

LINEAR ALGEBRA METHODS IN COMBINATORICS

Based on Gowers's Blog Post:

<https://gowers.wordpress.com/2008/07/31/dimension-arguments-in-combinatorics/>

The Story of

ODDTOWN & EVENTOWN



A B C D E F G H I J



Inhabitants of **eventown** keep themselves busy with clubs.



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Club formation, however, is subject to some regulations.



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Every club should be distinct and have an **even number of members**.

Between any two clubs, the number of **common members is even**.



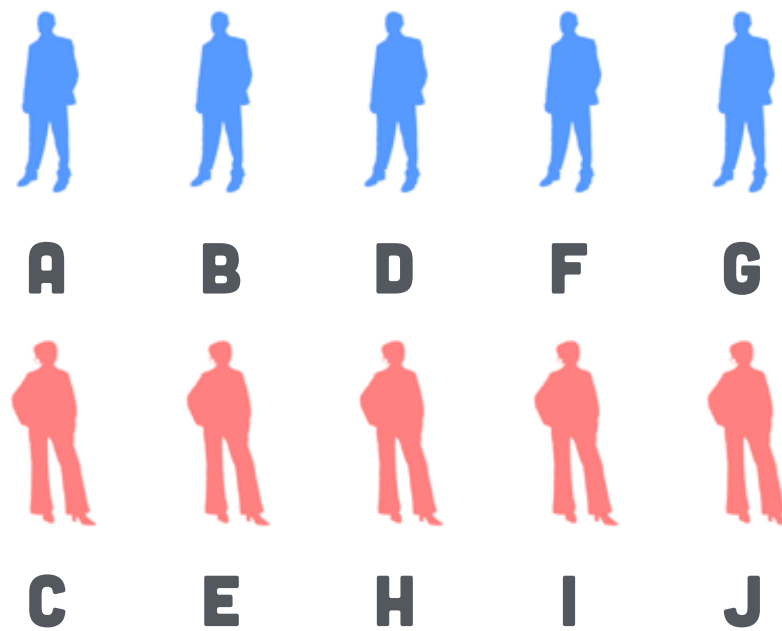
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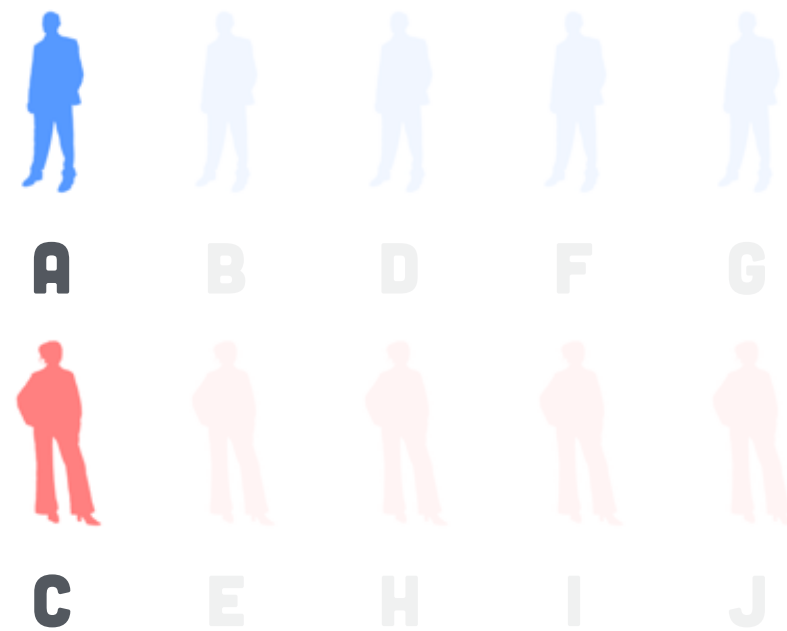
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Members of eventown want to form **as many clubs** as possible.



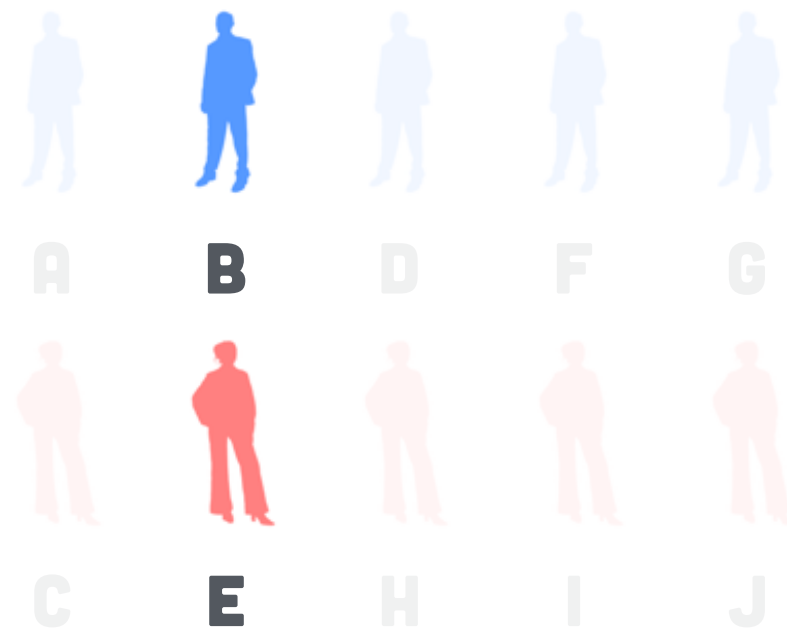
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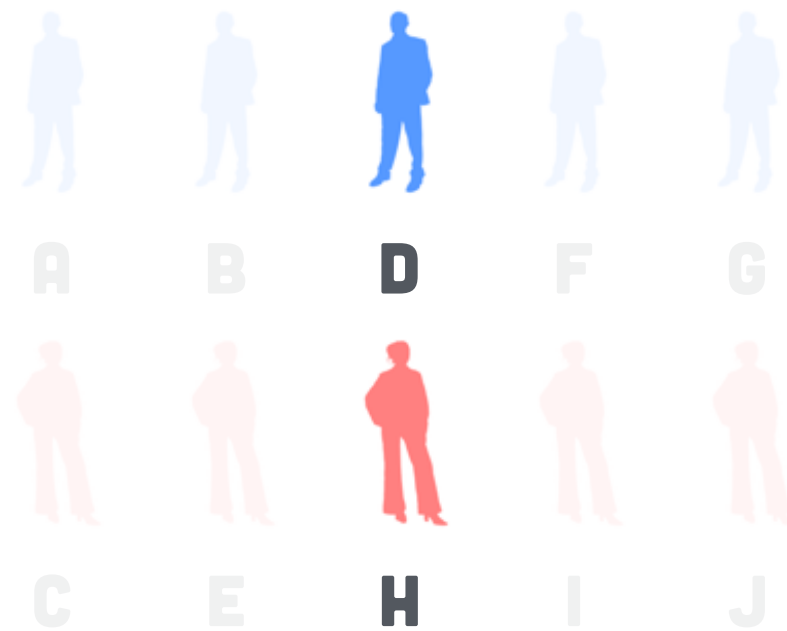
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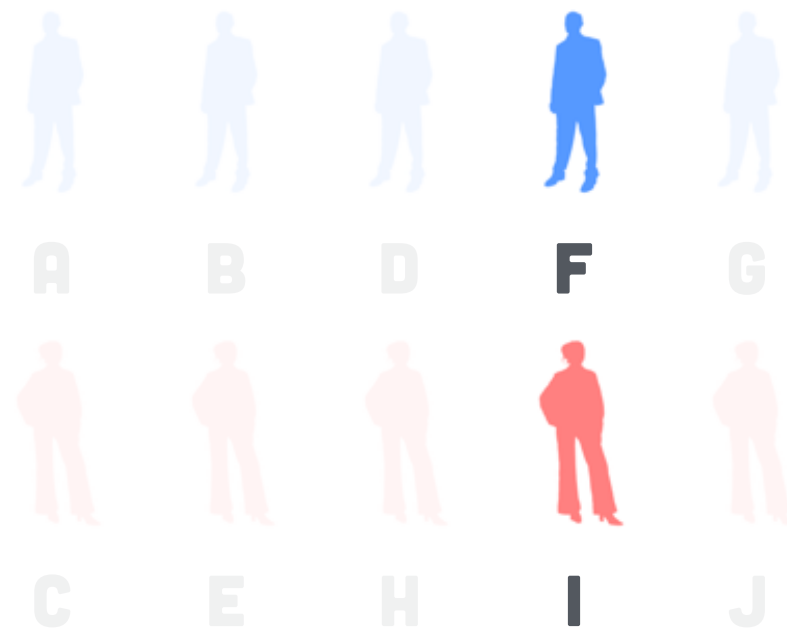
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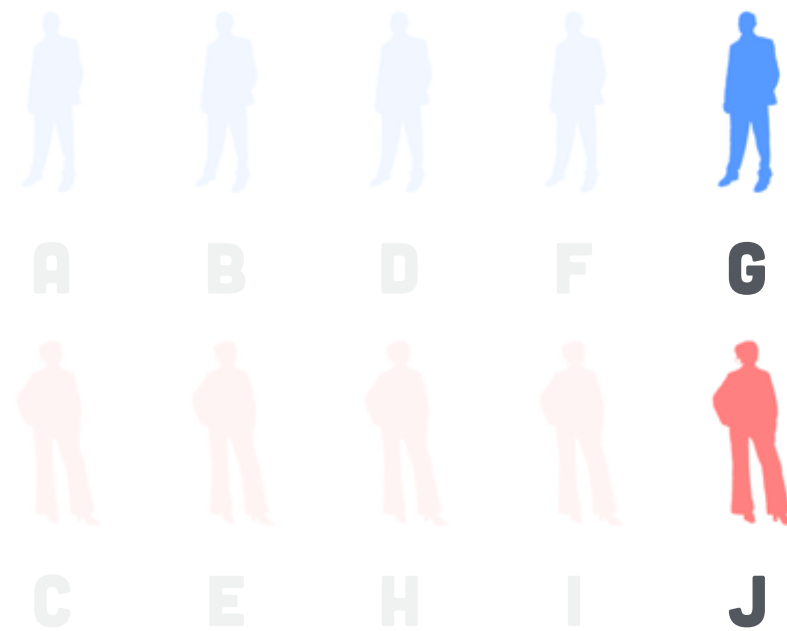
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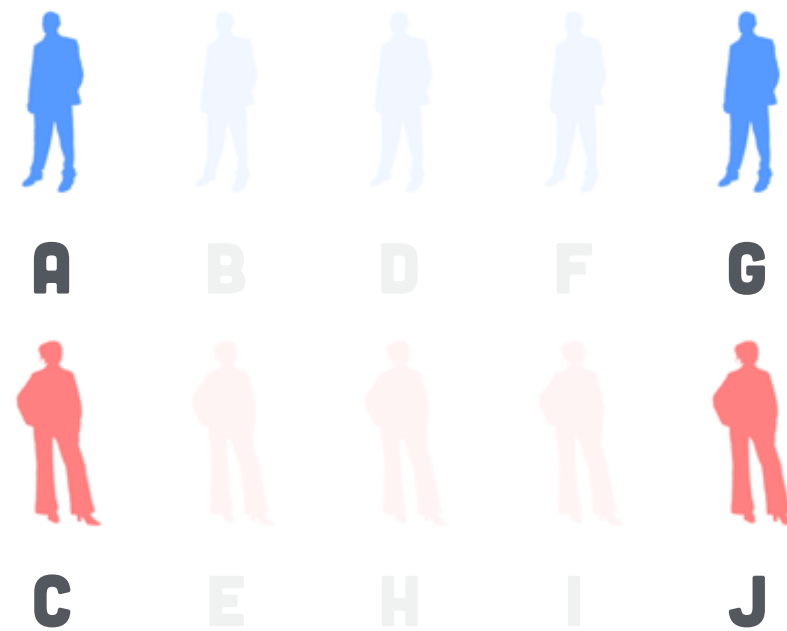
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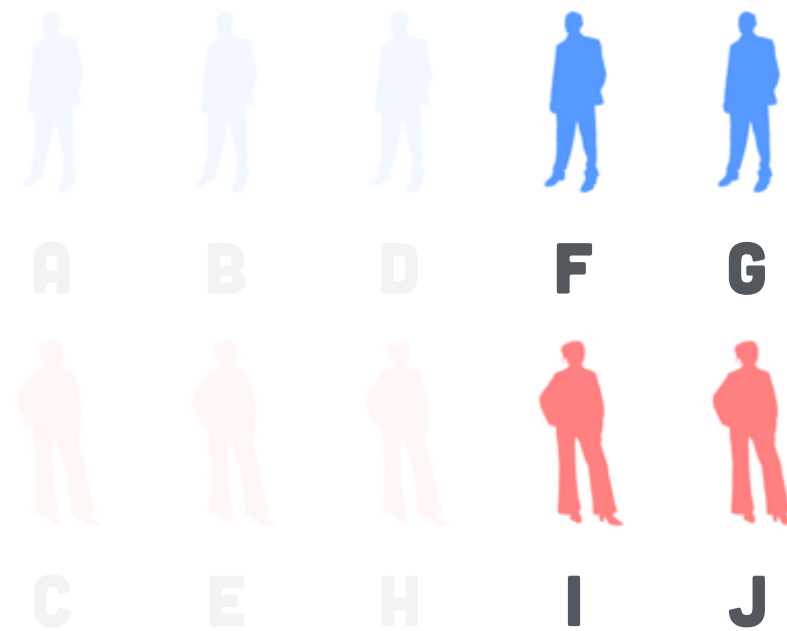
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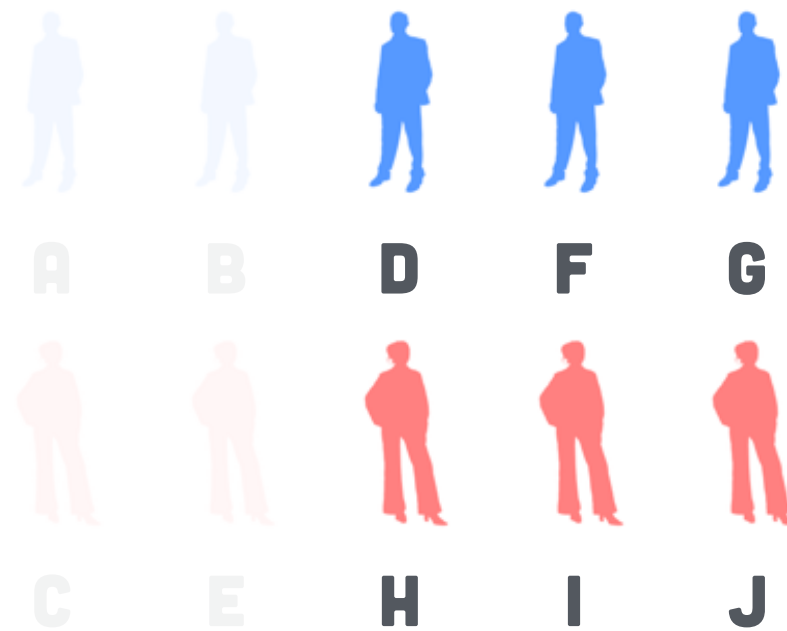
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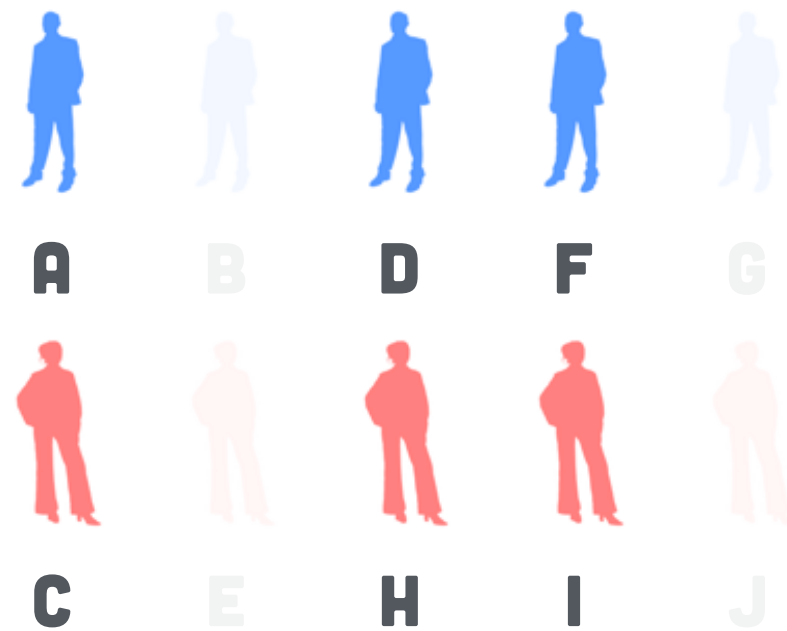
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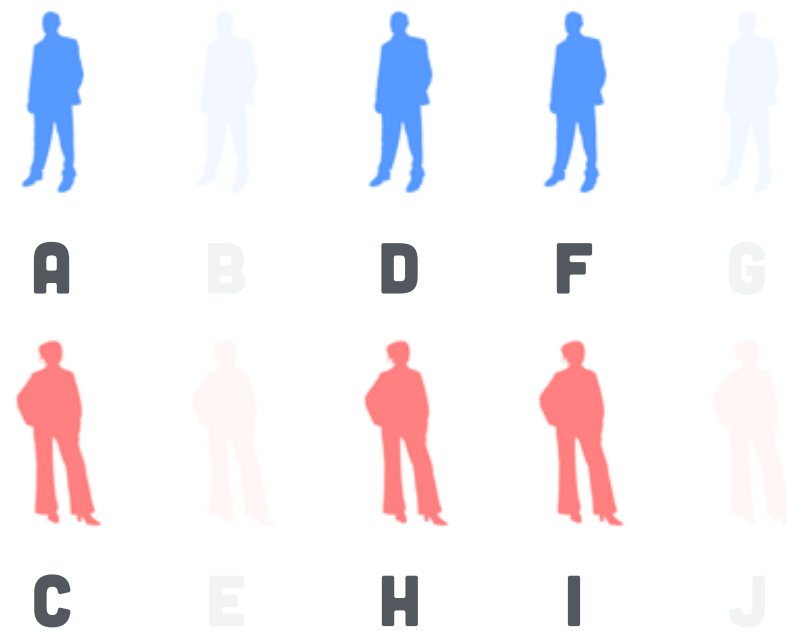
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n people : $n/2$ couples : $2^{(n/2)}$ clubs

