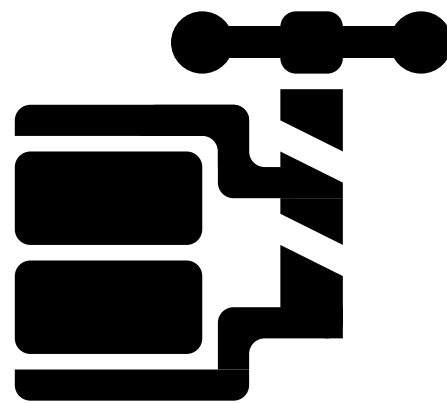


ITERATIVE COMPRESSION



June 6, 2017

ADMA Pre-conference school, SSN College

Input

A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Input

A graph on n vertices, m edges and an integer k .

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Question

Branch on cycles.

Input

A graph on n vertices, m edges and an integer k .

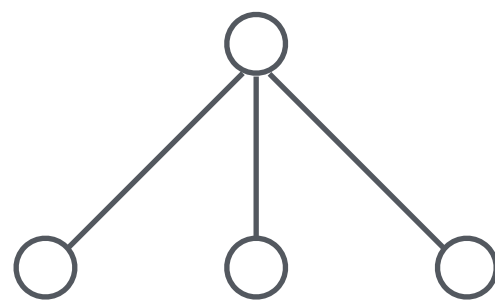
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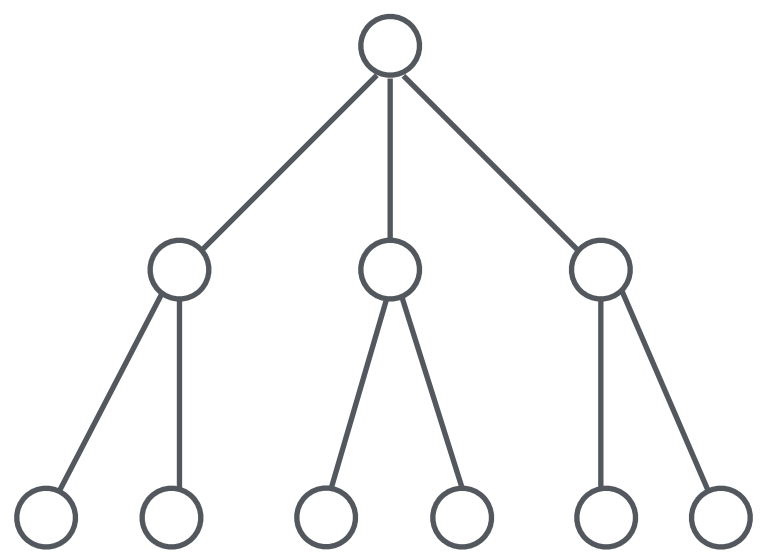
Question

