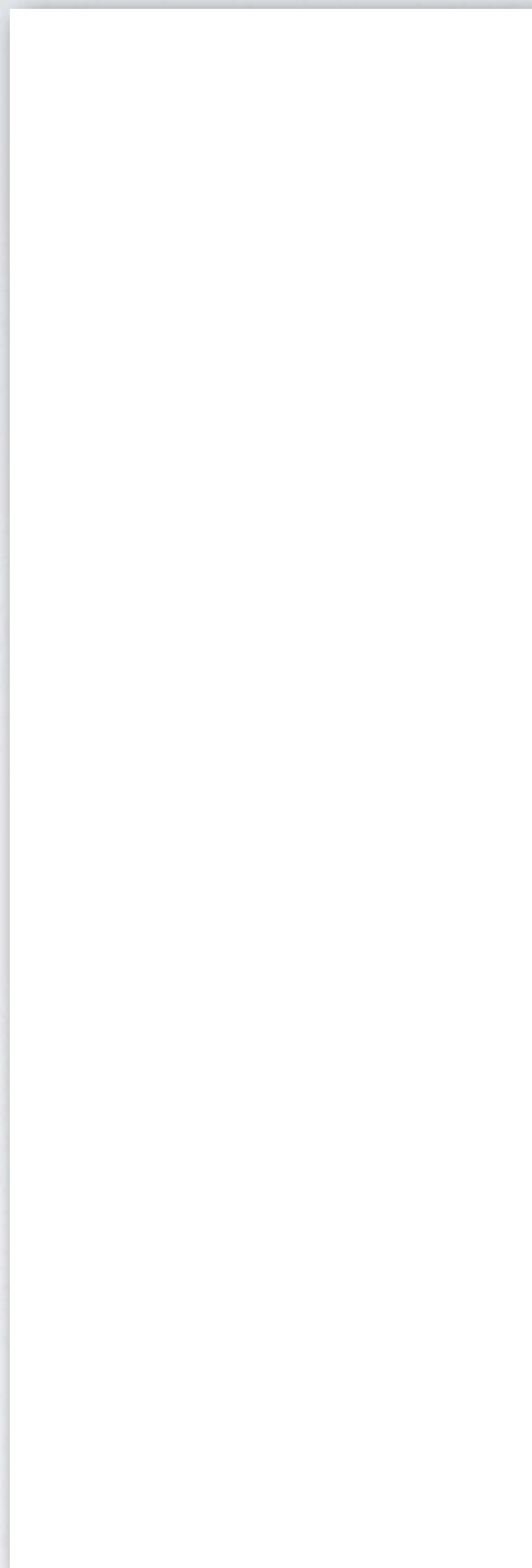
is a function $f : \{0, 1\}^* \times \mathbb{N} \rightarrow \{0, 1\}^* \times \mathbb{N}$

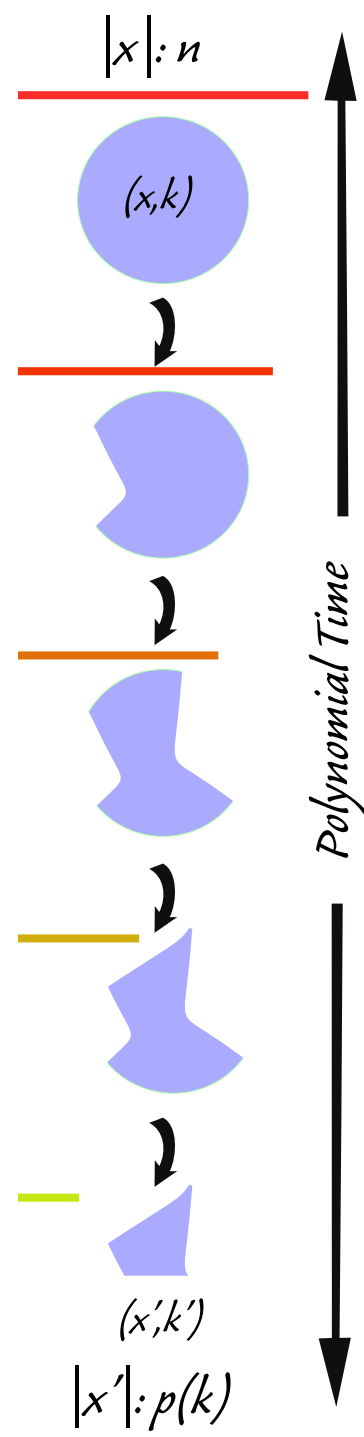
such that for all $(x, k), |x| = n$

$(f(x, k)) \in L$ iff $(x, k) \in L$

$|x'| = g(k)$ and $k' \leq k$

and f is polynomial time computable.





THE F-DELETION PROBLEM

A classic optimization question
often takes the following general form...

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often takes the following general form...

How “close” is a graph to having a certain property?

This question can be formalized in a number of ways,
and a well-studied version is the following:

What is the smallest number of vertices that need to be
deleted so that the remaining graph is
_____?

What is the smallest number of vertices that need to be deleted so that the remaining graph is independent?

What is the smallest number of vertices that need to be deleted so that the remaining graph is acyclic?

What is the smallest number of vertices that need to be deleted so that the remaining graph is planar?

What is the smallest number of vertices that need to be deleted so that the remaining graph is constant treewidth?

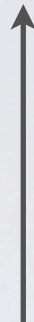
What is the smallest number of vertices that need to be deleted so that the remaining graph is
in X ?

$X = \text{a property}$

A property = an infinite collection of graphs

that satisfy the property.

A property = an infinite collection of graphs



that satisfy the property.

A property = an infinite collection of graphs

can often be characterized by a finite set of
forbidden minors

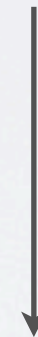
that satisfy the property.

A property = an infinite collection of graphs

whenever the family is closed under minors,
Graph Minor Theorem

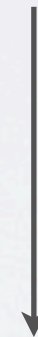
can often be characterized by a finite set of
forbidden minors

Independent = no edges



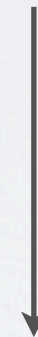
Forbid an edge as a minor

Acyclic = no cycles



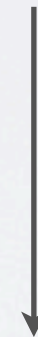
Forbid a triangle as a minor

Planar Graphs



Forbid a $K_{3,3}$, K_5 as a minor

Pathwidth-one graphs



Forbid T_2 , K_3 as a minor

Remove at most k vertices such that the remaining graph has no minor models of graphs from $\mathcal{F}$.

THE F-DELETION PROBLEM

Remove at most k vertices such that the remaining graph has no minor models of graphs from F .

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NP-Complete
(Lewis, Yannakakis)

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Polynomial Kernels?

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PLANAR THE F-DELETION PROBLEM

Remove at most k vertices such that the remaining graph has no minor models of graphs from F .

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NP-Complete
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PLANAR

THE F-DELETION PROBLEM

Remove at most k vertices such that the remaining graph has no minor models of graphs from F .
(Where F contains a planar graph.)

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Remark. We assume throughout that F contains connected graphs.

A Summary of Results

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- Planar F-deletion admits an approximation algorithm.

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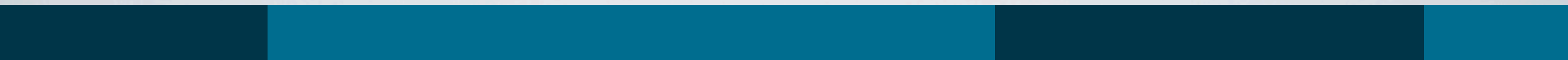
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- Some packing variants of the problem are not likely to have polynomial kernels.
- The kernelization complexity of Independent FVS and Colorful Motifs is explored in detail.



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THE PLANAR F-DELETION PROBLEM

Remove at most k vertices such that the remaining graph has no minor models of graphs from F .

The graphs in F are connected, and at least one of them is planar.

