

PC Seminar - 29 Sep 2020

On the complexity of

WINNER VERIFICATION & CANDIDATE WINNER  
for multiwinner voting rules

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Joint work with Chinmay Sonar & Palash Dey  
( IJCAI 2020. )

On the complexity of

WINNER VERIFICATION & CANDIDATE WINNER

for multiwinner voting rules

Is  $S \subseteq C$  an  
optimal committee?

Given  $c$  &  $k$ ,  
does  $c$  belong to some  
winning committee of size  $k$ ?

Chamberlin-Courant & Monroe  
voting rules

in the settings of rankings & approval ballots.

## Outline

- Motivation and background; definitions
- Some Complexity Results
- Restricted Domains - a quick detour

# Motivation

## Reasons to hold an election

Choose a president

Find a time for the  
PC Seminar Series

Choose the best paper  
at a conference

Figure out which Indian  
movie is to be sent to the Oscars

Shortlist students for  
a PhD program

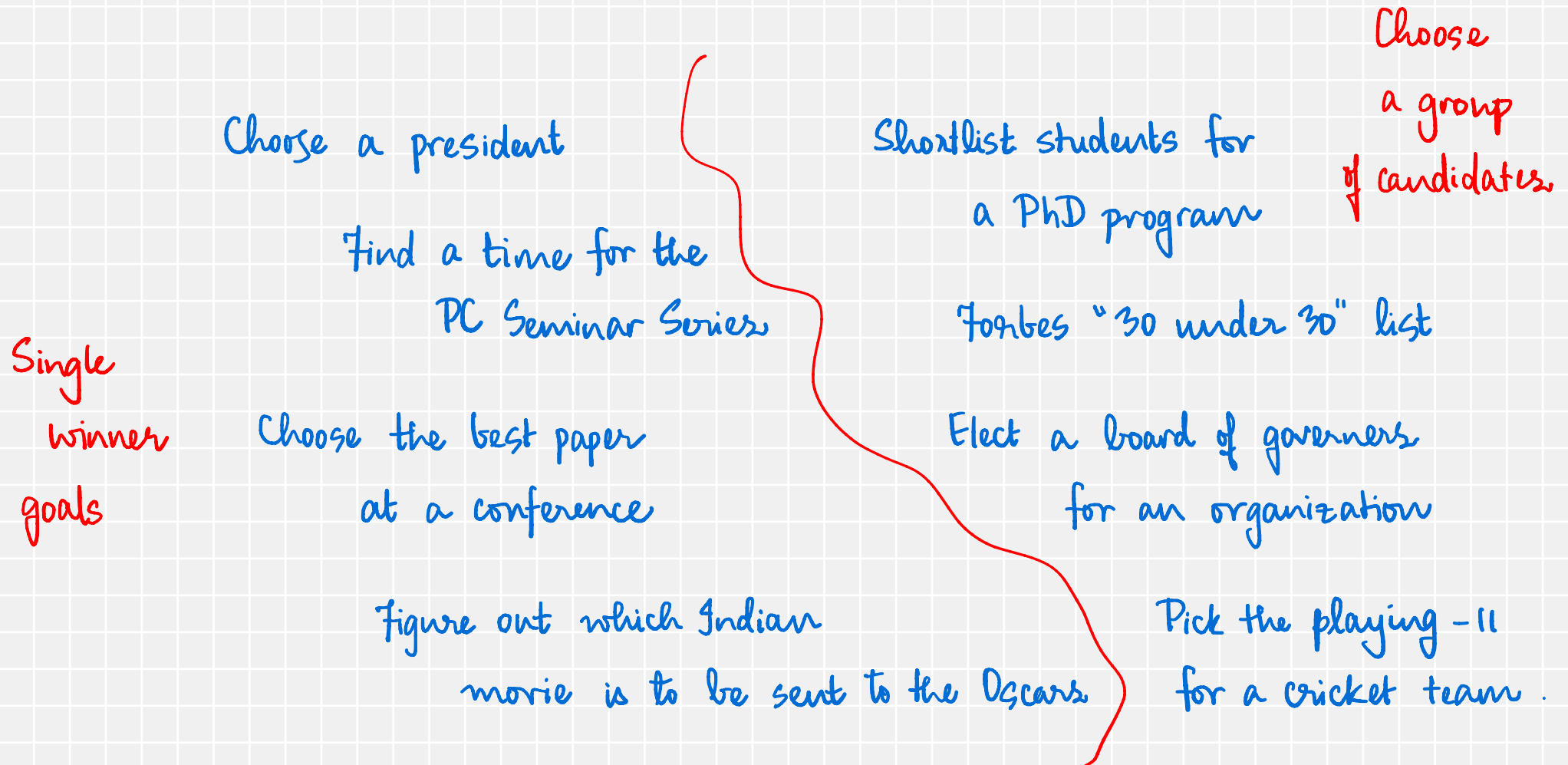
Forbes "30 under 30" list

Elect a board of governors  
for an organization

Pick the playing-11  
for a cricket team.