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Inhabitants of **oddtown** keep themselves busy with clubs.

Club formation, however, is subject to some regulations.

Every club should be distinct and
have an **odd number of members**.

Between any two clubs, the number
of **common members is even**.

Members of odd town want to form **as many clubs** as possible.



Each to his own: can form n “singleton” clubs.

Can we do better?

Every club should be distinct and have an **odd number of members**.

Between any two clubs, the number of **common members is even**.

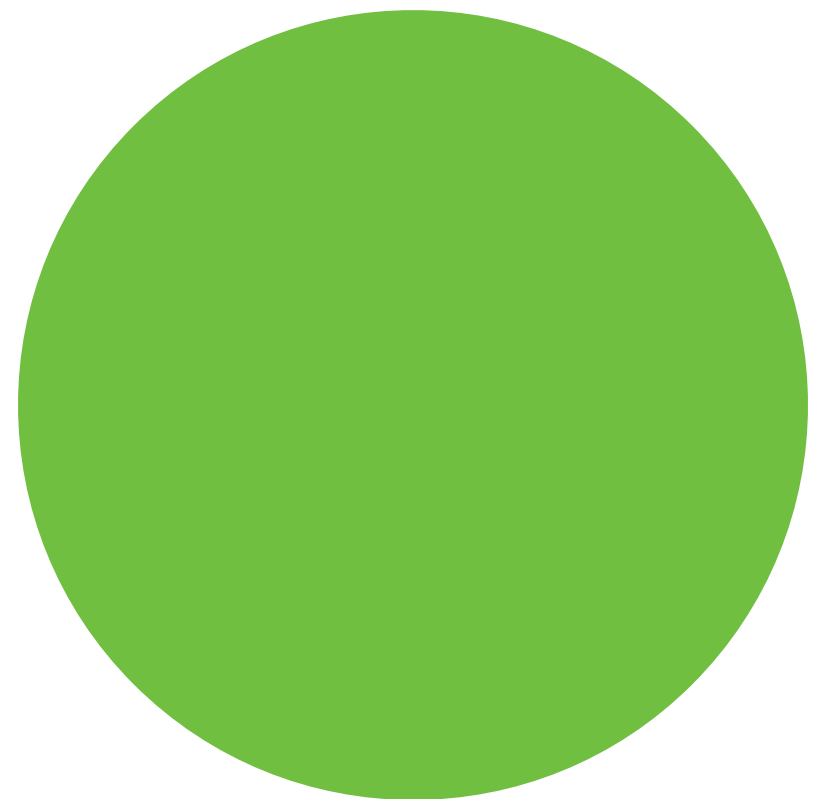
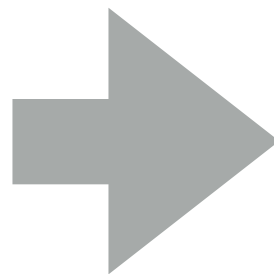
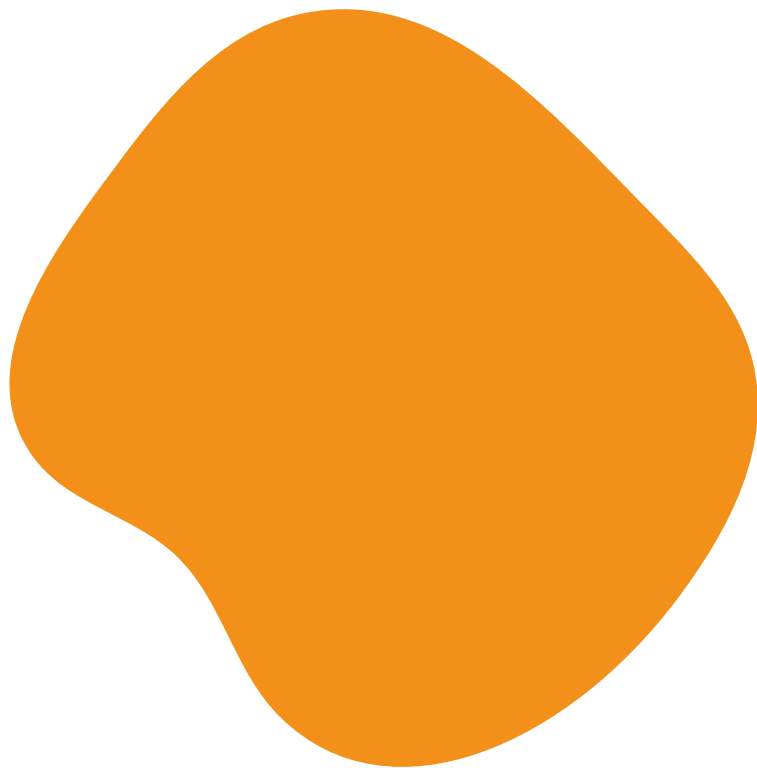
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DIMENSIONALITY PARADIGM

A collection X
for whose size
we would like
an upper bound.