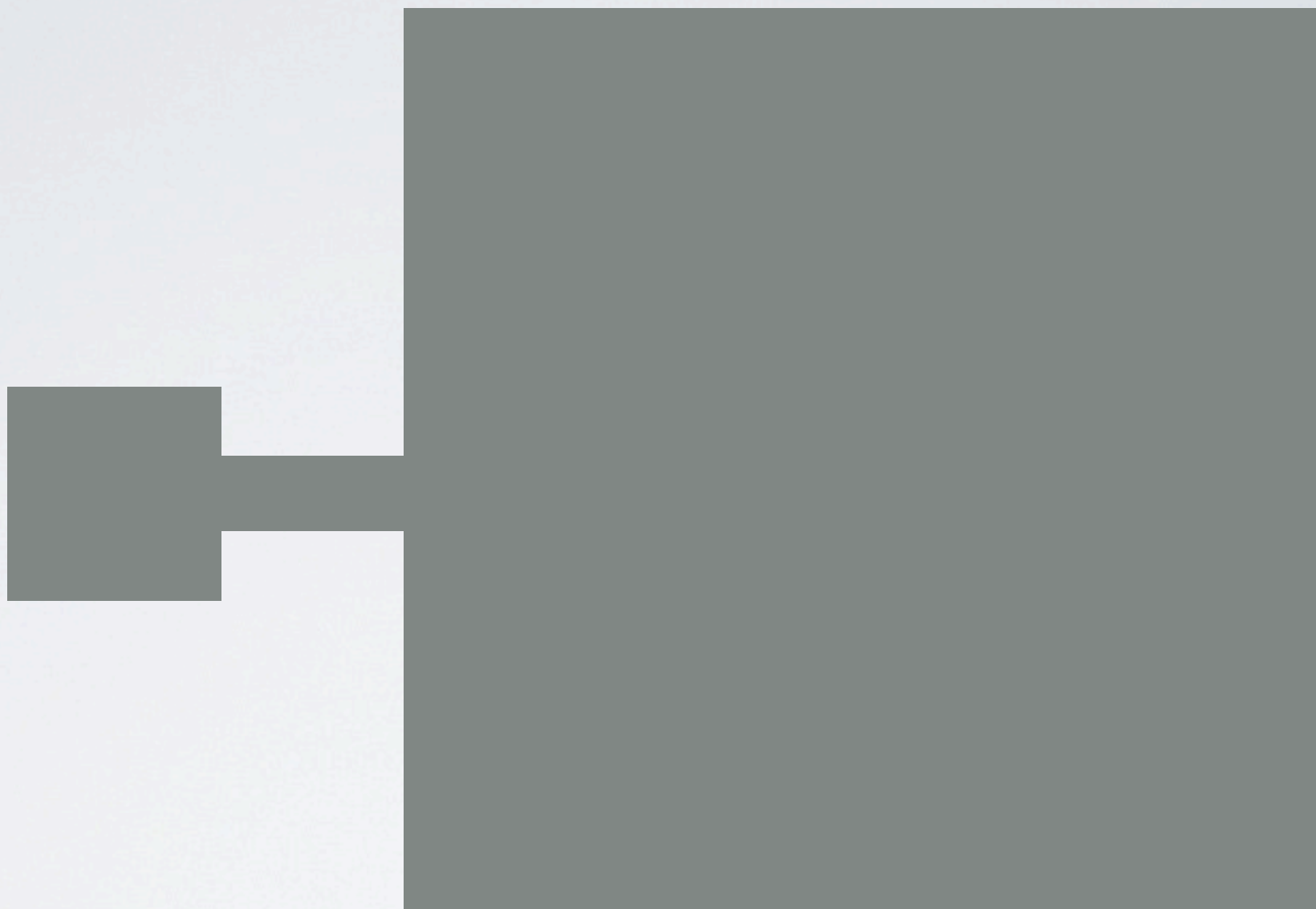
INGREDIENTS

1. Let H be a planar graph on h vertices.

 If the treewidth of G exceeds c_h
 then G contains a minor model of H .

2. The planar F -deletion problem can be solved
 optimally in polynomial time
 on graphs of constant treewidth.

3. Any YES instance of planar F -deletion
 has treewidth at most $k + c_h$.



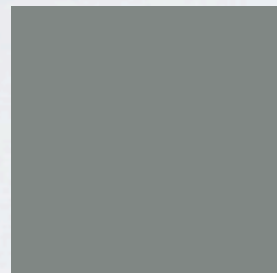
Constant treewidth



Large enough to guarantee a minor model of H , but still a constant - so that the problem can be solved optimally in polynomial time.

(Fact 1 & 2)

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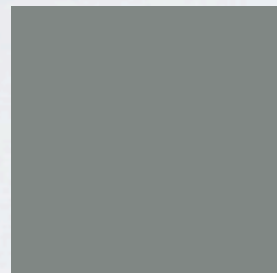


The Rest of the Graph

“Small” Separator

Bounded in terms of k
(Fact 3)

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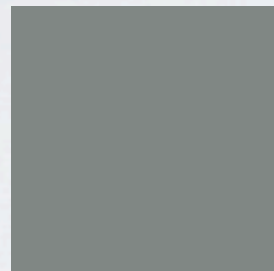


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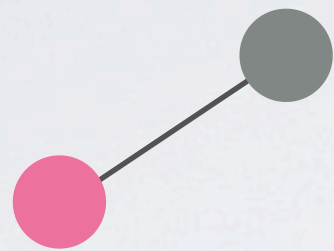
Recurse