# Motivation

## Reasons to hold an election



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Single  
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\* Shortlisting

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## Types of multiwinner elections.

Excellence.

Choose the "best  $k$  options".

A cricket team or an award shortlist.

Diversity

Cover as much ground as possible.

Facility location; Movie/product rec.

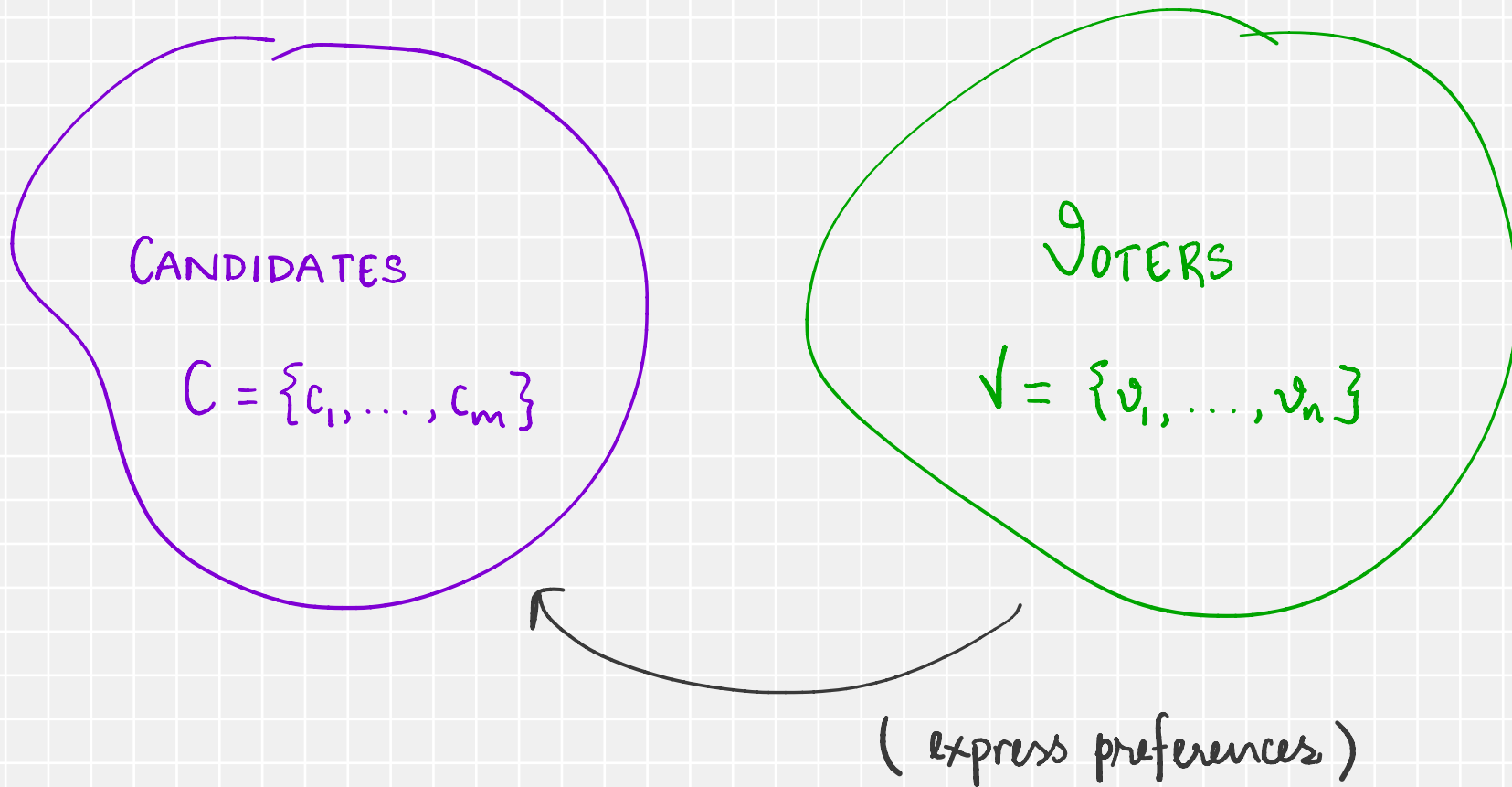
Proportionality.

Make sure everyone is represented fairly.