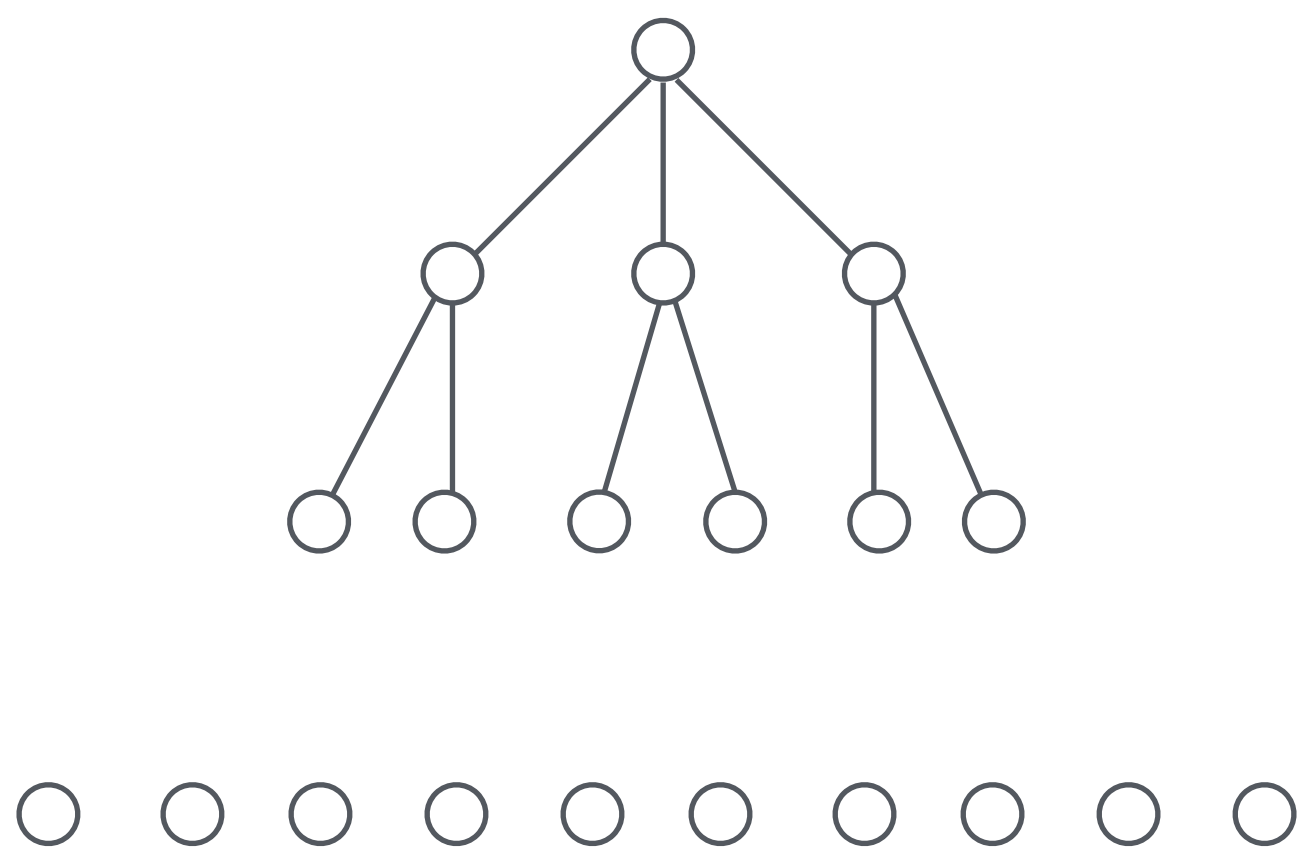
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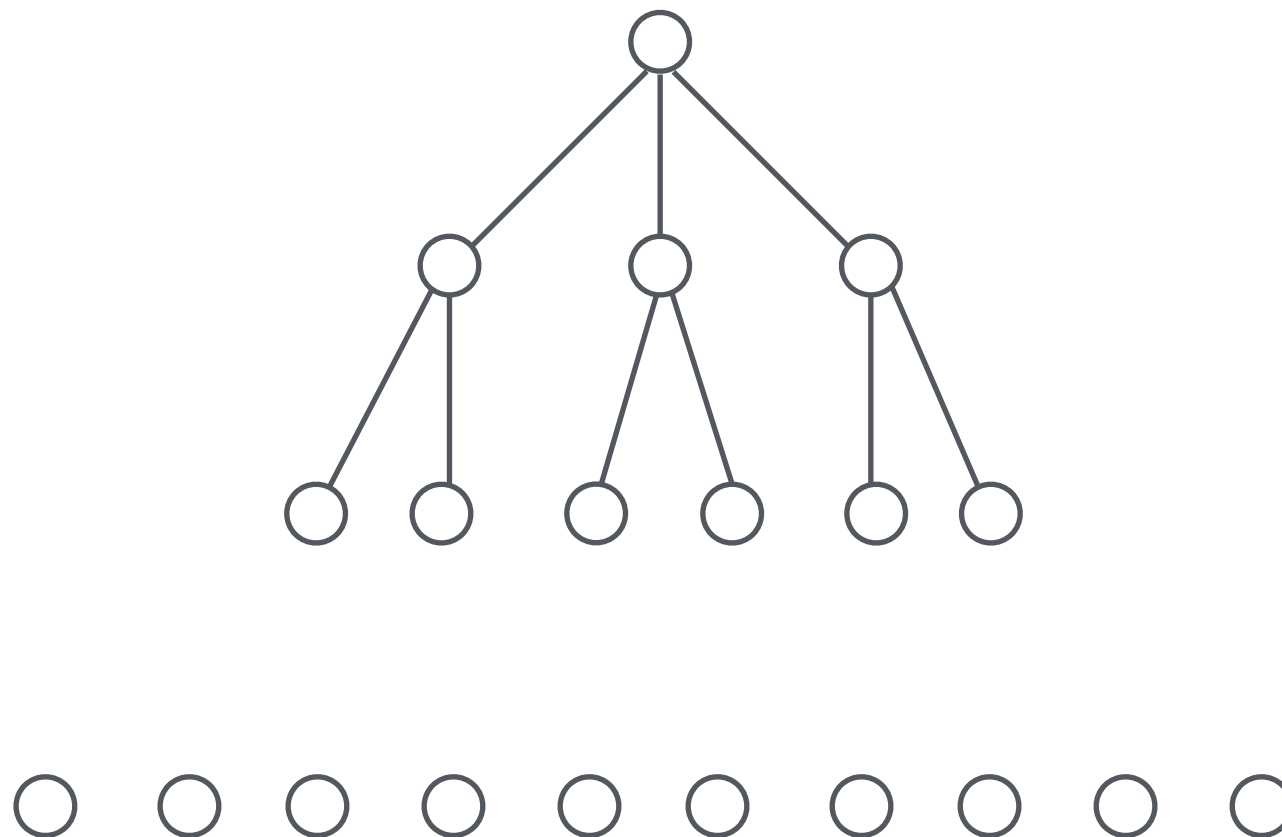
If there is always a cycle of length at most t ,
this gives us an algorithm with running time $O(t^k)$.