ANALYSIS

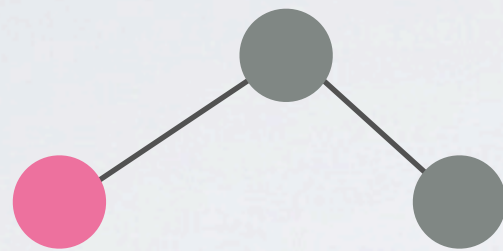
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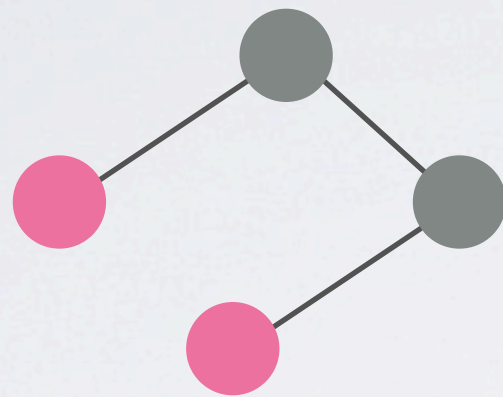
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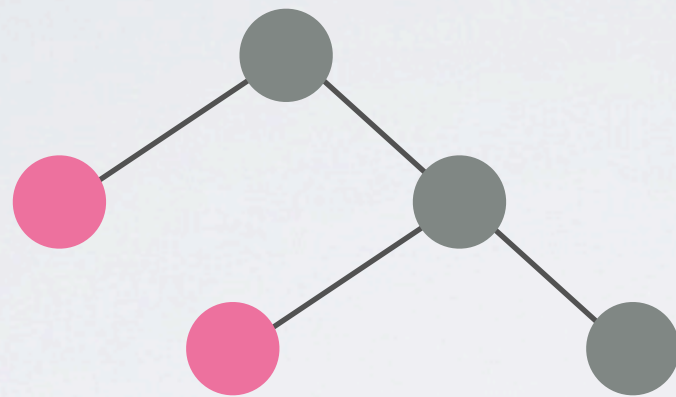
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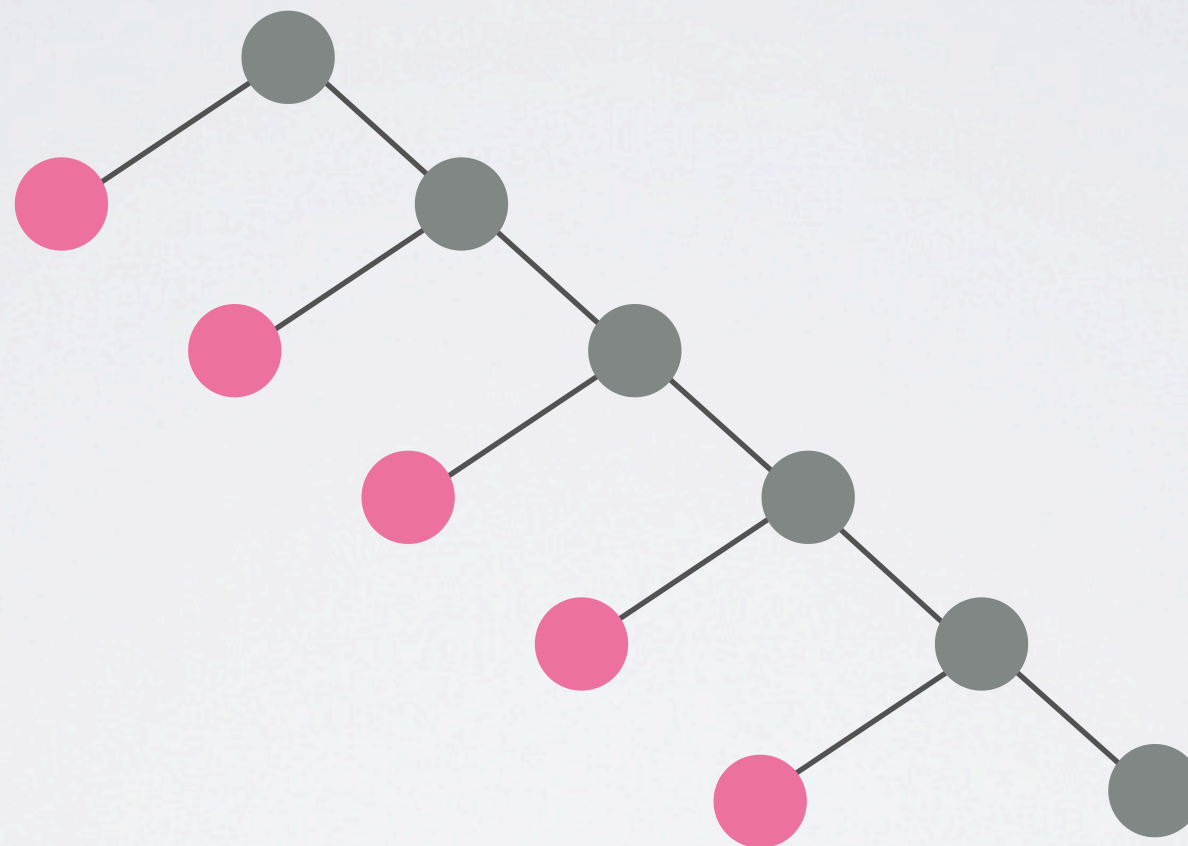