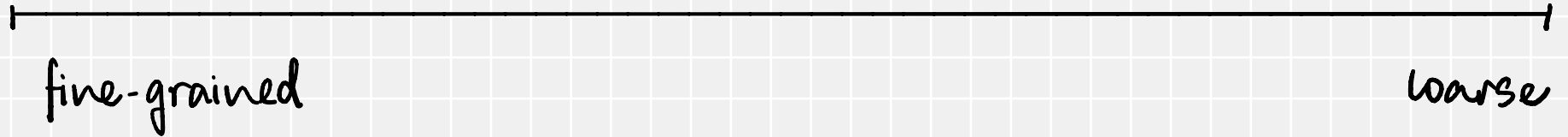
Parliamentary Elections



# Key Elements of a Multiwinner Voting instance.



# Expressing preferences



associate a  
utility with  
each candidate.

associate a  
position with  
each candidate

associate a  
"binary feeling"  
with each candidate

# Expressing preferences

fine-grained

associate a  
utility with  
each candidate.

associate a  
position with  
each candidate

(rankings)

coarse

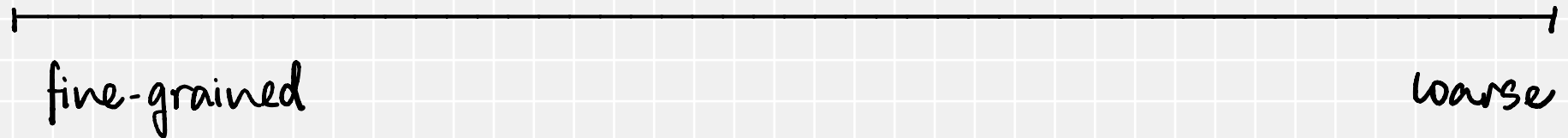
associate a  
"binary feeling"  
with each candidate

(approval ballots)

# Expressing preferences

more expressive

less expressive



fine-grained

coarse

associate a  
utility with  
each candidate.

associate a  
position with  
each candidate  
(rankings)

associate a  
"binary feeling"  
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(approval ballots)

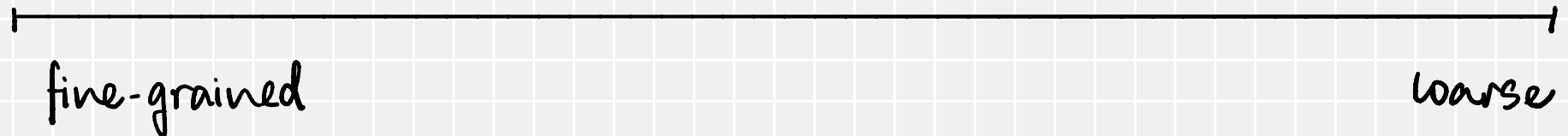
# Expressing preferences

higher cognitive load &  
elicitation complexity

fairly easy  
to specify

more expressive

less expressive



fine-grained

coarse

associate a  
utility with  
each candidate.

associate a  
position with  
each candidate

(rankings)

associate a  
"binary feeling"  
with each candidate

(approval ballots)

# Rankings

|             |       |     |       |     |       |     |       |     |       |     |       |     |       |
|-------------|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|
| $\succ_U :$ | $c_1$ | $>$ | $c_3$ | $>$ | $c_4$ | $>$ | $c_7$ | $>$ | $c_5$ | $>$ | $c_2$ | $>$ | $c_6$ |
| pos :       | 1     |     | 2     |     | 3     |     | 4     |     | 5     |     | 6     |     | 7     |



single-candidate positions.