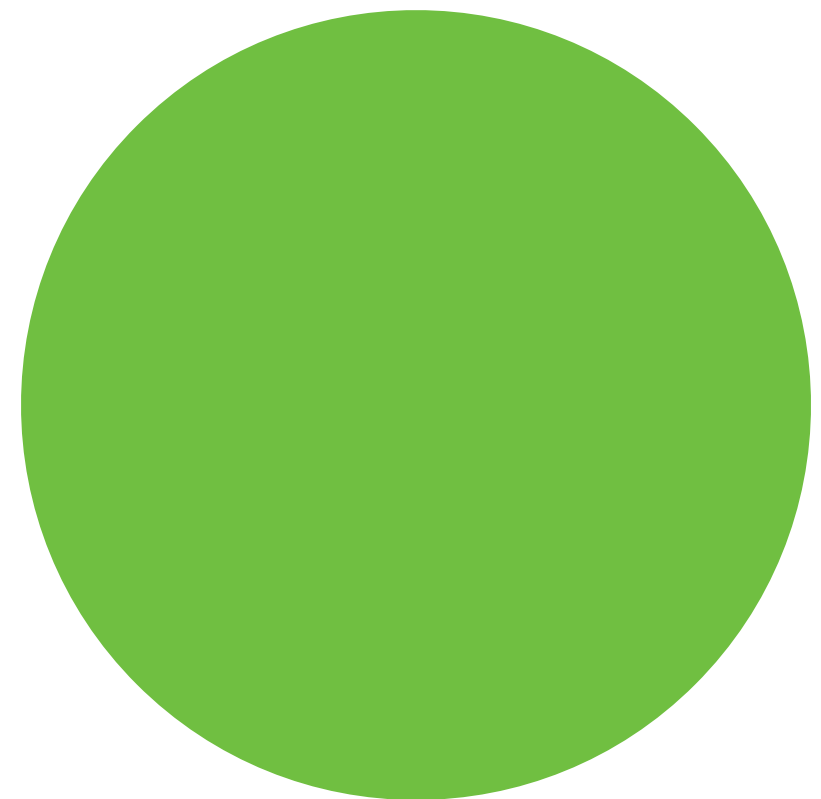
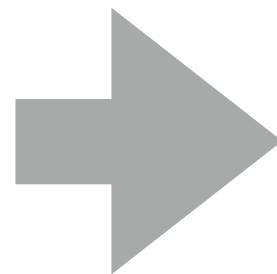
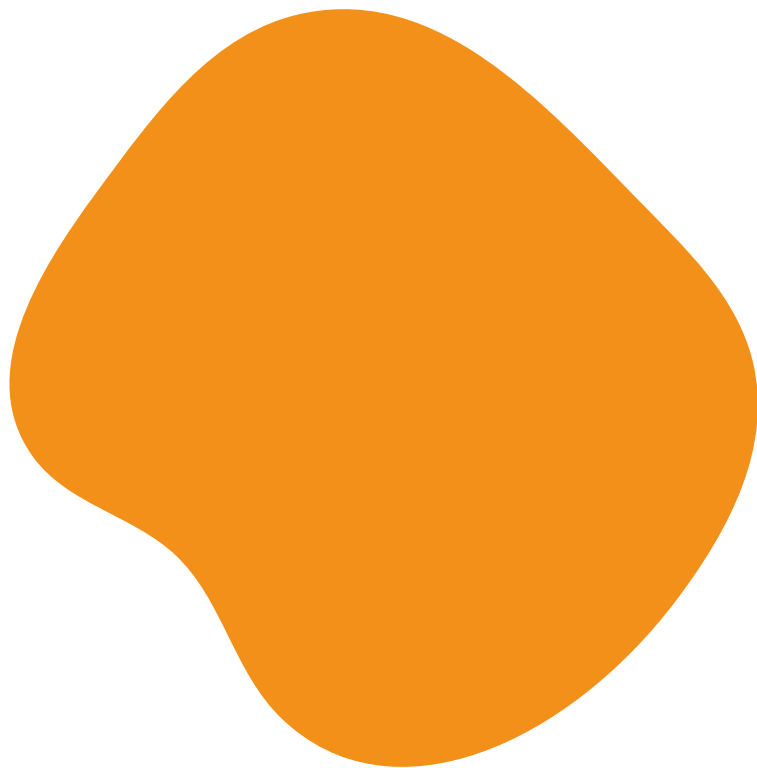
A linear space Y
where objects of X correspond
to **linearly independent**
vectors.



The size of X is now at most the dimension of Y .

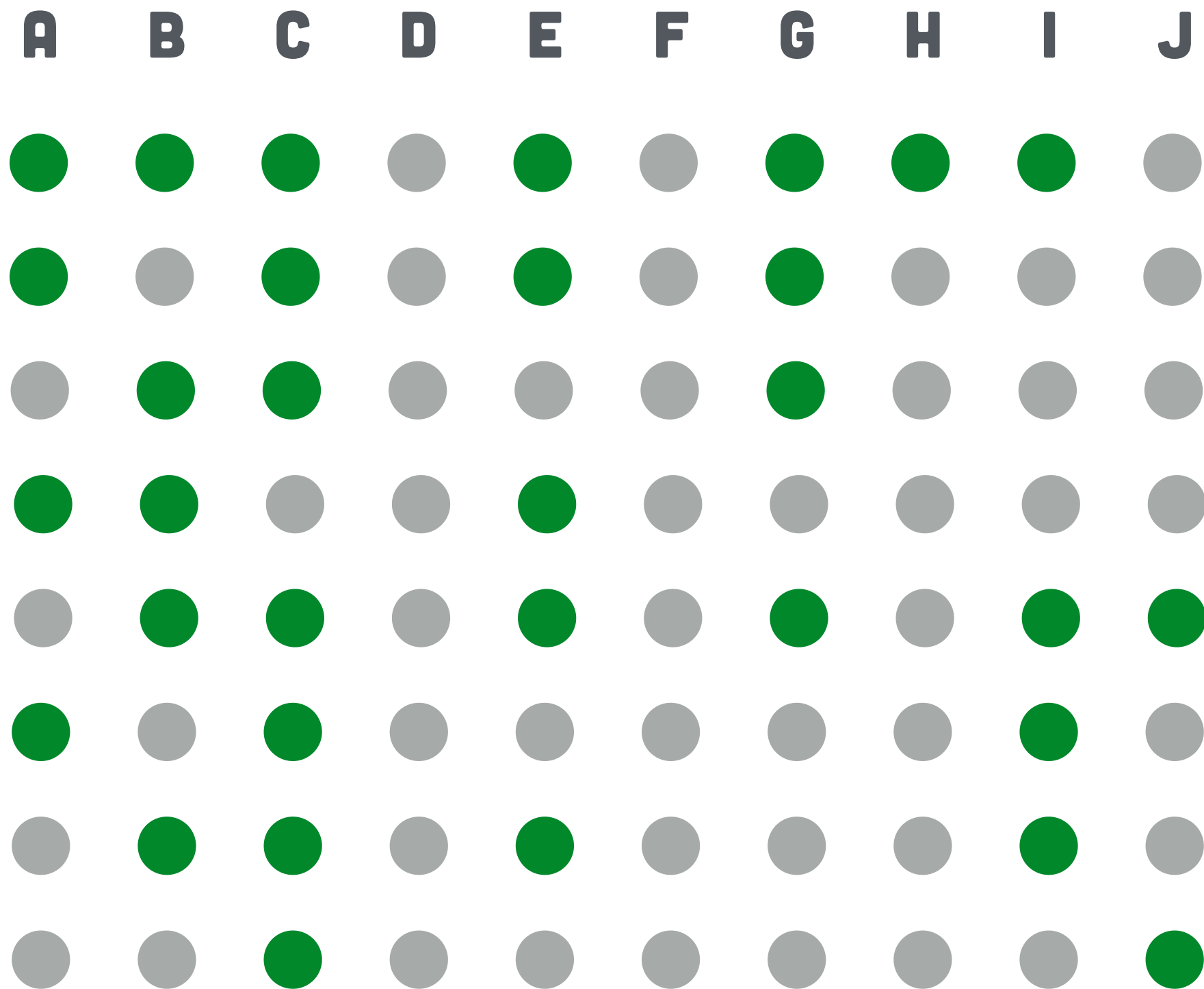
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Map each club to its characteristic vector. We're going to $\mathbb{F}_2^n$.

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Claim: If the clubs were indeed following oddtown rules
— sizes odd, intersections even —
then these characteristic vectors are linearly independent.



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$$\mathbf{a}_1 \mathbf{f}_1 + \mathbf{a}_2 \mathbf{f}_2 + \dots + \mathbf{a}_i \mathbf{f}_i + \dots + \mathbf{a}_k \mathbf{f}_k = \mathbf{0}$$



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Multiply throughout by $\mathbf{f}_i$ to obtain:



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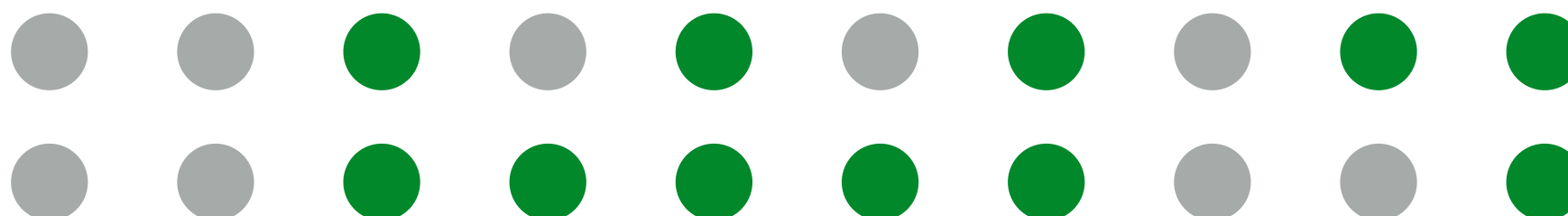


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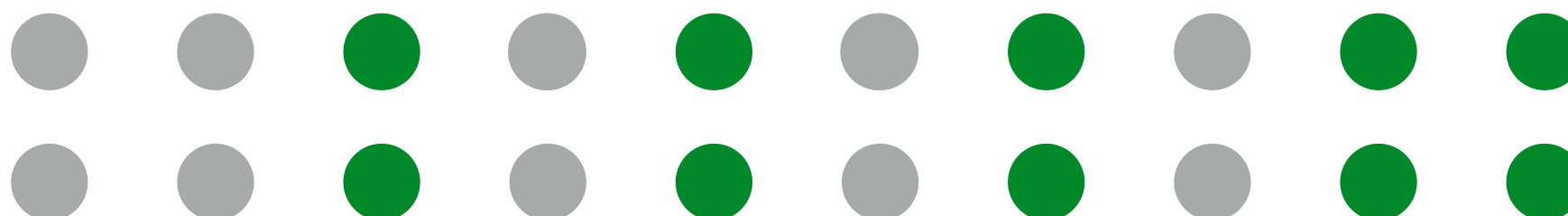


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$$a_i = 0$$



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Between any two clubs, the number of **common members is even**.

Members of odd town want to form **as many clubs** as possible.



Each to his own: can form n “singleton” clubs.

The best we can hope for!

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The Story of

GOSSIP TOWN...



Inhabitants of **gossiptown** keep themselves busy with clubs.

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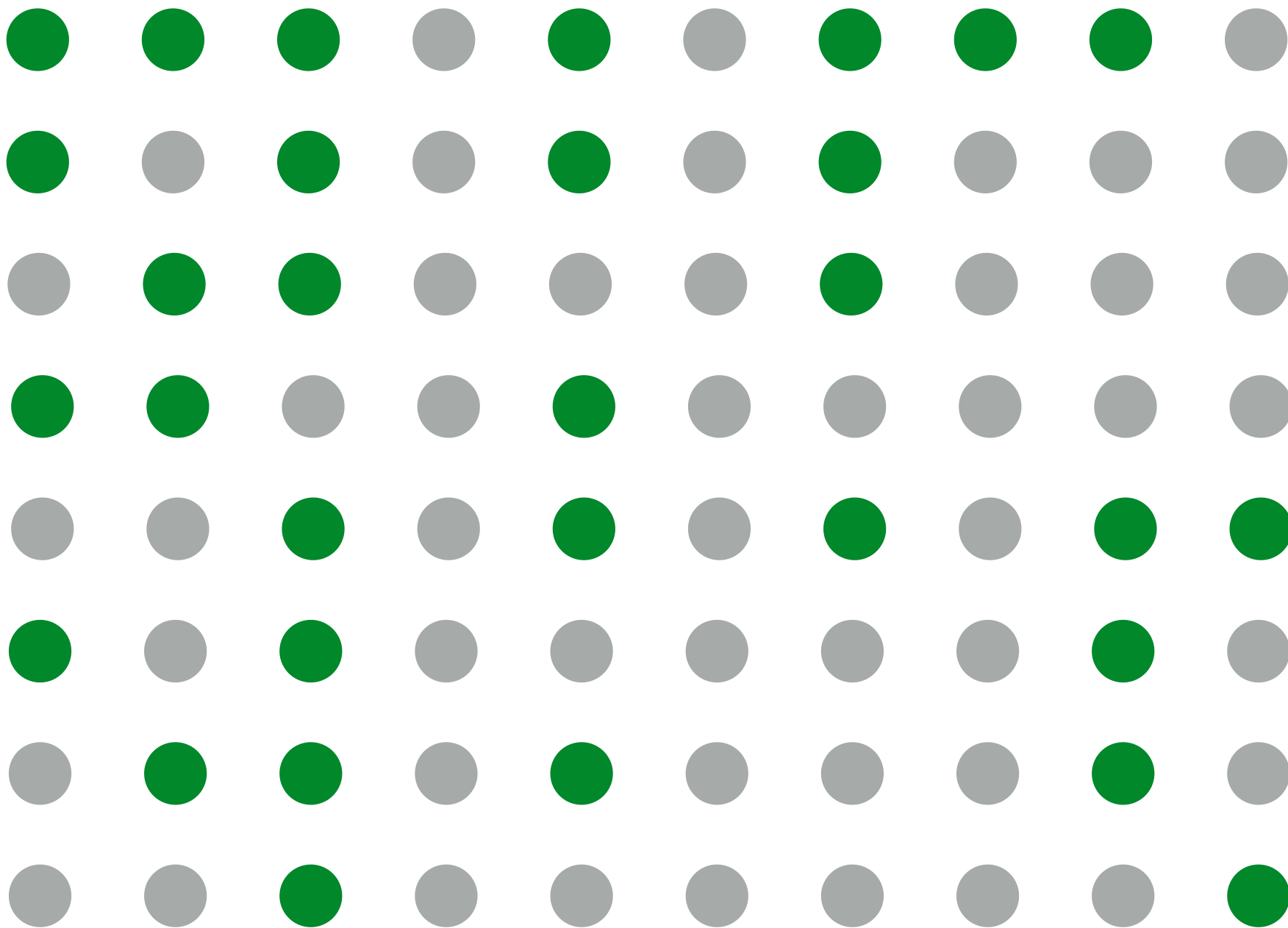
There is no **gossip network**
of size more than d .