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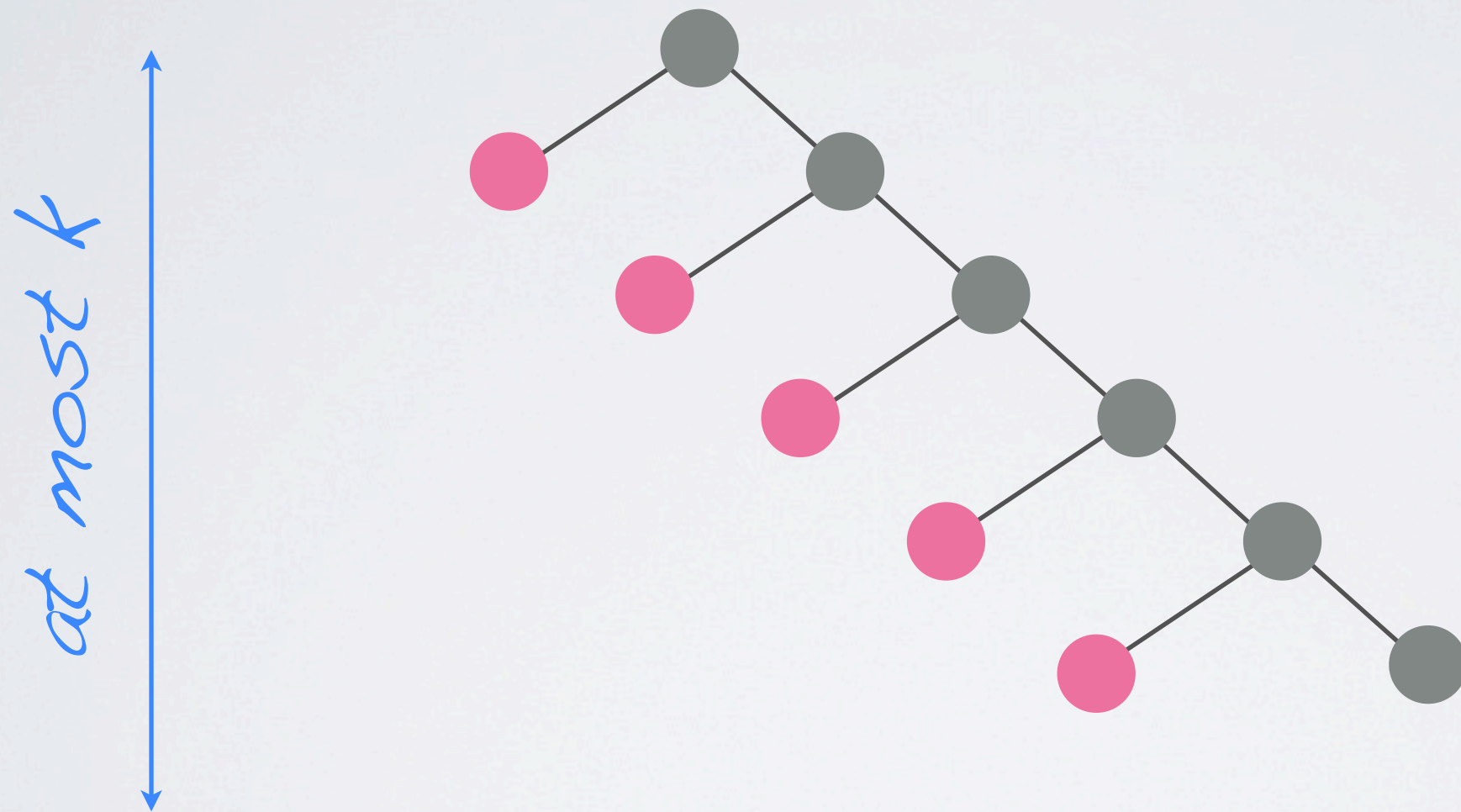
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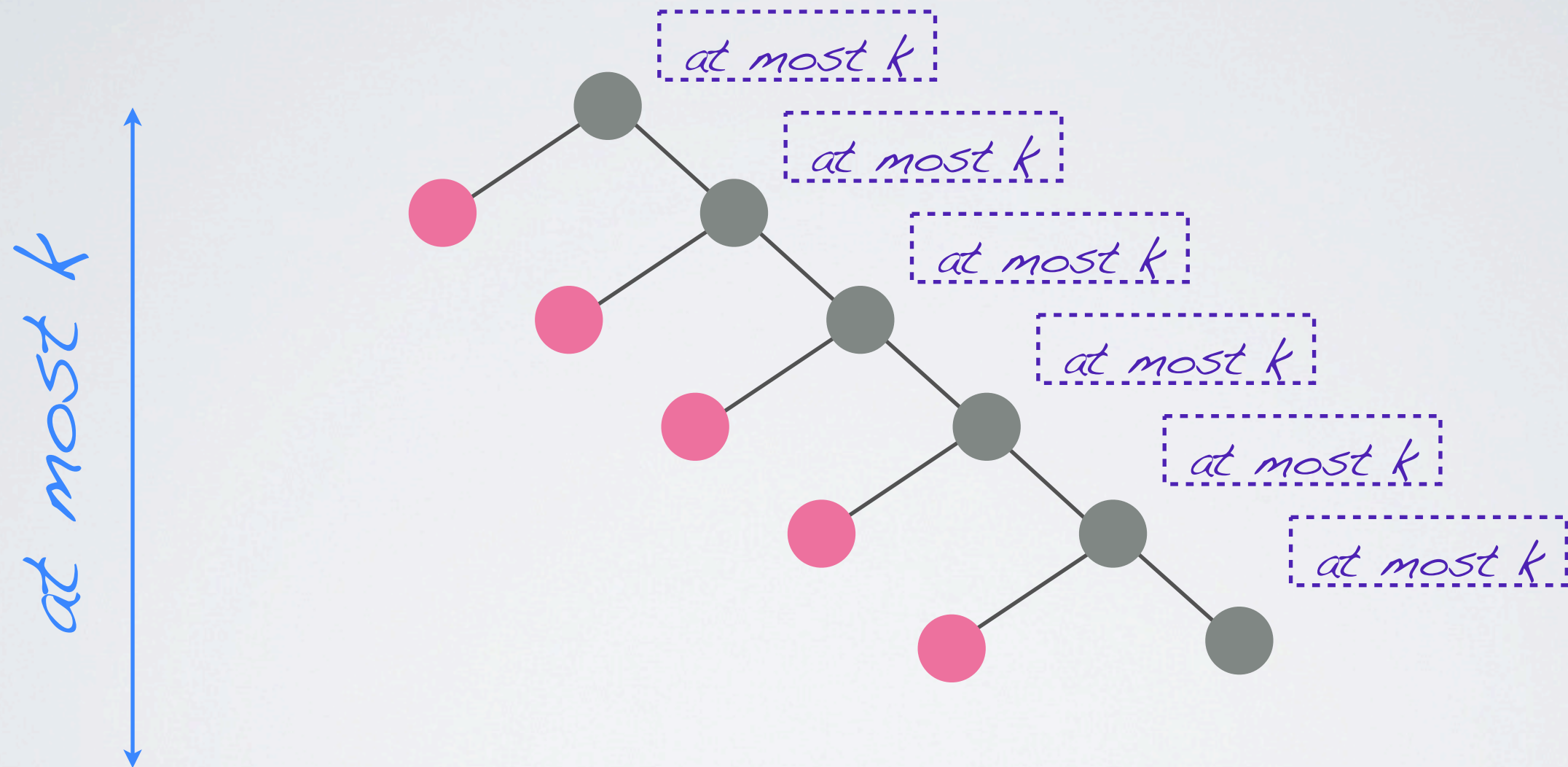
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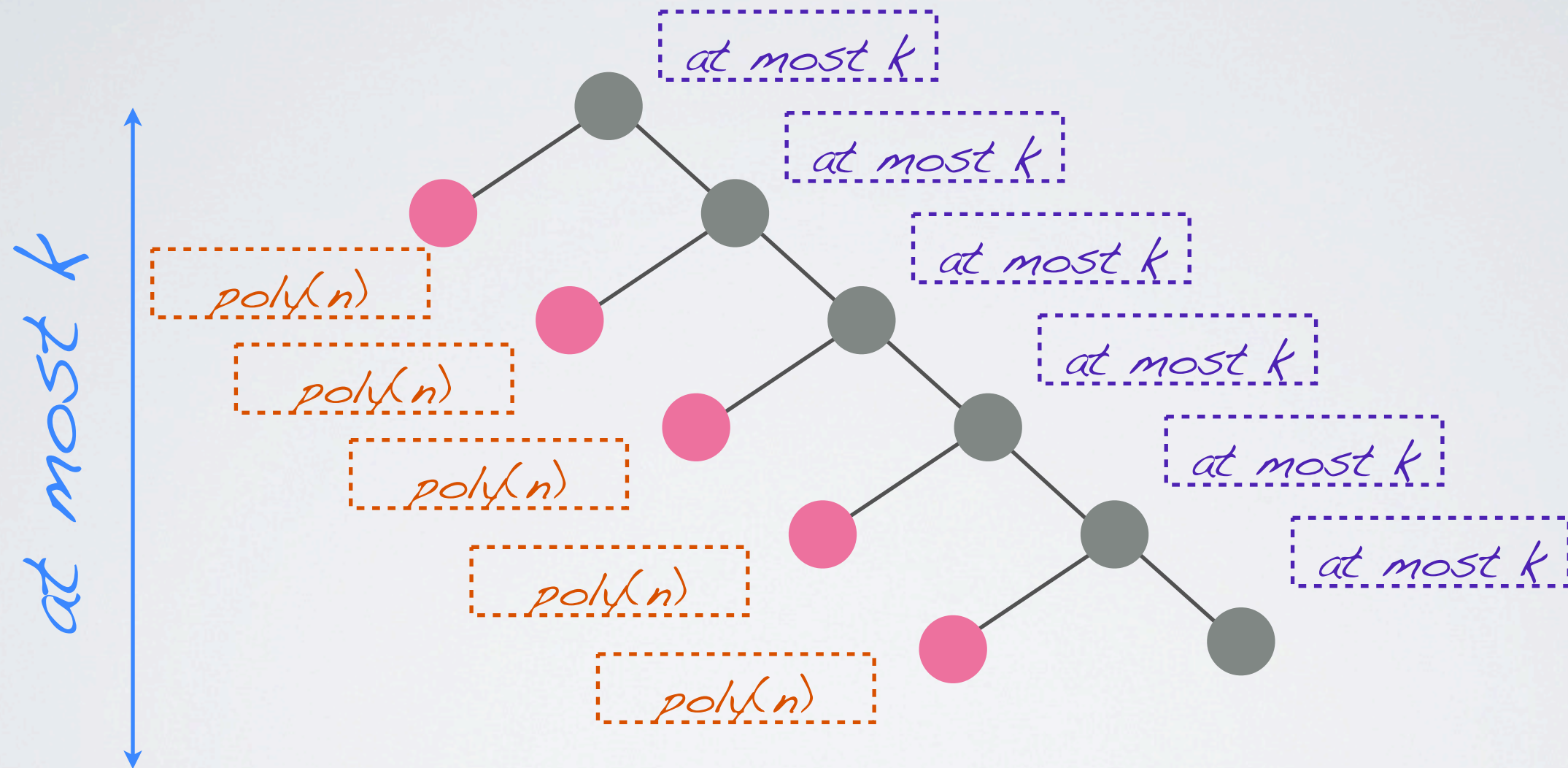
ANALYSIS