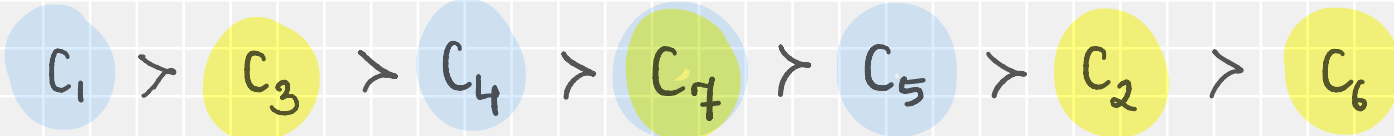
# Rankings

$\succ_U :$        $c_1 > c_3 > c_4 > c_7 > c_5 > c_2 > c_6$

$\text{pos} :$       1      2      3      4      5      6      7

$\text{pos}(S) :$       2      4      6      7

# Rankings

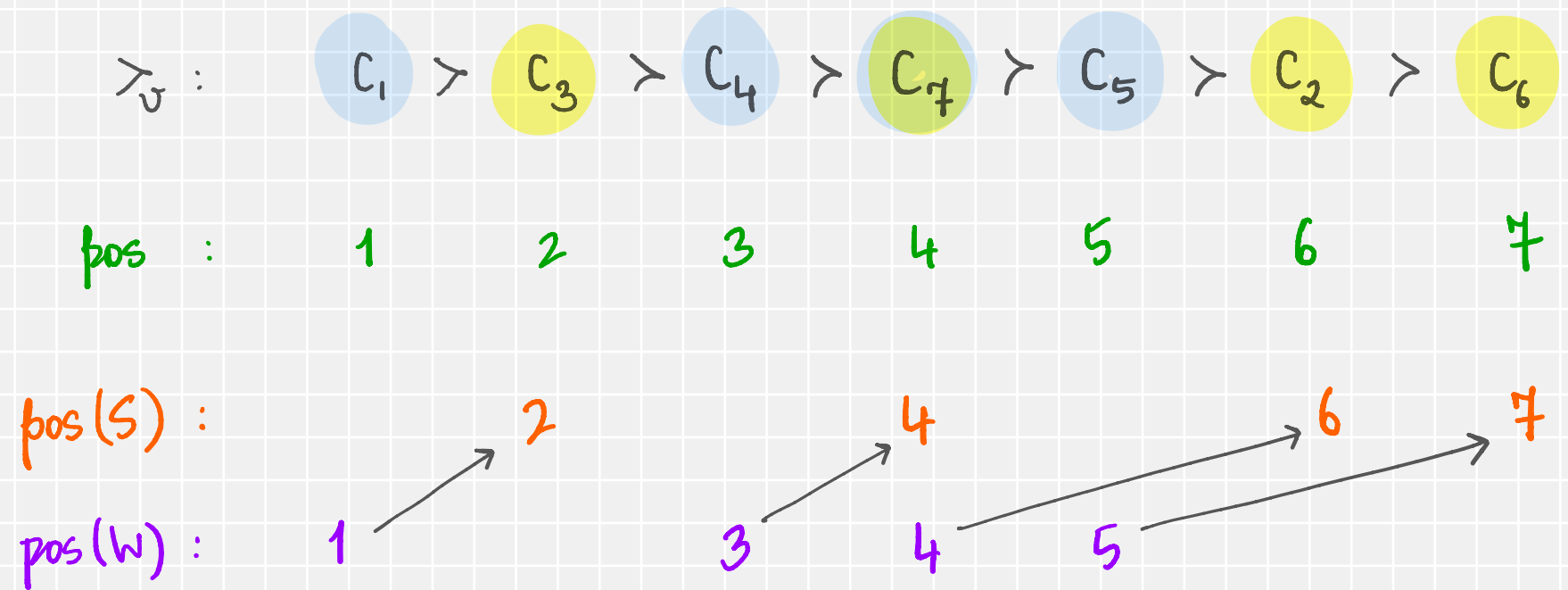
$\succ_U :$    $c_1 > c_3 > c_4 > c_7 > c_5 > c_2 > c_6$

$\text{pos} :$  1 2 3 4 5 6 7

$\text{pos}(S) :$  2 4 6 7

$\text{pos}(W) :$  1 3 4 5

# Rankings

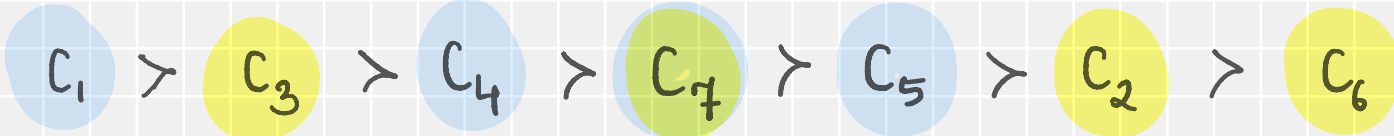


$W$  dominates  $S$ .

# Rankings

positions  $\mapsto$  scores

if I dominates J ;  $\text{score}(I) \geq \text{score}(J)$ .

$\succ_S :$    $C_1 > C_3 > C_4 > C_7 > C_5 > C_2 > C_6$

pos :  1 2 3 4 5 6 7

pos(S) :  1 2 3 4 5 6 7

pos(W) :  1 2 3 4 5 6 7

W dominates S.

# Rankings

positions  $\mapsto$  scores

if I dominates J ;  $\text{score}(I) \geq \text{score}(J)$ .

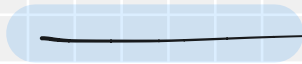
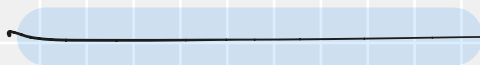
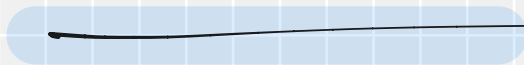
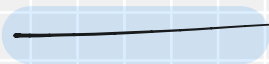
$\succ_J :$   $C_1 > C_3 > C_4 > C_7 > C_5 > C_2 > C_6$