Members of gossiptown want to form **as many clubs** as possible.



A set of people X form a **gossip network** if:
for *any* subset Y of X , there is a club that
allows **all of Y** and **none of $X \setminus Y$** .

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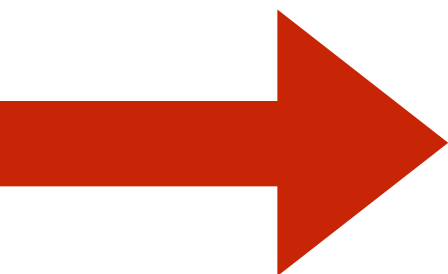
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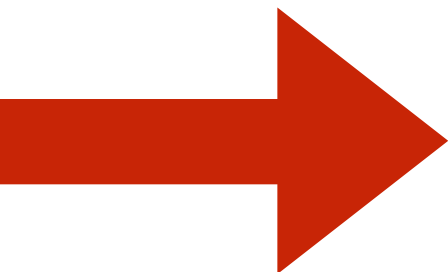
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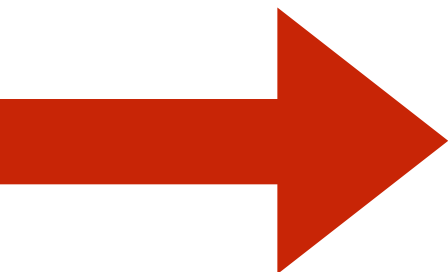
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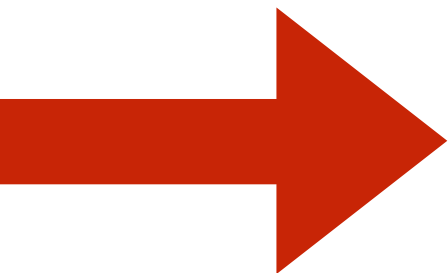
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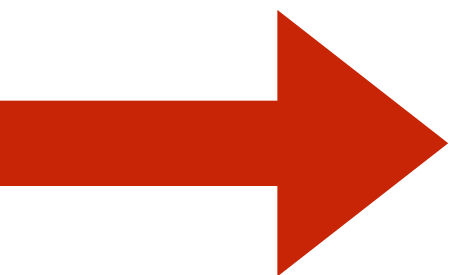
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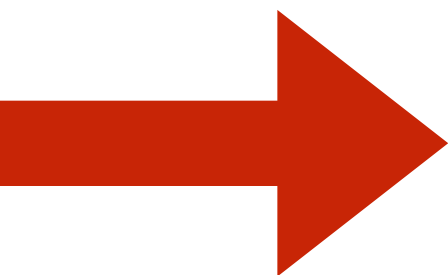
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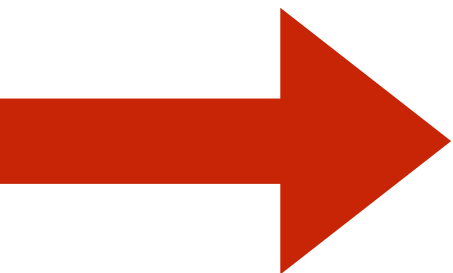
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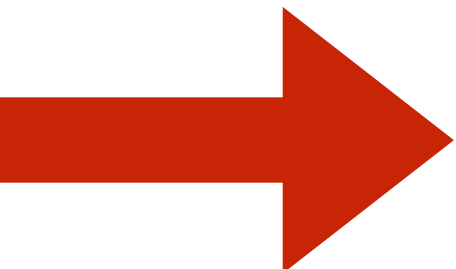
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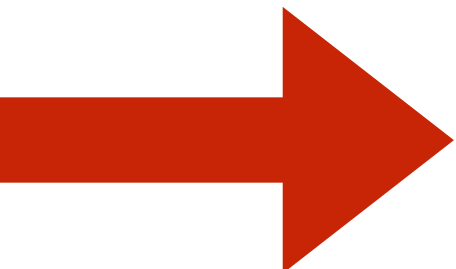
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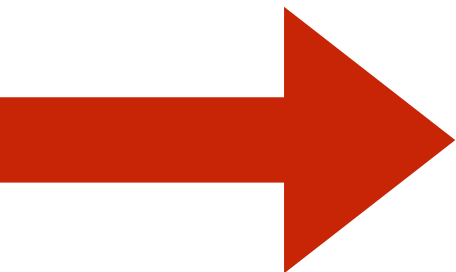
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While abiding by the rules,
how many clubs can we have?

(no gossip networks of size more than d)

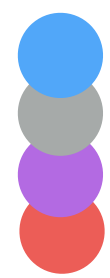
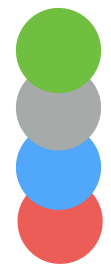
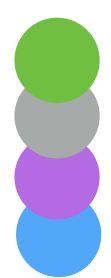
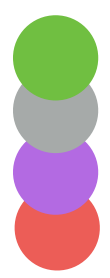
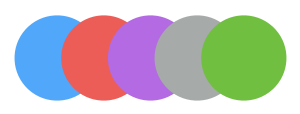
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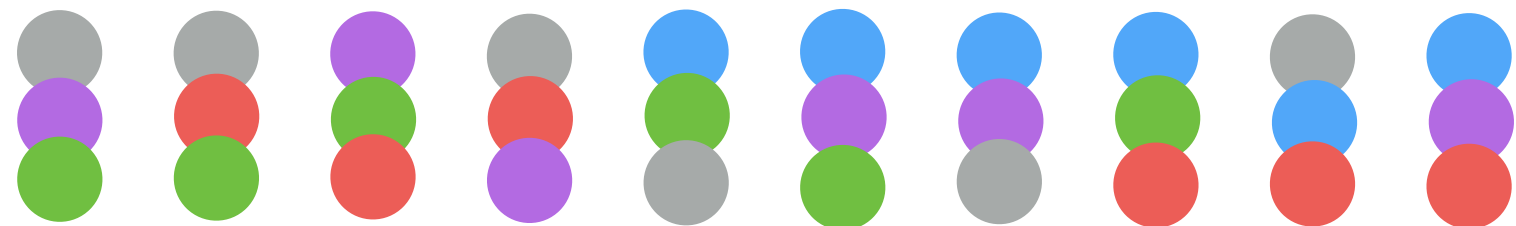
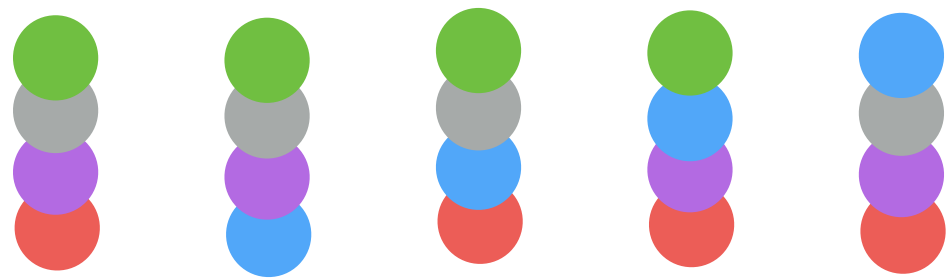
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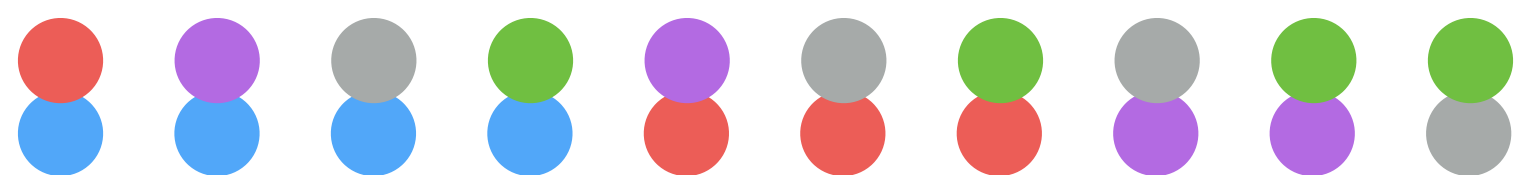
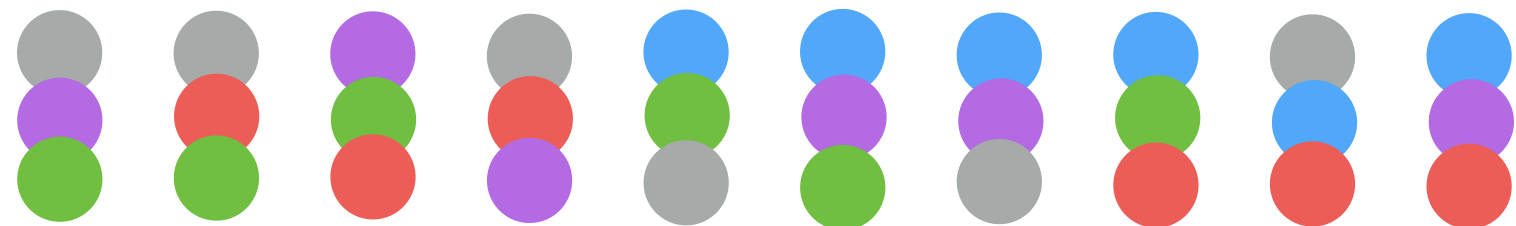
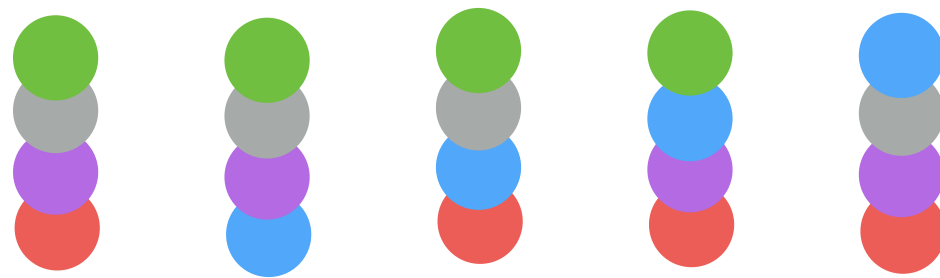
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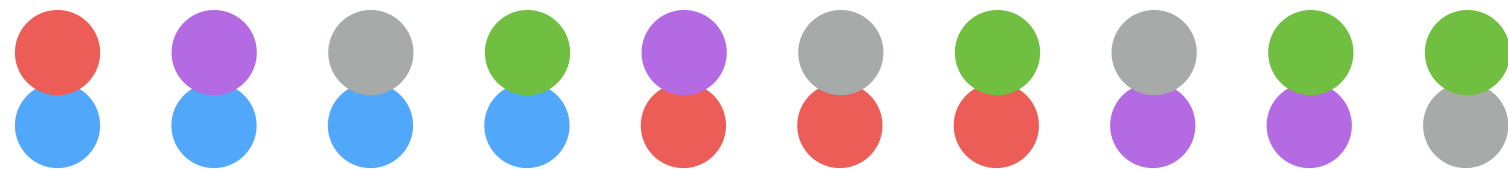
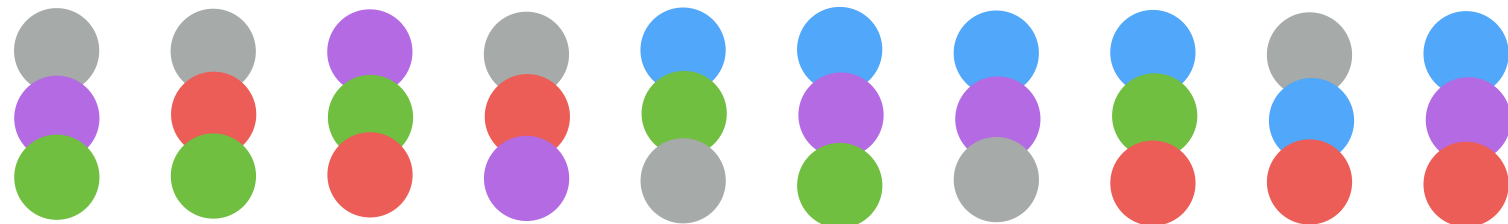
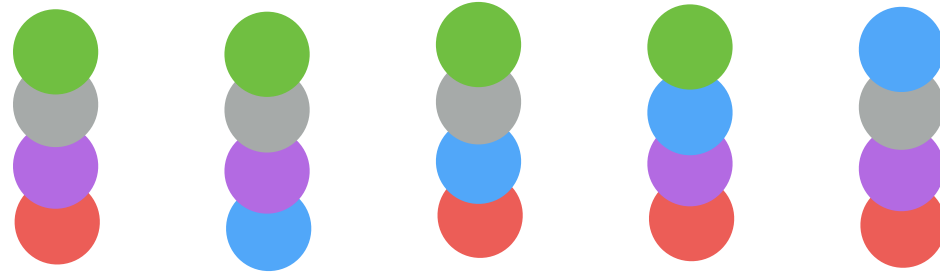
Simply indulge *every* subset of
size at most d in a club of its own!