Any graph with **minimum degree three**
has a cycle of length $2(\log n) + 1$.

If there is always a cycle of length $O(\log n)$, this gives us
an algorithm with running time
 $O((\log n)^k)$.

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$$k(\log \log n) \leq \log n$$

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$$k \leq \frac{\log n}{\log \log n}$$

$$k(\log \log n) \leq \log n$$

$$(\log n)^k \leq n$$

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an algorithm with running time
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If there is always a cycle of length $O(\log n)$, this gives us
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We will prove: $\log \log n < k$

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If not, then: $\log n < k(\log \log n)$

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$$k \leq \log \log n$$

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$$\frac{\log n}{\log \log n} < k$$

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$$k \leq \log \log n$$

$$\text{If not, then: } \log n < k(\log \log n) \leq (\log \log n)^2$$

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$$\frac{\log n}{\log \log n} < k$$

$$\log n < k(\log \log n) < k^2$$

We will prove: $\log \log n < k$

$$k \leq \log \log n$$

$$\text{If not, then: } \log n < k(\log \log n) \leq (\log \log n)^2$$

...this is a contradiction.

If there is always a cycle of length $O(\log n)$, this gives us
an algorithm with running time
 $O((\log n)^k)$.

$$\text{If } k \leq \frac{\log n}{\log \log n}, \text{ then... } (\log n)^k \leq n$$

$$\text{If } \frac{\log n}{\log \log n} < k, \text{ then... } (\log n)^k \leq (k^2)^k$$

So we have a $2^{O(k \log k)} \text{poly}(n)$ algorithm here.

Input

A graph $G = (V, E)$ with n vertices, m edges, and k .

Is there a vertex cover of size at most k ?