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ANALYSIS





How do we get here?



1. Let H be a planar graph on h vertices.

 If the treewidth of G exceeds c_h
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2. The planar F -deletion problem can be solved
 optimally in polynomial time
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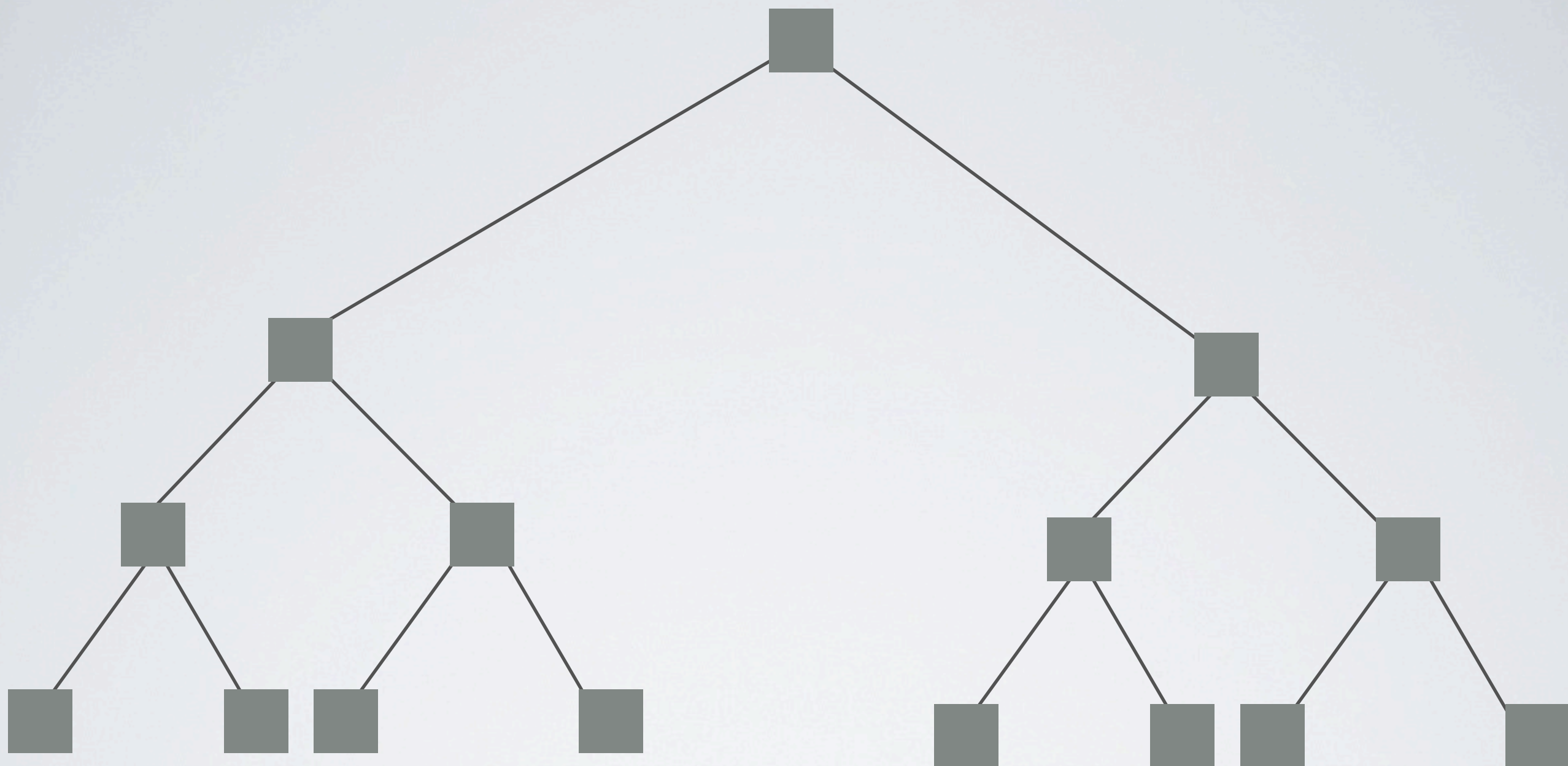