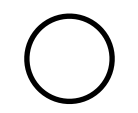
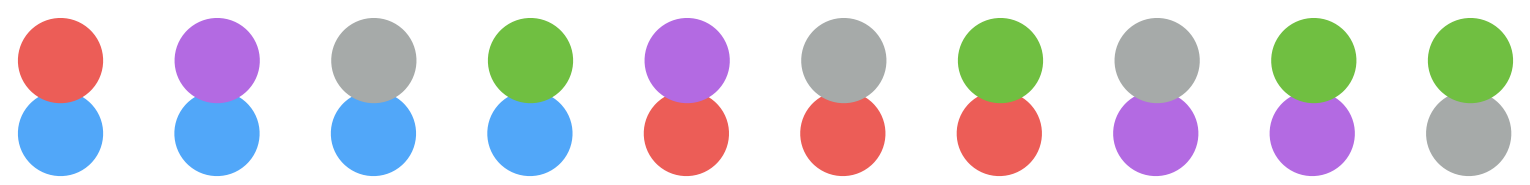
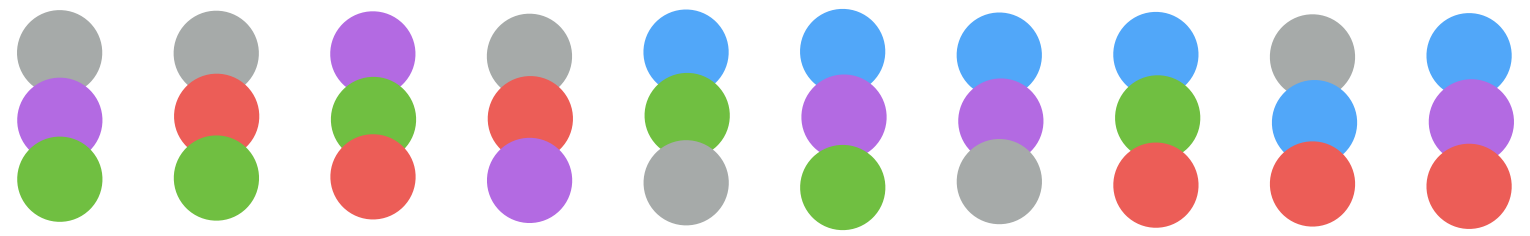
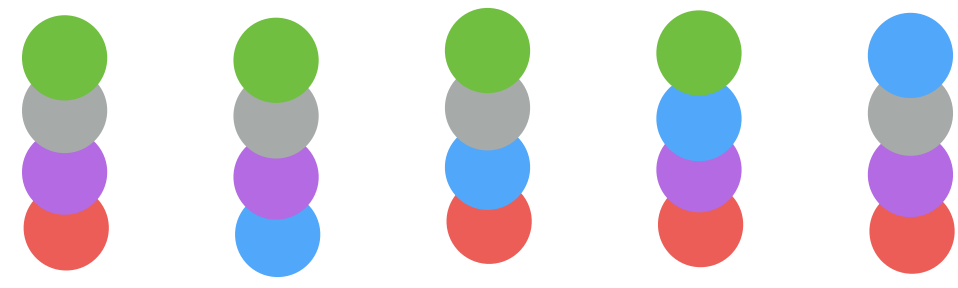
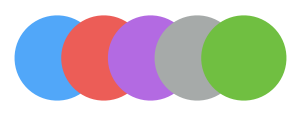












$\{a\ b\ c\ d\ e\}$

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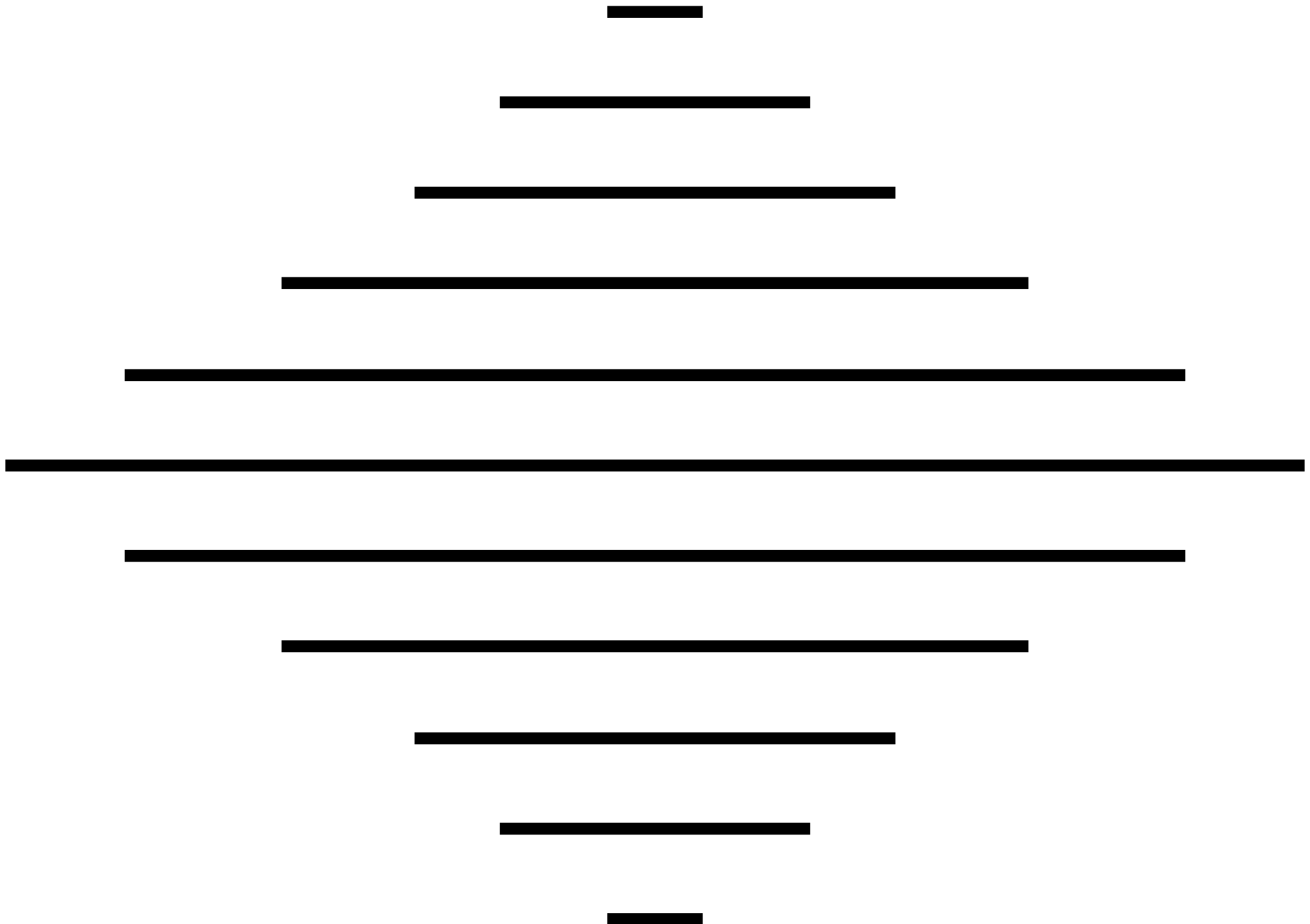
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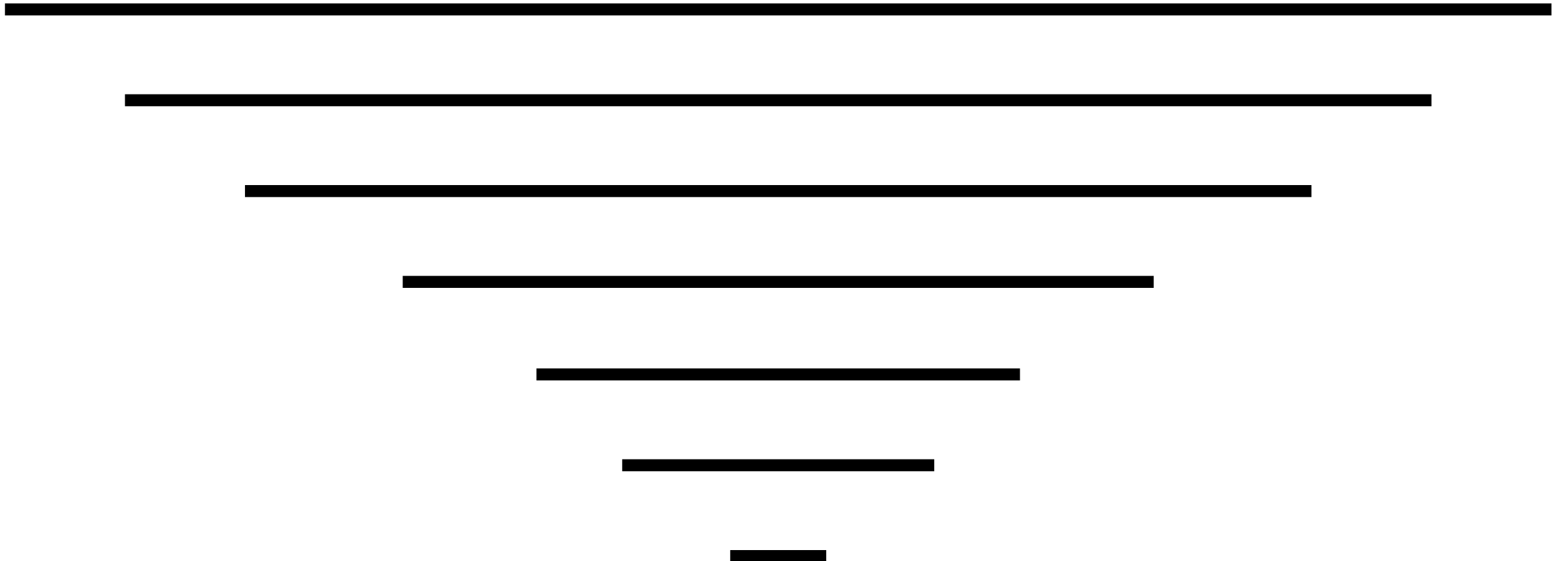
BOOLEAN LATTICE