(m-i) Borda Score :  $\begin{matrix} 6 & 5^* & 4 & 3 & 2 & 1 & 0 \end{matrix}$

$\text{pos}(S) :$   $\begin{matrix} & 2 & & 4 & & 6 & 7 \end{matrix}$

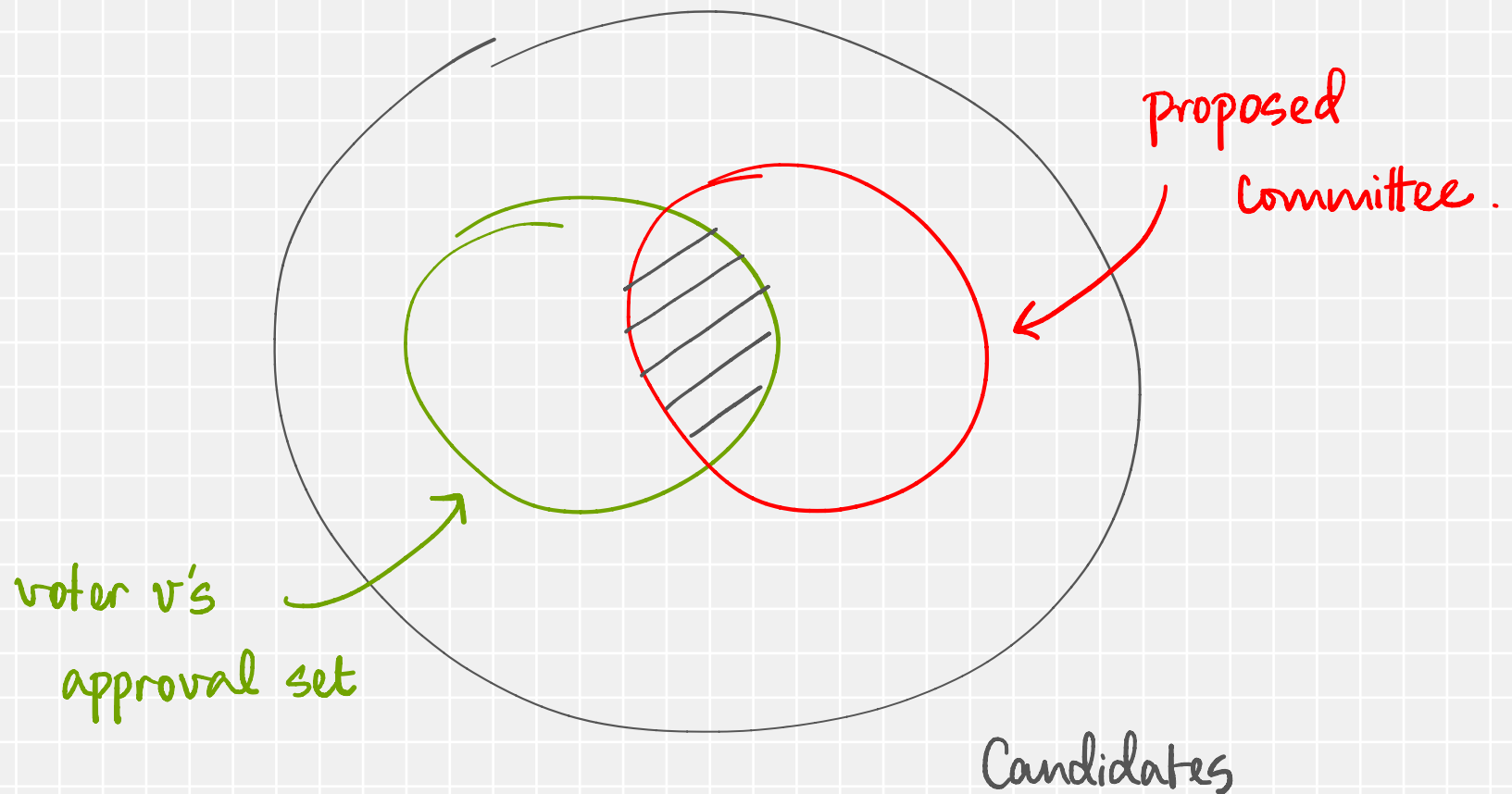
Chamberlin-Courant Score  $\mapsto$  Borda score of the highest-ranked candidate.