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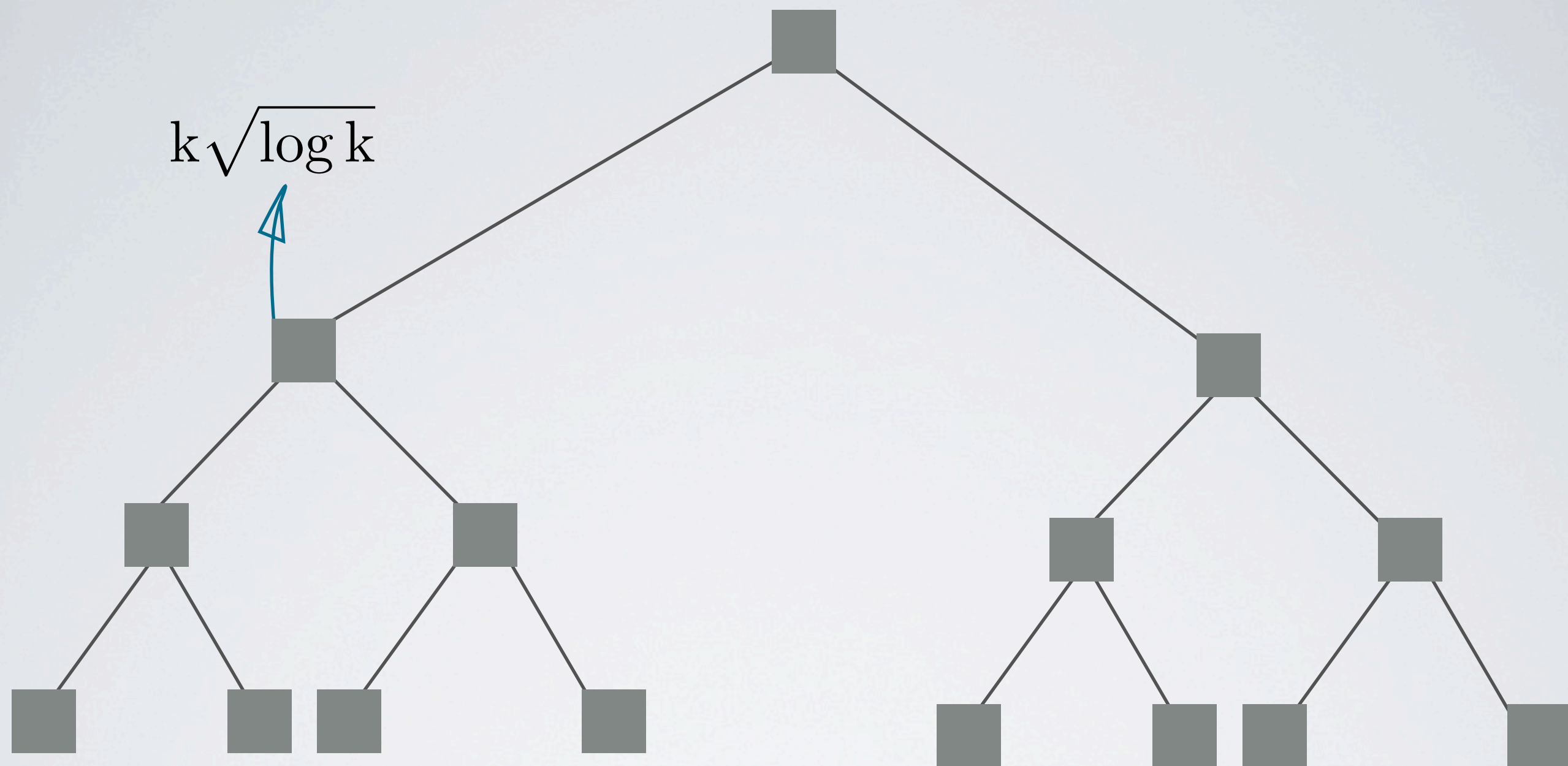
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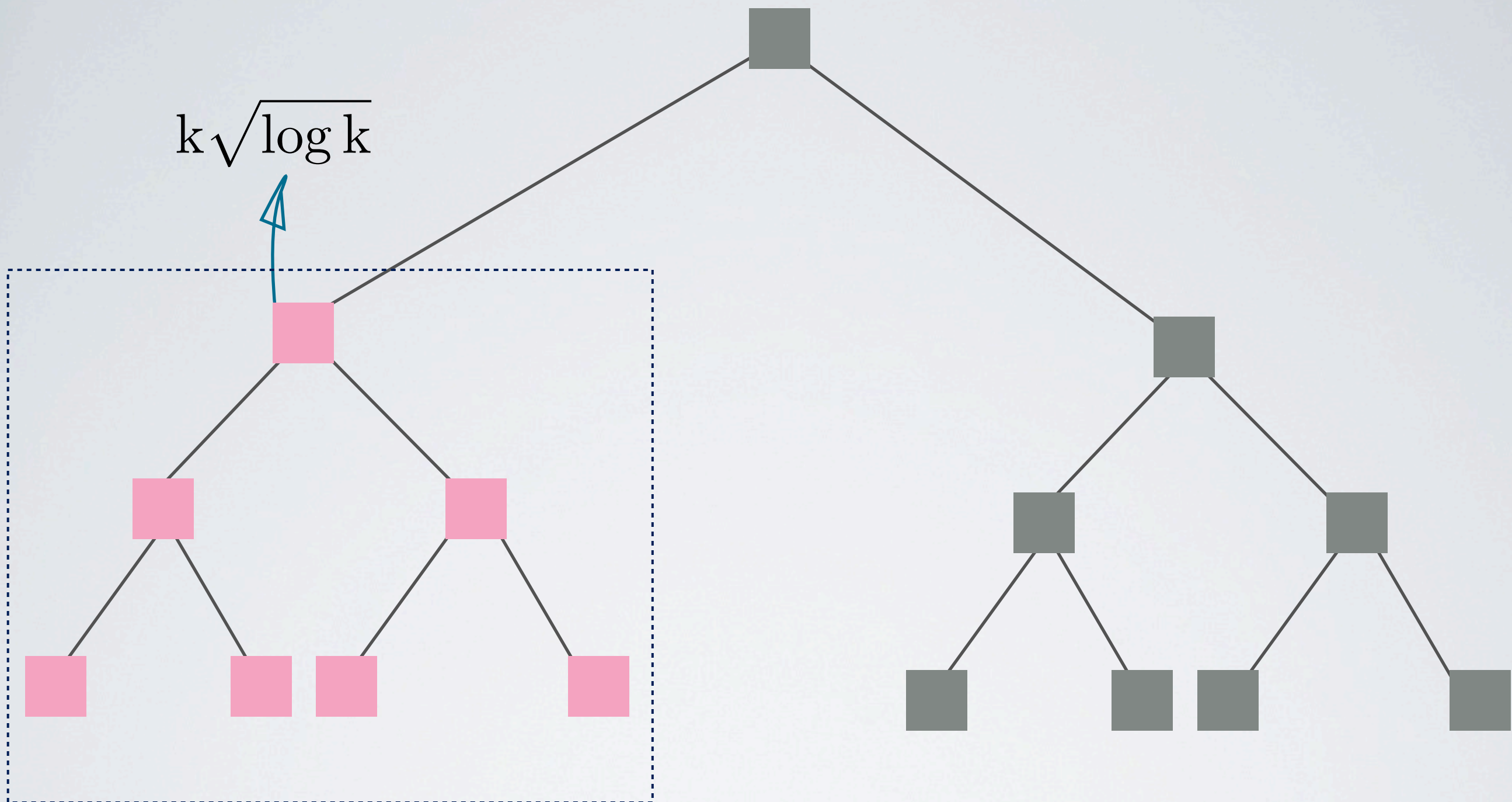
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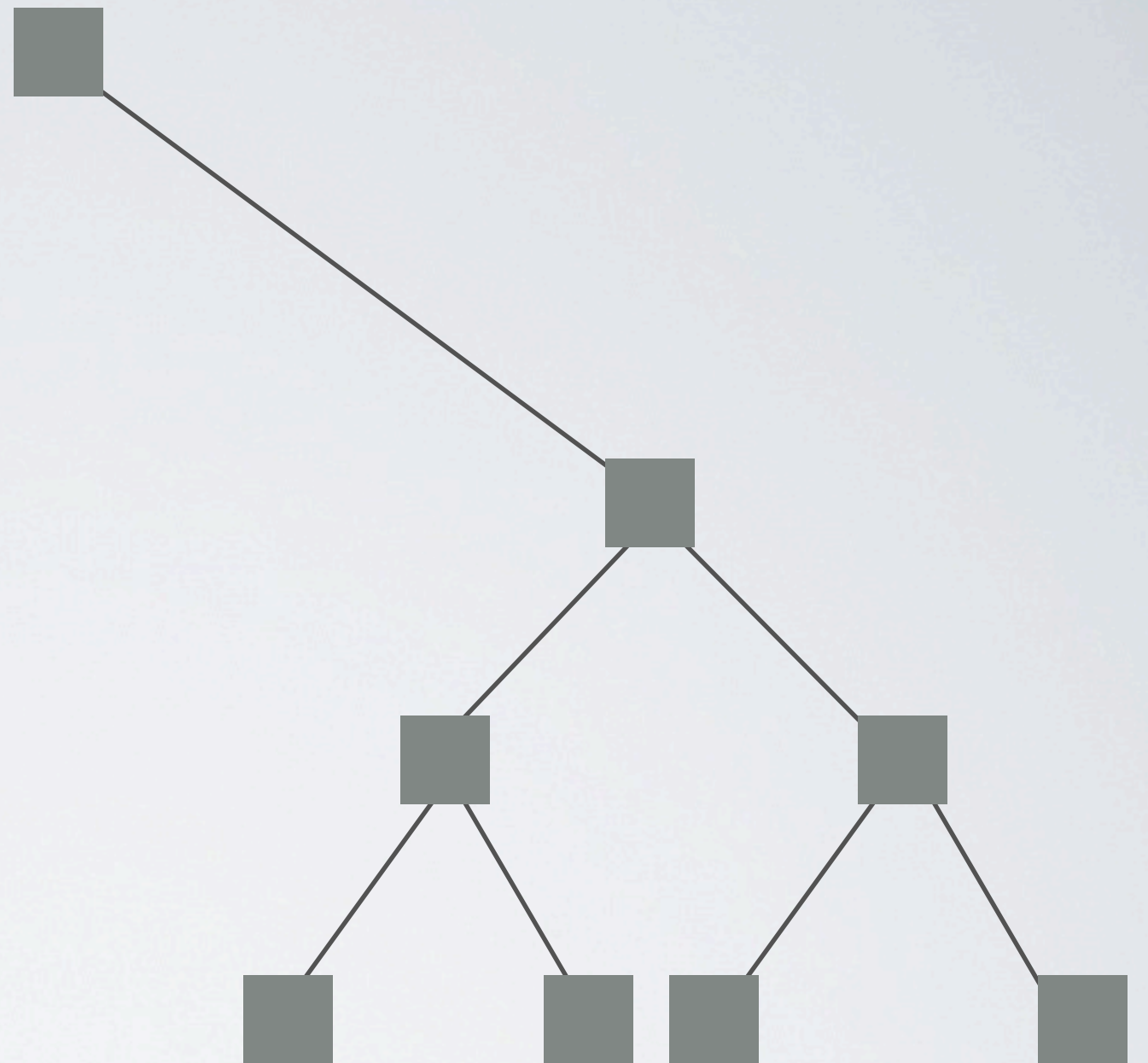
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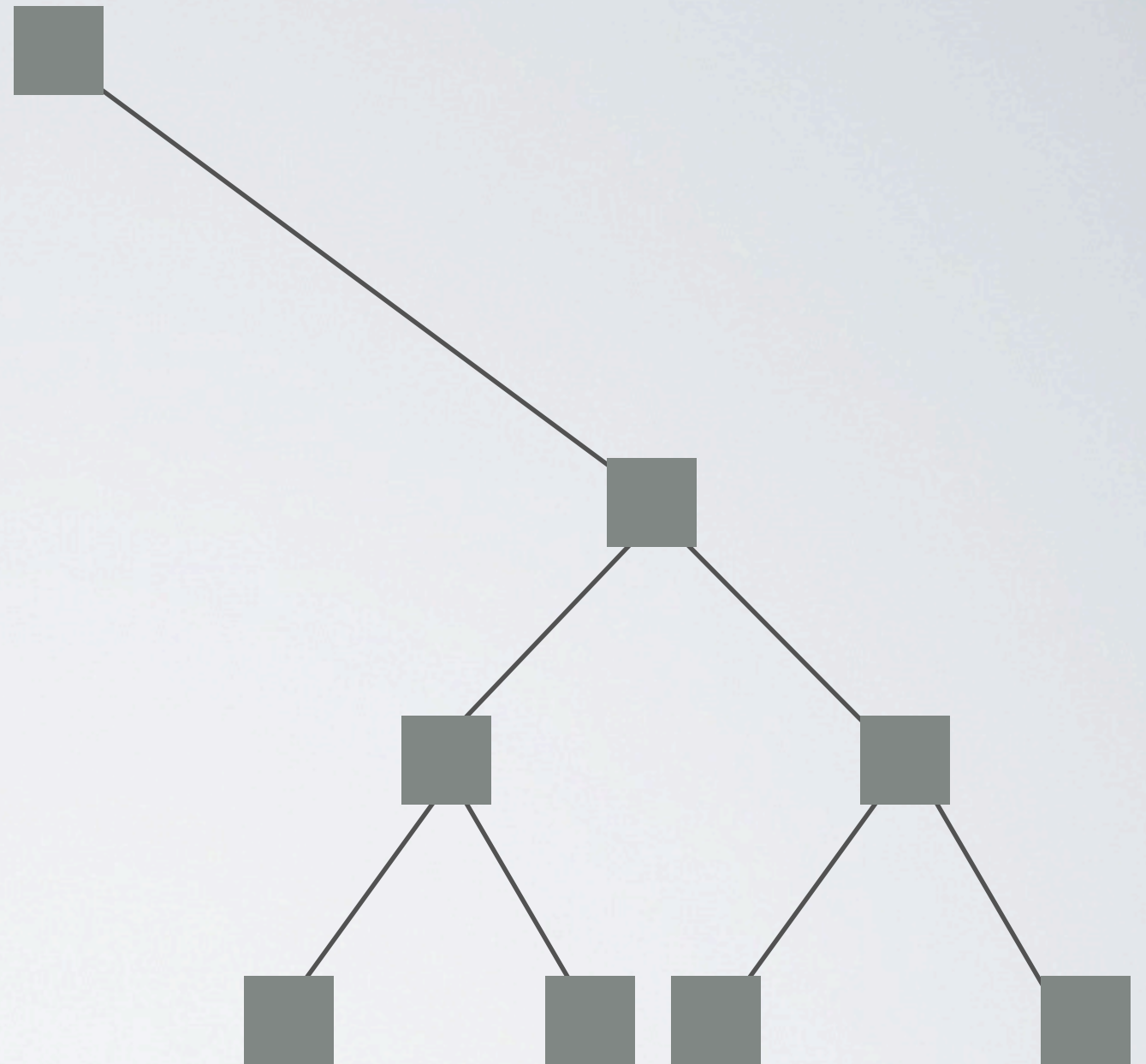