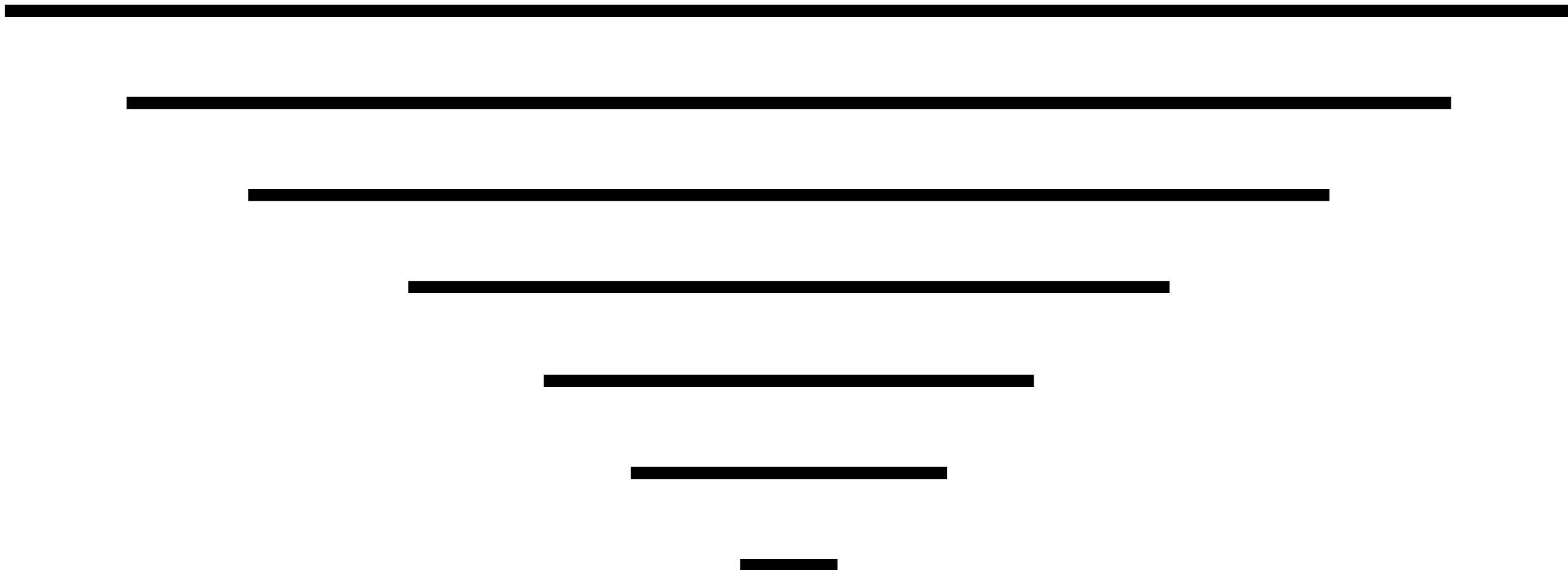
BOOLEAN LATTICE

[up to subsets of size at most d.]

$$S_d = \sum_{i=1}^d \binom{n}{i}$$

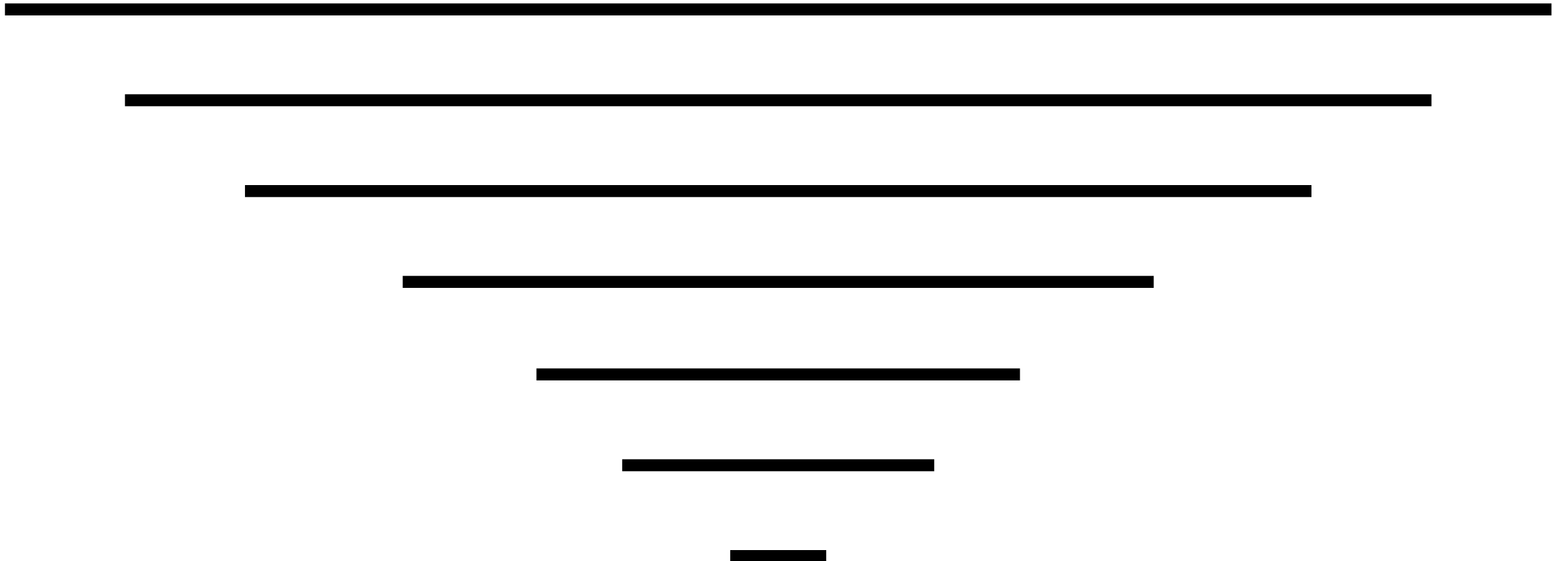


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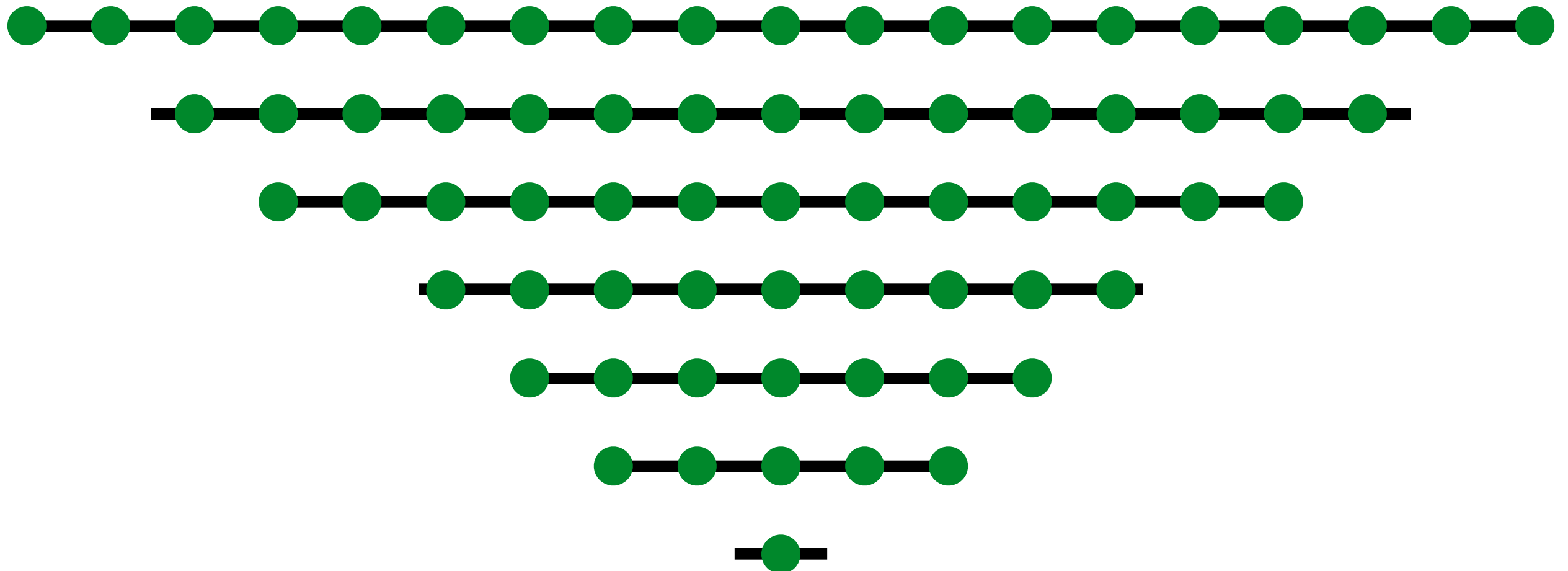
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The plan: Map clubs to 0/1 vectors, as before,
but now these vectors lie in **S_d -dimensional space**.



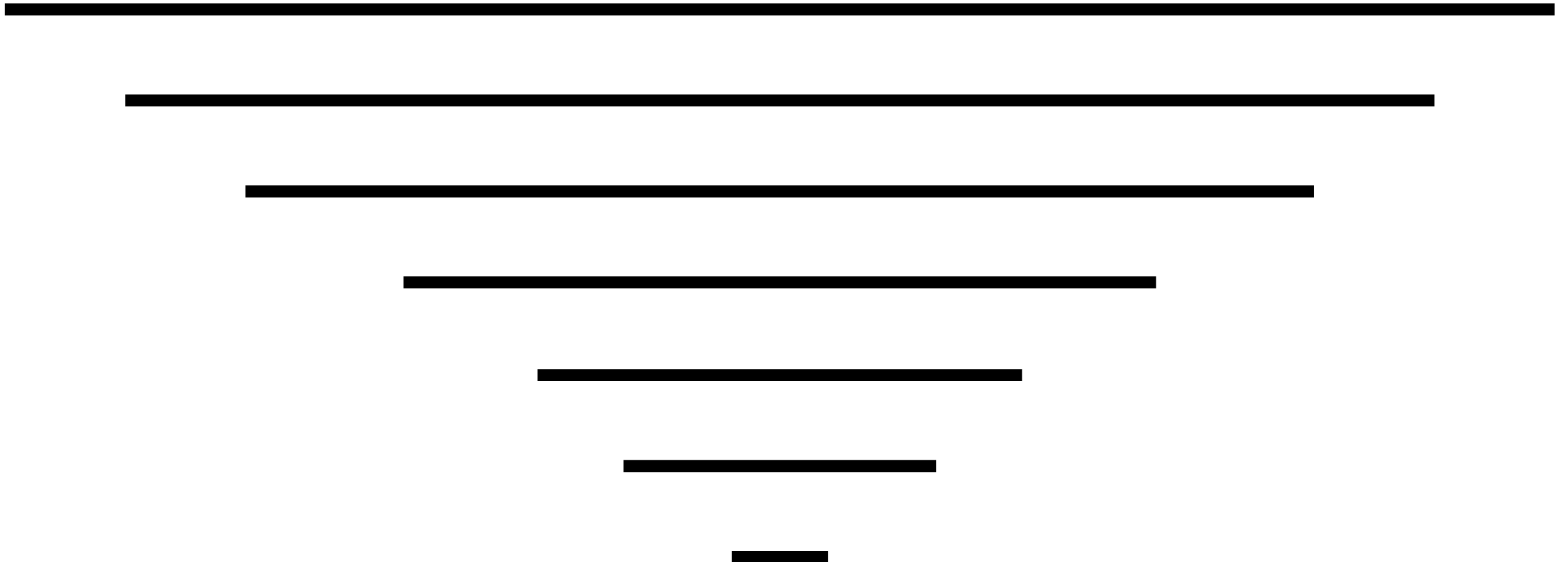
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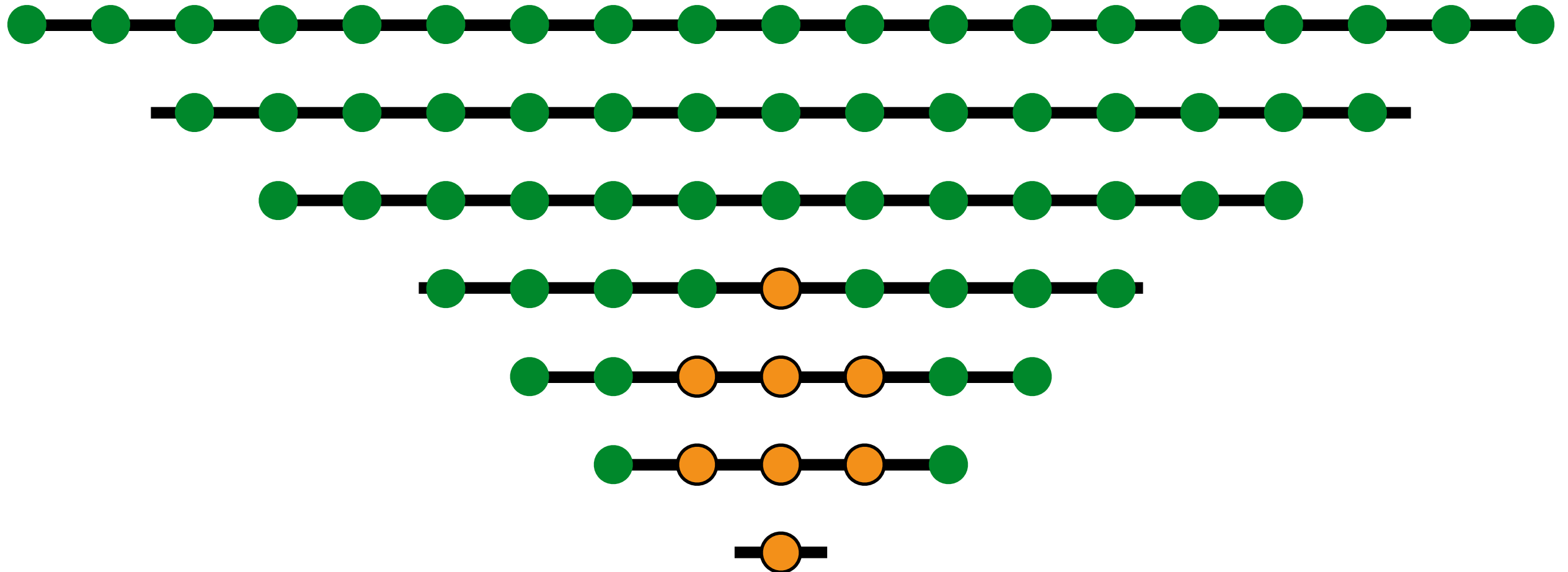
The vector corresponding to a club X , say g_X ,
is **one on subsets of X** , and zero everywhere else.