$\leq 92$



Approval Ballots

$$\text{Score}(w) = \sum_{i \in \text{voters}} w(|w \cap c_i|)$$

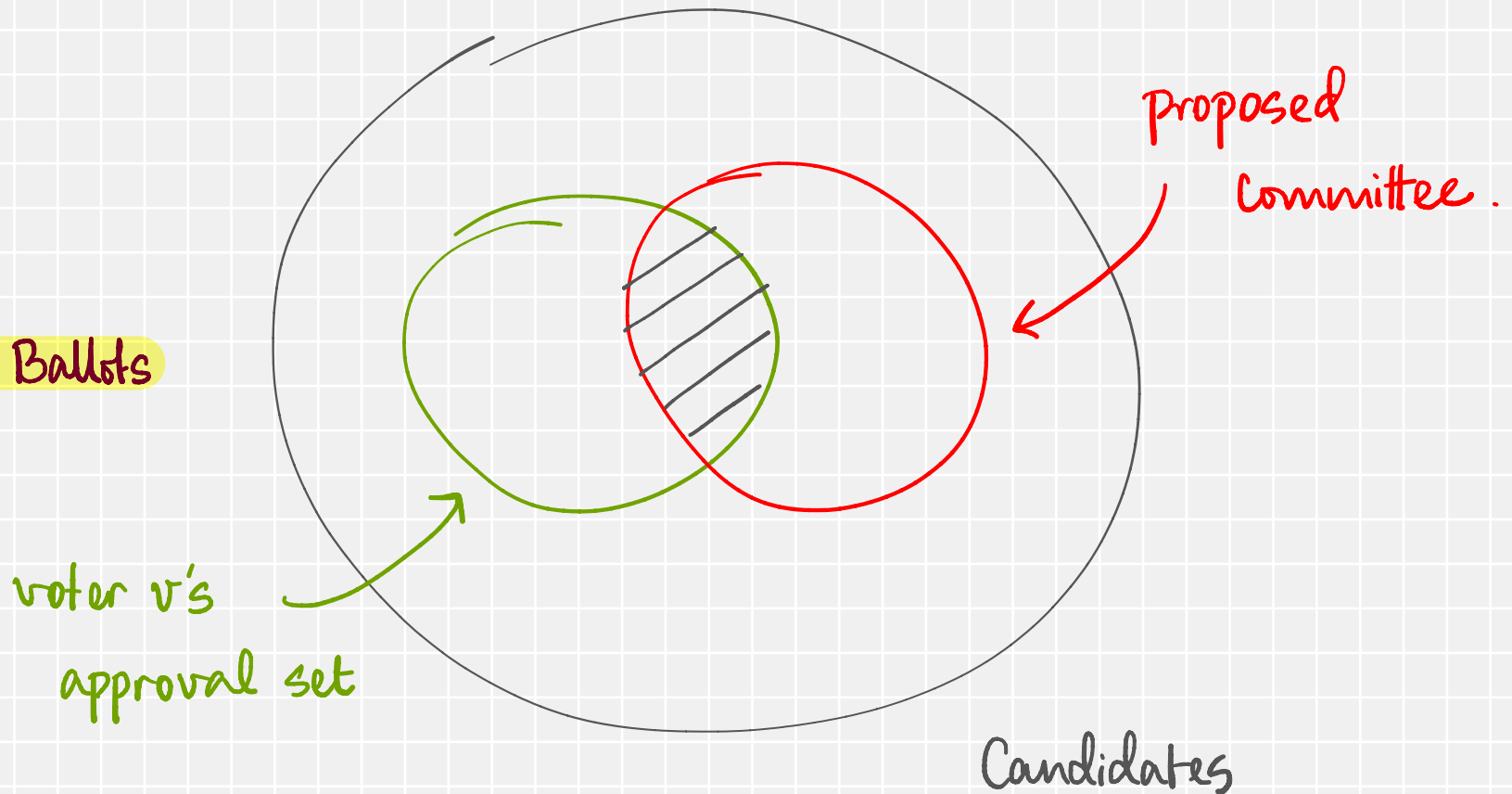


## Approval Ballots

$$\text{Score}(w) = \sum_{i \in \text{voters}} w(|w \cap c_i|)$$

$$w(x) = x$$

Approval Ballots



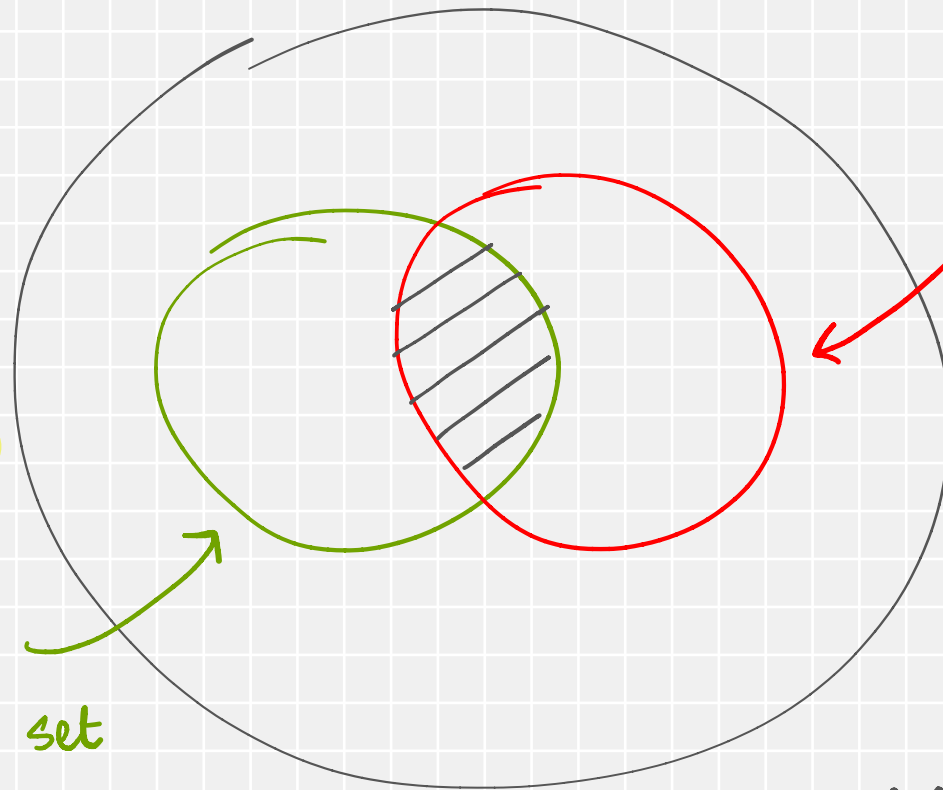
# Approval Ballots

$$\text{Score}(w) = \sum_{i \in \text{voters}} w(|w \cap c_i|)$$

$$w(x) = \min\{1, x\}$$

Chamberlin-Courant

voter  $v$ 's  
approval set



proposed  
committee.

Candidates

## Natural Questions

1. Find a winning committee .
2. Verify if a committee is winning .
3. Check if a candidate belongs to a winning committee .
4. Given a committee, is it "robust" ?

Chamberlin-Courant  
wrt approval ballots.

$\equiv$  Max-Cover problem<sup>★</sup>.

W-hard in  $k$   
even for restricted  
scenarios;

FPT-approximation  
algorithms are  
known.

Candidates



$(U, \mathcal{F}, k)$ : Choose  $k$  sets to cover  
as much of  $U$  as possible.



voters



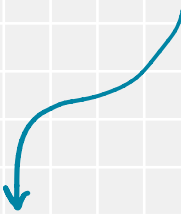
committee  
size

CC-Winner-Verification is co-NP hard.