Repeat.

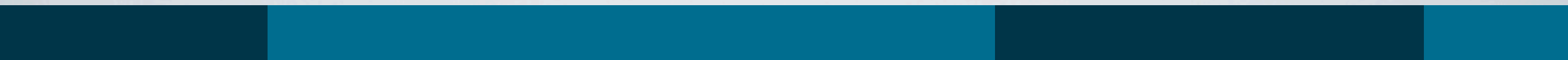


The solution size is proportional to $k^2 \sqrt{\log k}$

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Can be improved to $k(\log k)^{3/2}$ with the help of bootstrapping.

Running the algorithm through
values of k between 1 and n (starting from 1)
leads to an approximation
for the optimization version of the problem.



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Remove at most k vertices such that the remaining graph has no minor models of graphs from F .

Polynomial Kernels?

NP-Complete
(Lewis, Yannakakis)

FPT
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CONJECTURE

THE F-DELETION PROBLEM

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The problem admits polynomial kernels when F contains a planar graph.

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Remove at most k vertices such that the remaining graph has no minor models of graphs from F .

On Claw free graphs

The problem admits  polynomial kernels when F contains a planar graph.

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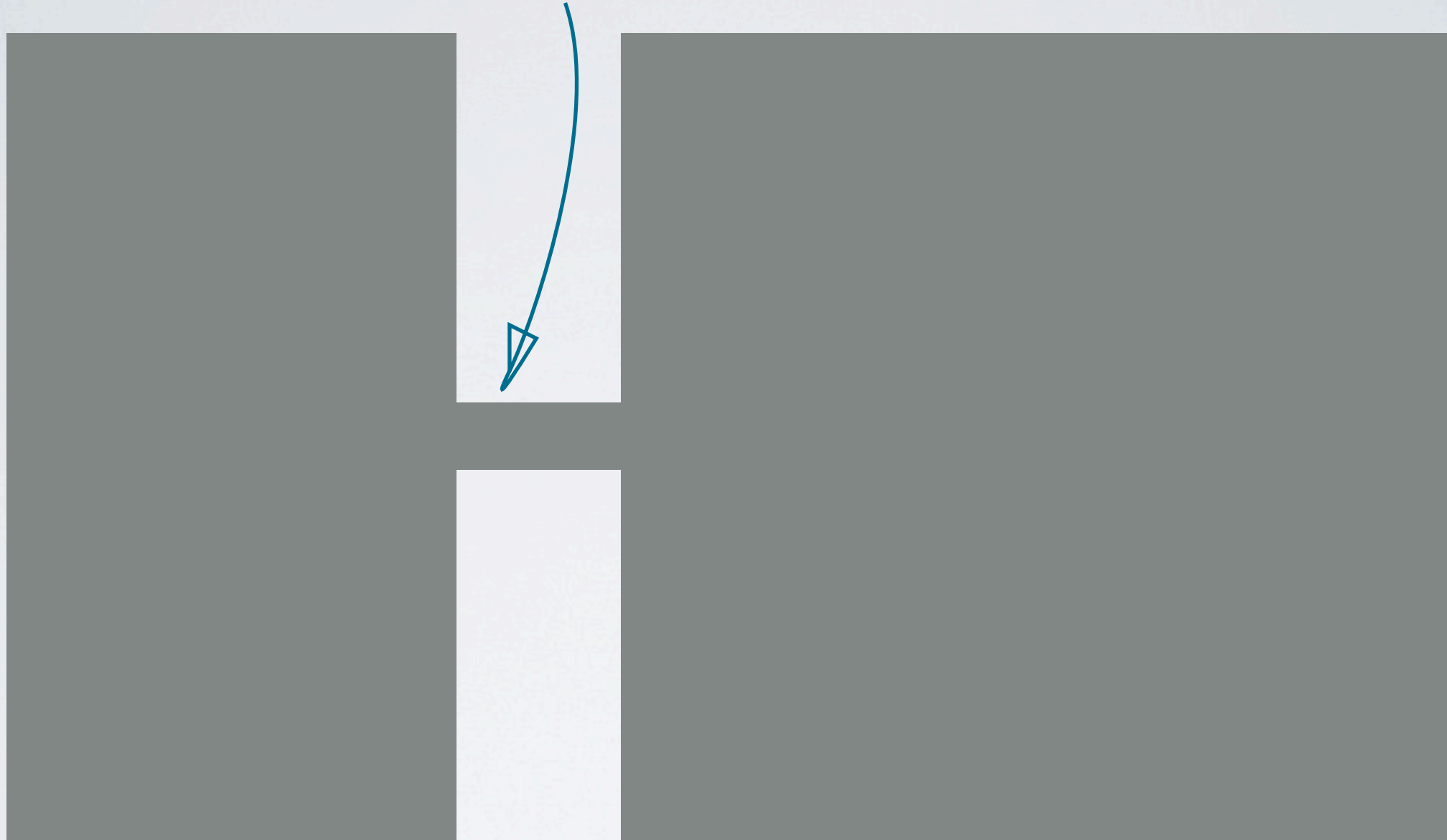
particular

The problem admits polynomial kernels when F contains a planar graph.