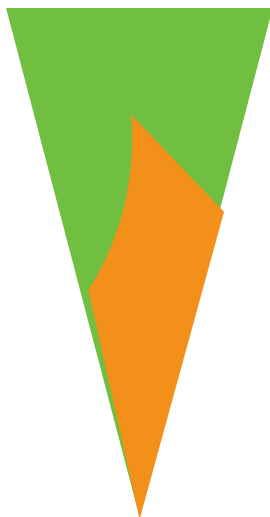
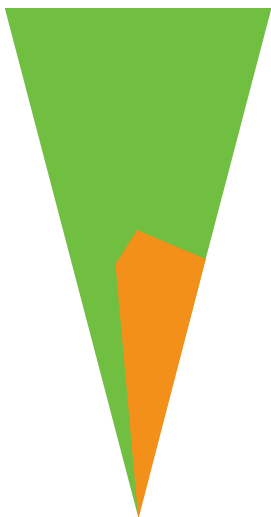
That is, for an index Y ,
 $g_X(Y) = 1$ iff Y is a subset of X .



The vector corresponding to a club X , say g_X ,
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Vectors corresponding to clubs.

Claim: Linearly independent if there are no gossip networks of size more than d .



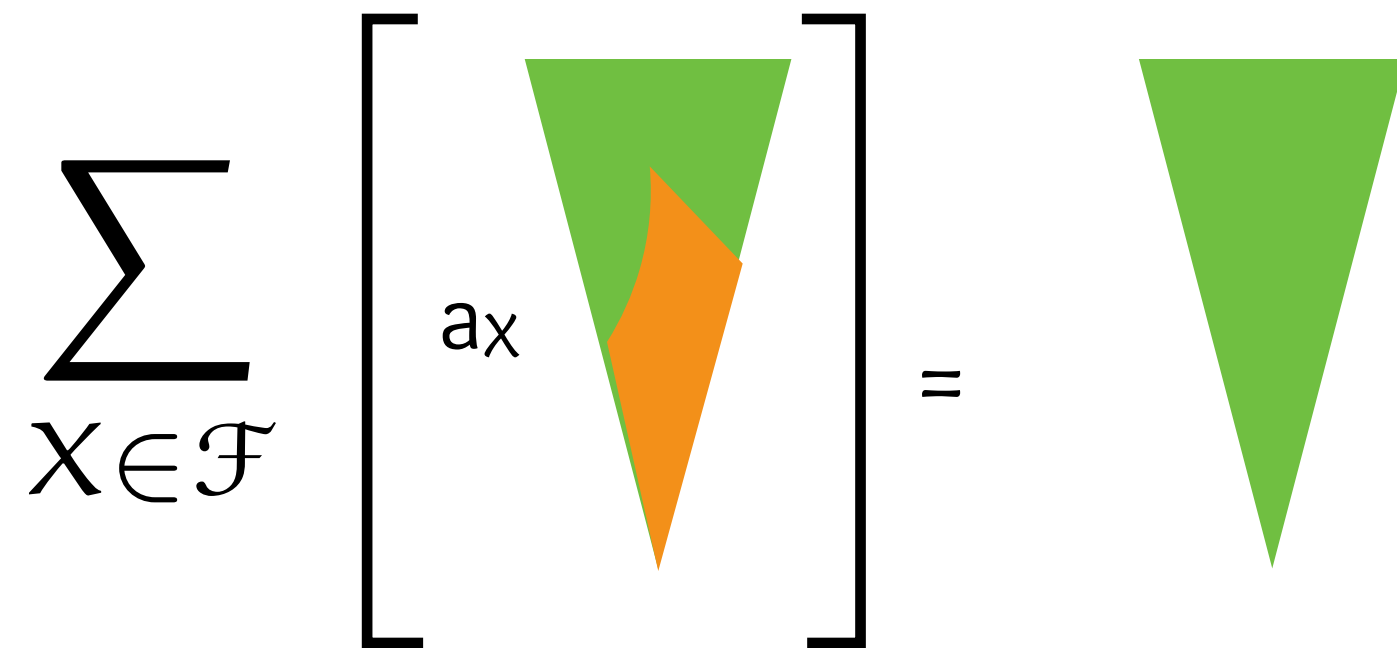
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Suppose not.

There are coefficients a_χ (not all zero)
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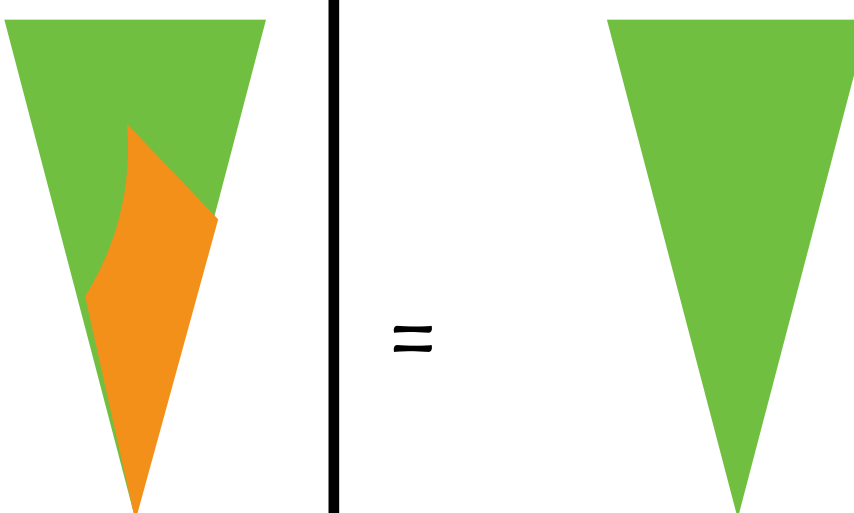
There are coefficients a_x (not all zero)
such that:

$$\sum_{X \in \mathcal{F}} \left[a_x \text{ (green triangle with orange region)} \right] = \text{green triangle}$$


The diagram illustrates the Carathéodory theorem. On the left, a summation over a family of sets $\mathcal{F}$ is shown. Each term in the sum is a coefficient a_x multiplied by a region. The regions are represented by green triangles, with one triangle containing an orange region. These regions are enclosed in large square brackets. An equals sign follows, pointing to a single green triangle on the right, which represents the convex hull of the union of the regions on the left.

$$\sum_{X \in \mathcal{F}} \left[\begin{array}{c} \text{ax} \\ \text{green triangle with orange overlap} \end{array} \right] = \text{green triangle}$$

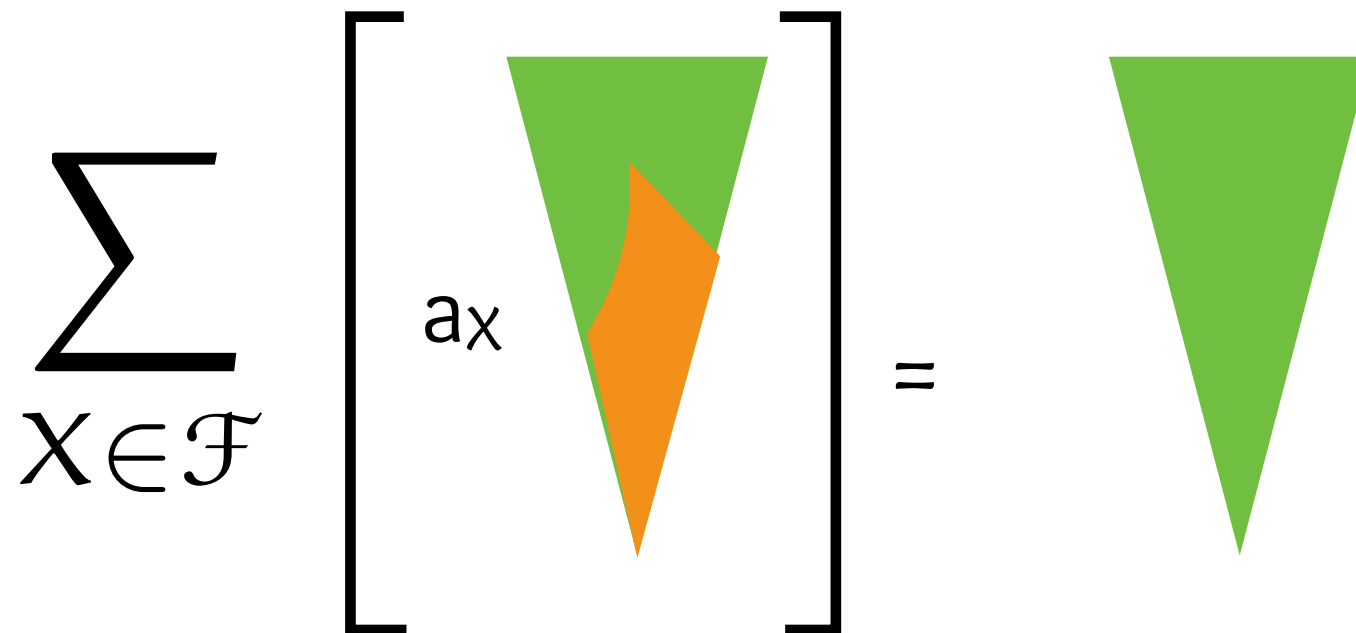
For any set Z , define:

$$\sum_{X \in \mathcal{F}} \left[a_X \right] =$$


The diagram illustrates the definition of a sum over a family of sets. On the left, a summation symbol $\sum$ is followed by the index set $X \in \mathcal{F}$. To the right of the summation is a large square bracket containing the term a_X . To the right of the bracket is an equals sign. To the right of the equals sign is a large green inverted triangle. The large green inverted triangle is composed of several smaller green inverted triangles, each of which contains a smaller orange inverted triangle. This visualizes the concept of a sum over a family of sets, where the result is the union of all sets in the family.