c-3 Hitting Set  $\mapsto (U, \mathcal{F}, k)$

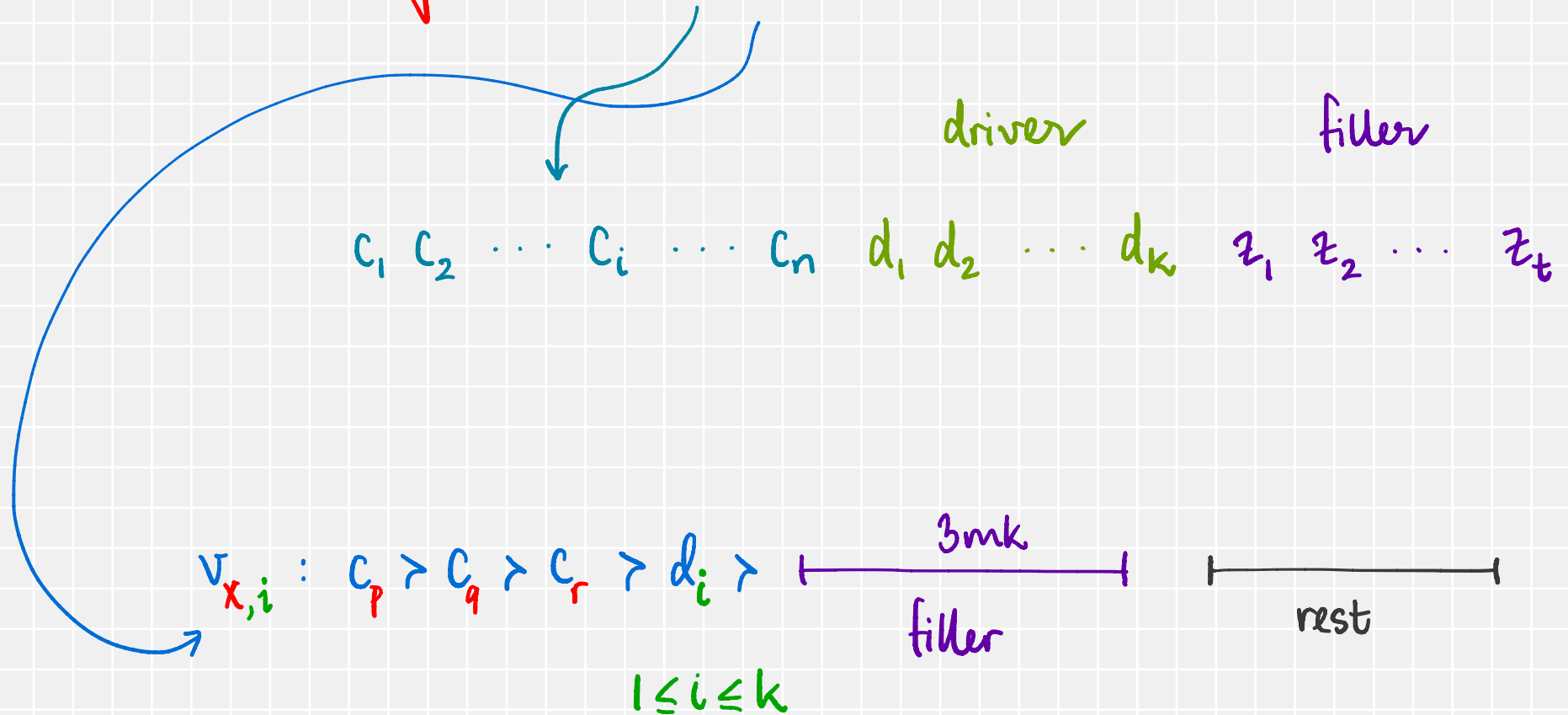
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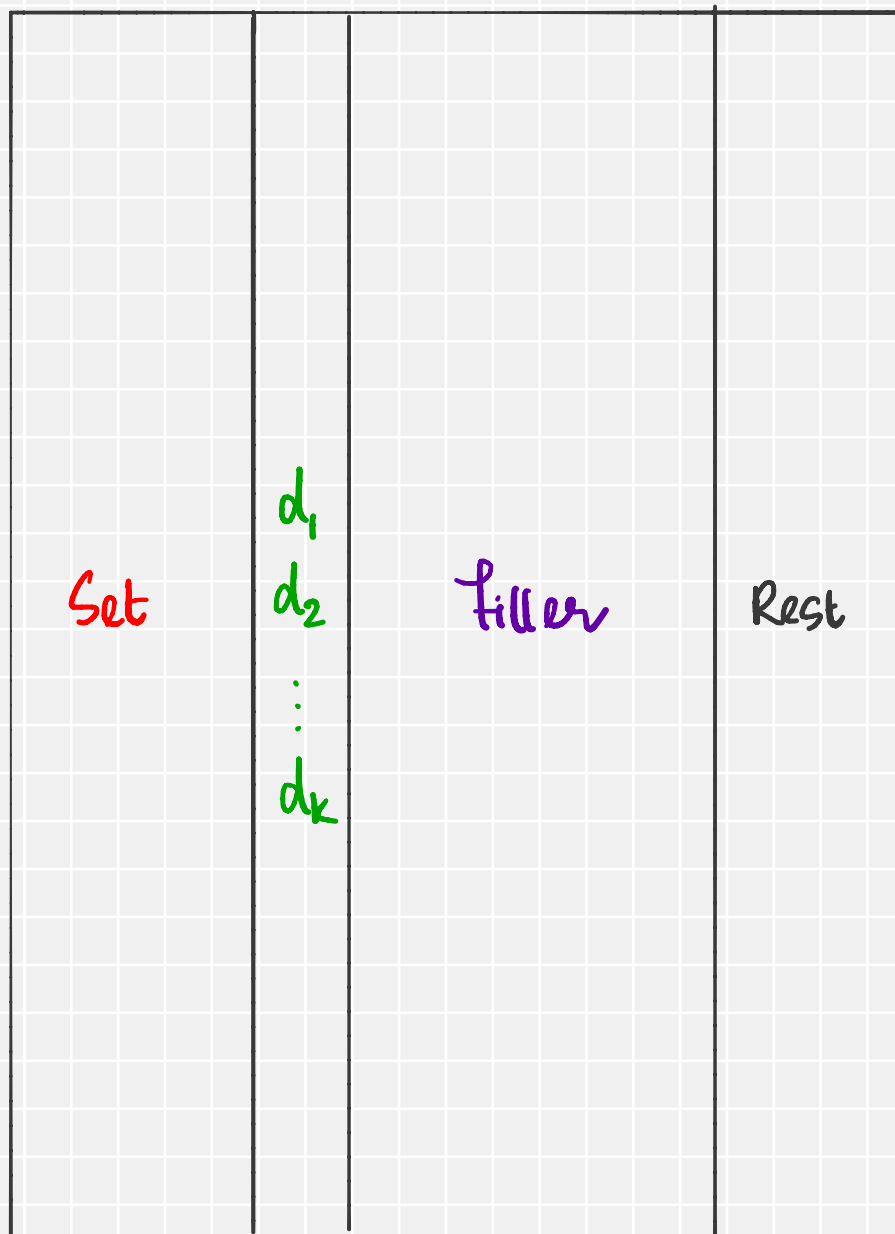
  
 $c_1 \ c_2 \ \dots \ c_i \ \dots \ c_n$     $d_1 \ d_2 \ \dots \ d_k$     $z_1 \ z_2 \ \dots \ z_t$   
driver                      filler

CC-Winner-Verification is co-NP hard.

C-3 Hitting Set  $\mapsto (U, \mathcal{F}, k)$



every set  
has a  
block of  
k voters



No HS of size  $\leq k$

iff  $D$  is a winning  
committee

CC-Score  
of a hitting set  
 $\leq 2mk$

CC-Score of  
 $\{d_1, \dots, d_k\}$   
 $= 3mk$

Vertex-Lover Member.

$G; v \in V$

Does there exist a minVC  
containing  $v$ ?

$\textcircled{4}_2^P$ -complete

\* an useful starting point

## Restricted Domains

Rankings: Single-peaked & single crossing domains

Approval ballots: interval-like domains

Distance-to-triviality parameterizations