PROTRUSION-BASED REDUCTIONS

THE IDEA

A Boundary of Constant Size



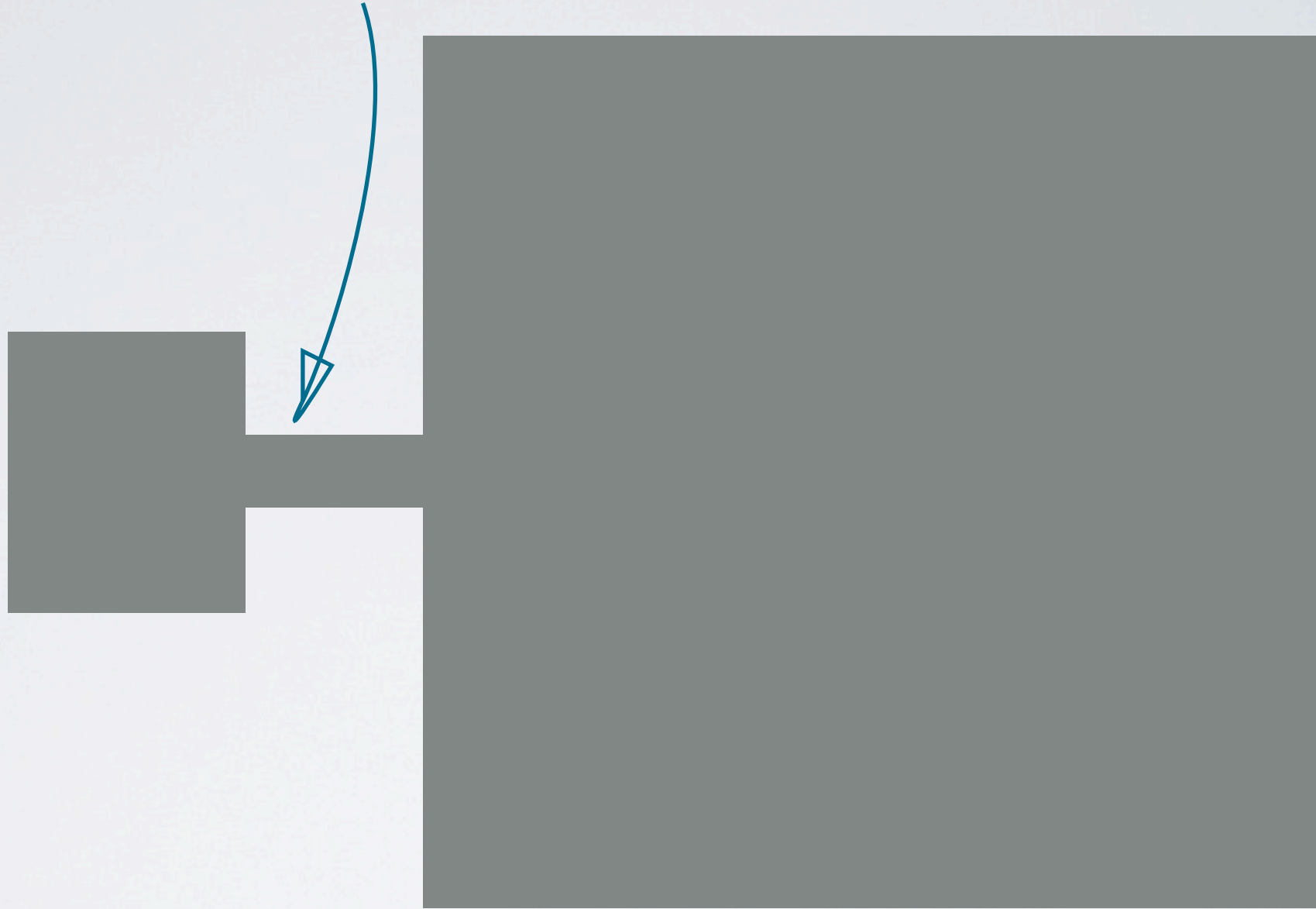
Constant Treewidth

A Boundary of Constant Size

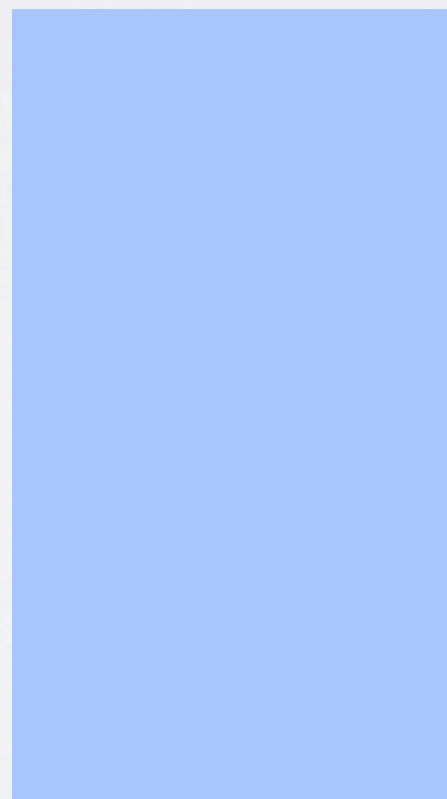


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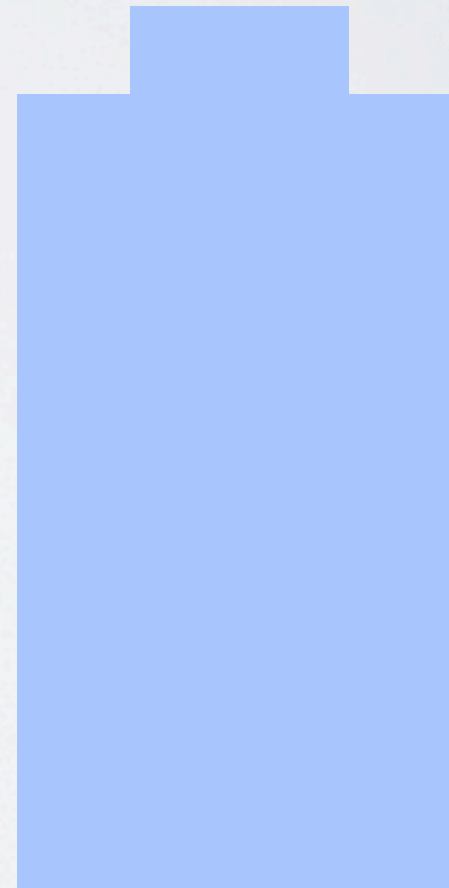


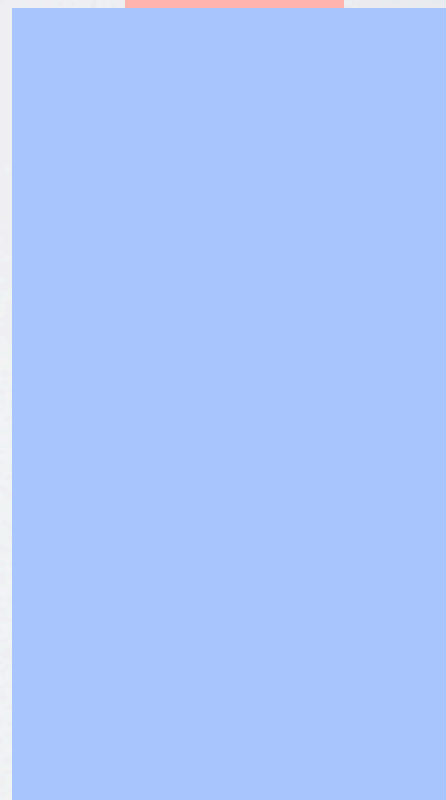
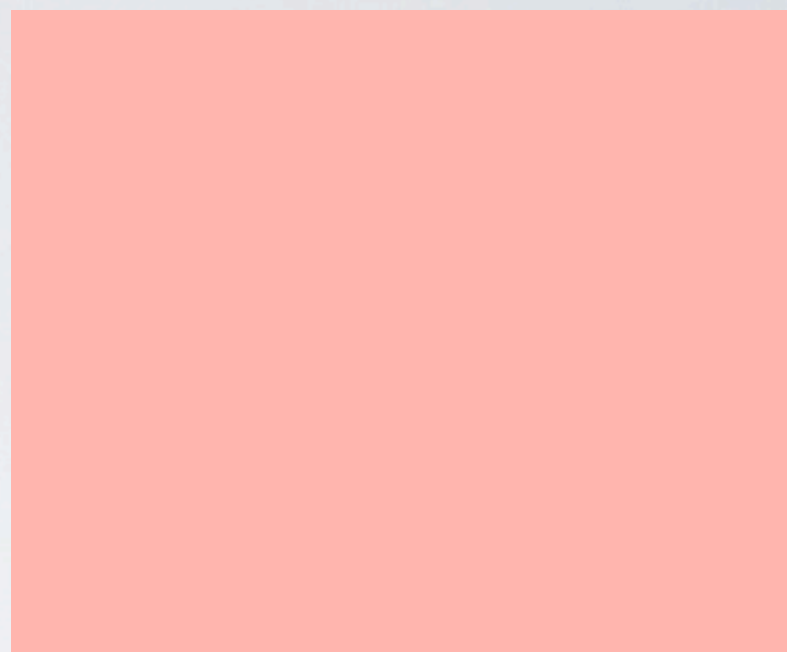
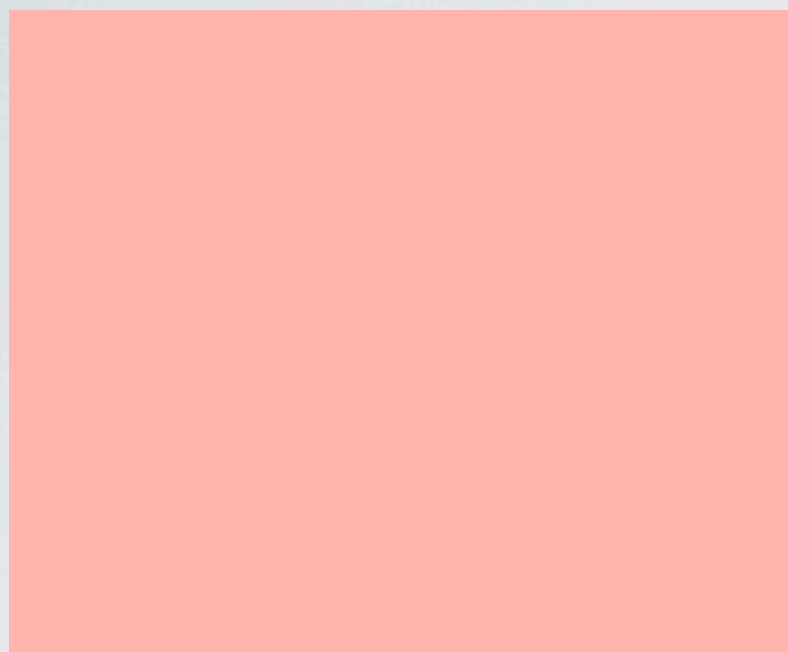
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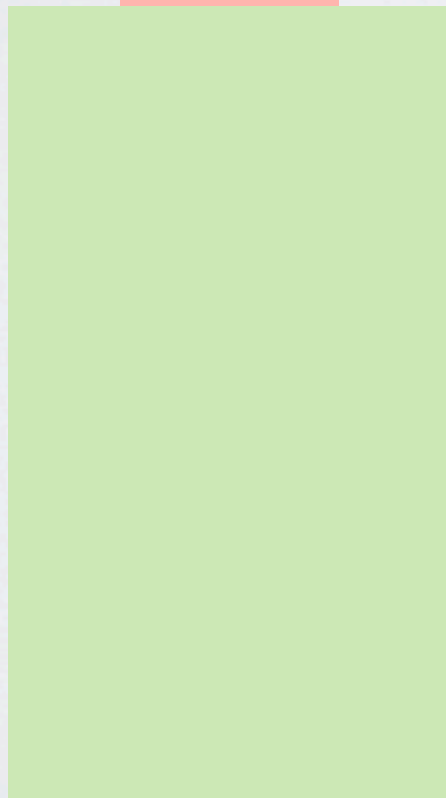




The space of t-boundaried graphs
can be broken up into equivalence classes
based on how they “behave” with
the “other side” of the boundary.







The value of the
optimal solution
is the same
up to a constant.

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For some problems,
the number of equivalence classes is finite,
allowing us to replace protrusions in graphs.

For the protrusion-based reductions to take effect,
we require **subgraphs of constant treewidth**
that are separated from the rest of the graph by
a constant-sized separator.

Approximation Algorithm

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F-hitting Set

Constant Treewidth

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Restrictions like claw-freeness.

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FURTHER DIRECTIONS

- What happens when we drop the planarity assumption?
- What happens if there are graphs in the forbidden set that are not connected?
- Are there other infinite classes of graphs (not captured by finite sets of forbidden minors) for which the same reasoning holds?
- How do structural requirements on the solution (independence, connectivity) affect the complexity of the problem?

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THANK YOU!