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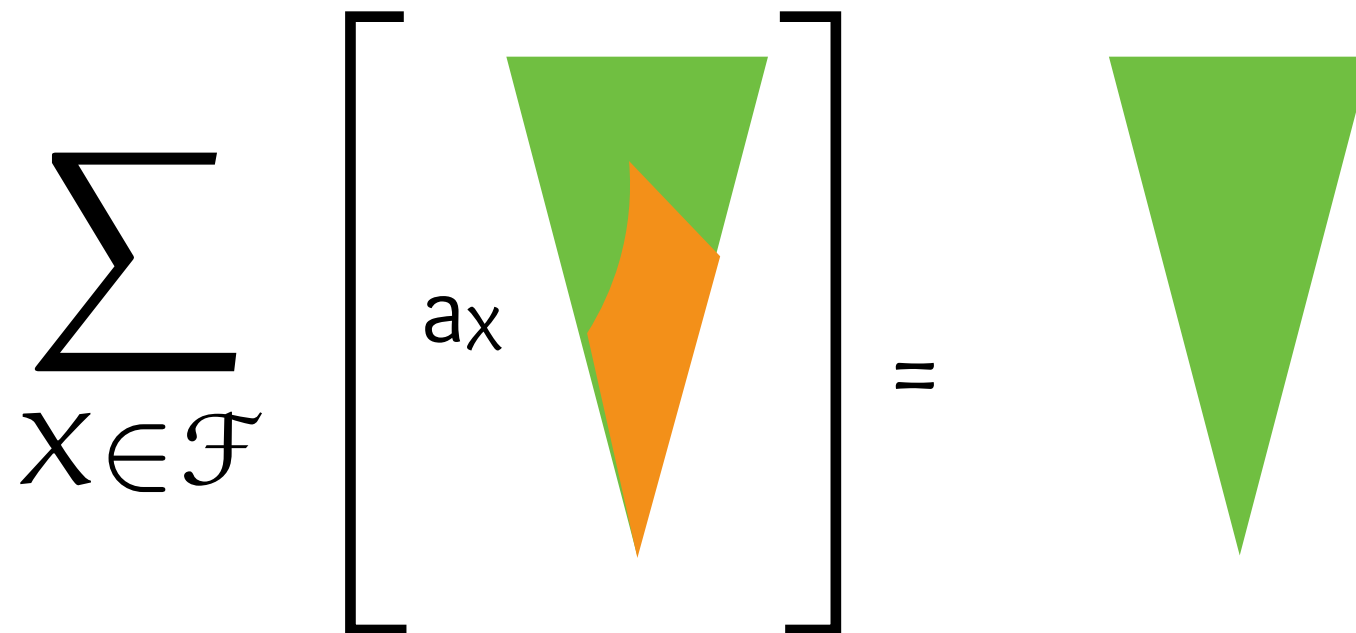
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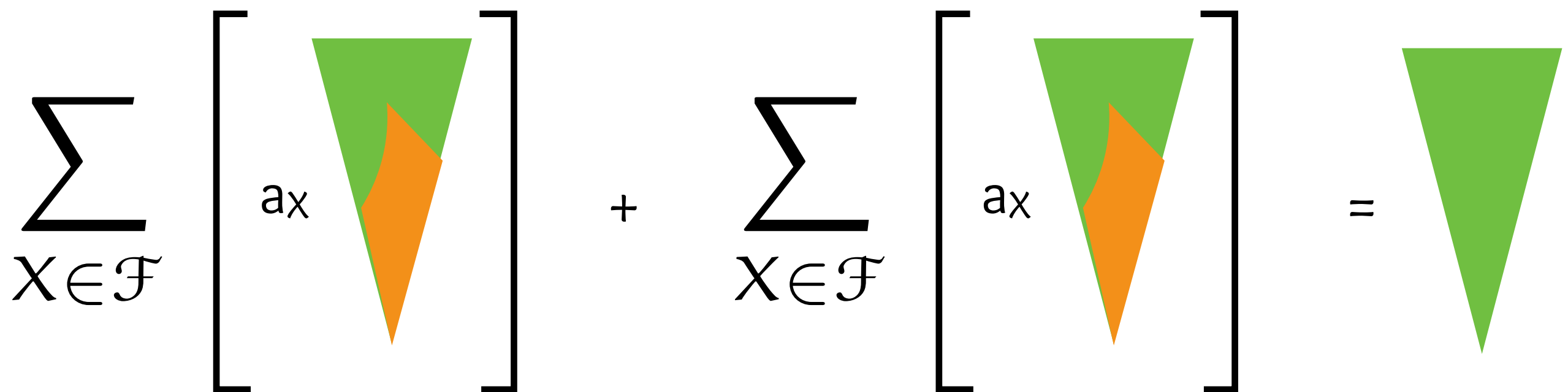
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$$\sum_{X \in \mathcal{F}} \left[a_X \text{ (triangle with orange bottom)} \right] + \sum_{X \in \mathcal{F}} \left[a_X \text{ (triangle with orange bottom)} \right] = \text{(solid green triangle)}$$


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$$\sum_{\substack{X \in \mathcal{F} \\ X \text{ allows } Z}} \left[a_X \cdot \text{triangle} \right] + \sum_{\substack{X \in \mathcal{F} \\ X \text{ disallows } Z}} \left[a_X \cdot \text{triangle} \right] = \text{triangle}$$

The diagram illustrates the identity $\sigma(Z) = 0$ for $|Z| < d+1$. It shows the sum of two terms, each represented by a bracketed expression. The first term is a sum over $X \in \mathcal{F}$ where X allows Z (indicated by blue text). The second term is a sum over $X \in \mathcal{F}$ where X disallows Z (indicated by red text). Each term is multiplied by a_X and a triangle. The triangles are green with an orange region removed. The final result is a single green triangle.

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$$\sum_{\substack{X \in \mathcal{F} \\ X \text{ allows } Z}} a_X \left[\text{diagram} \right] + \sum_{\substack{X \in \mathcal{F} \\ X \text{ disallows } Z}} a_X \left[\text{diagram} \right] = \text{diagram}$$

The diagram illustrates the decomposition of the sum $\sigma(Z)$ into two parts based on whether a set X contains Z . The first part, $\sum_{X \in \mathcal{F}; Z \subseteq X} a_X$, is represented by a sum over sets X that "allow" Z . The second part, $\sum_{X \in \mathcal{F}; Z \not\subseteq X} a_X$, is represented by a sum over sets X that "disallow" Z . The final result is a single green triangle, indicating that the sum of the two parts is zero.

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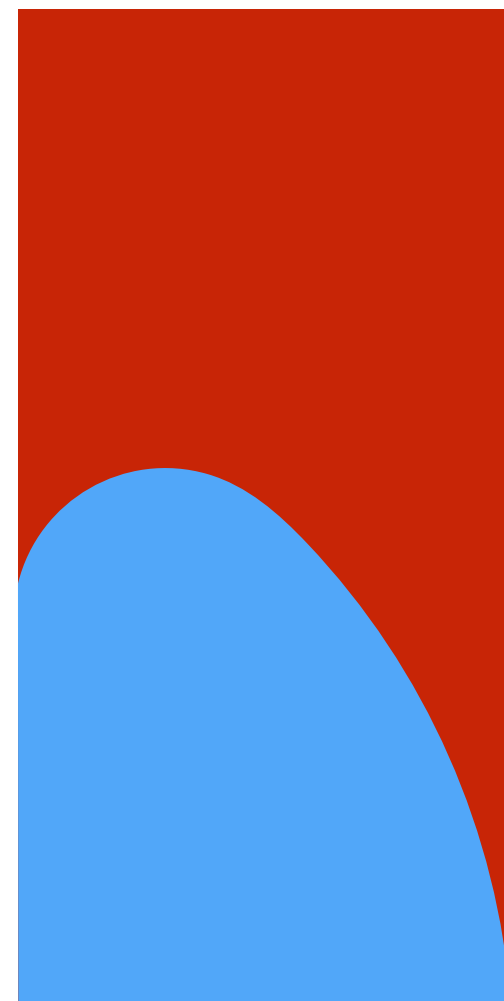
$$\sum_{X \in \mathcal{F}; Z \subseteq X} a_X - \sum_{X \in \mathcal{F}; Z \cup \{a\} \subseteq X} a_X - \sum_{X \in \mathcal{F}; Z \cup \{b\} \subseteq X} a_X + \sum_{X \in \mathcal{F}; Y \subseteq X} a_X$$

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 &= \sum_{Z \subseteq W \subseteq Y} \left\{ (-1)^{|W \setminus Z|} \sigma(W) \right\} \\
 &\neq 0
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The Story of

GOSSIP TOWN...

EPILOGUE

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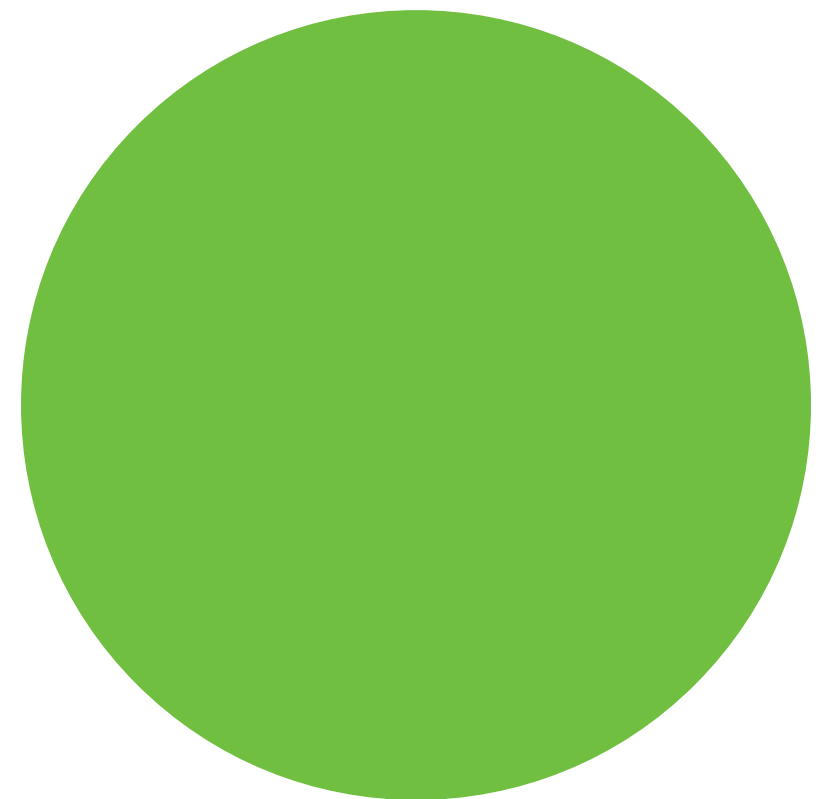
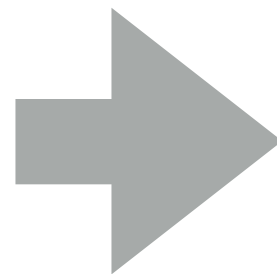
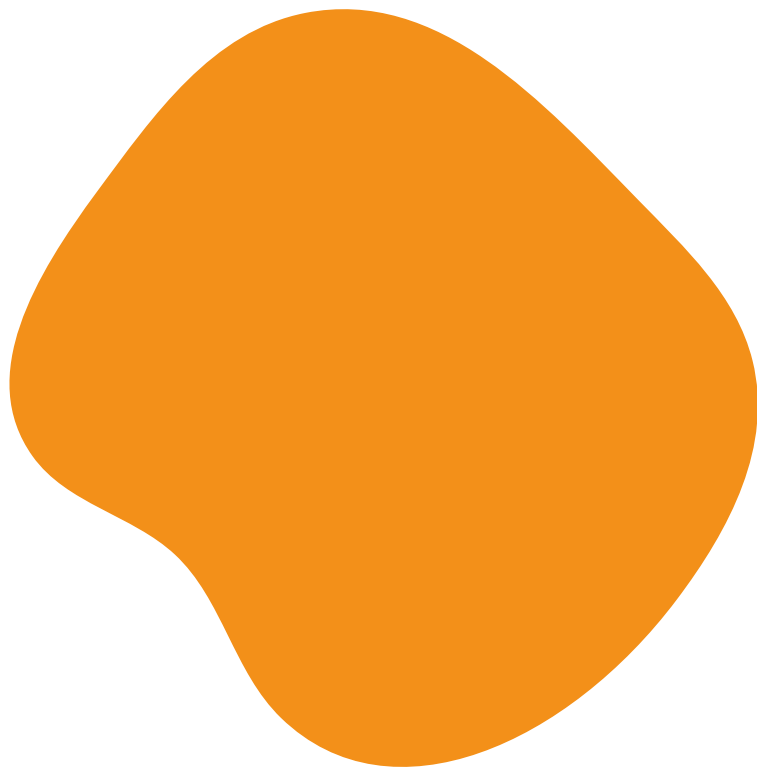
We have just seen that any family of more than S_d sets
must shatter a set of size $(d+1)$.

The size of a largest set shattered by a family F
is known as the **VC-Dimension of F .**

The size of X is now at most the dimension of Y .

A collection X
for whose size
we would like
an upper bound.

A linear space Y
where objects of X correspond
to **linearly independent**
vectors.



THANK YOU!