Question

VERTEX COVER

Input

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VERTEX COVER

Input

A graph $G = (V, E)$ with n vertices, m edges, and k .

Is there a vertex cover of size at most k ?

Question



A vertex cover of size $(k+1)$.

VERTEX COVER

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A graph G with a vertex cover of size $k+1$.

Is there a vertex cover of size at most k ?

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A vertex cover of size $(k+1)$.

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VERTEX COVER

Input

A graph G with a vertex cover of size $k+1$.

Is there a vertex cover of size at most k ?

Question



A vertex cover of size $(k+1)$.

Let us “guess” how a vertex cover of size at most k interacts with this one.

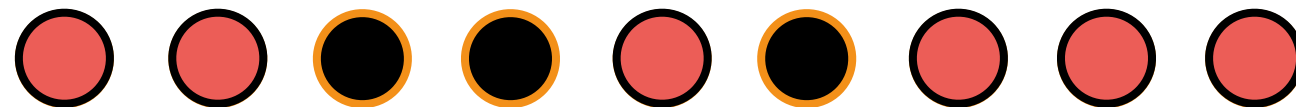
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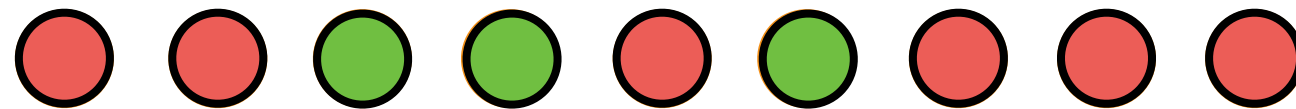
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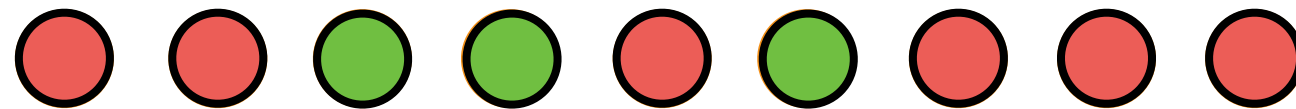
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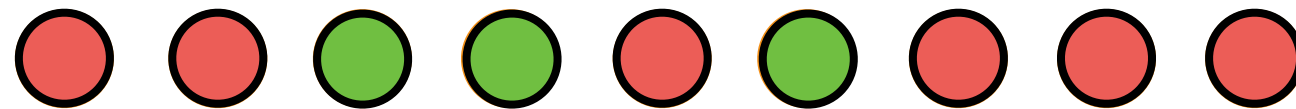
VERTEX COVER

Input

A graph G with a vertex cover of size $k+1$.

Is there a vertex cover of size at most k ?

Question



A vertex cover of size $(k+1)$.

Can there be an edge between
two red vertices?

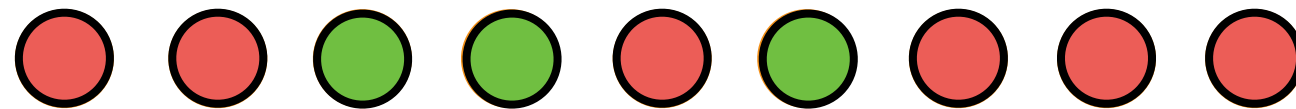
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A vertex cover of size $(k+1)$.



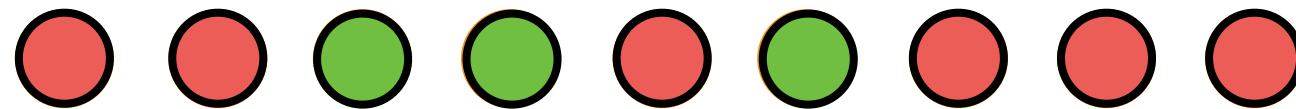
VERTEX COVER

Input

A graph G with a vertex cover of size $k+1$.

Is there a vertex cover of size at most k ?

Question



A vertex cover of size $(k+1)$.

Now we need to make up for the work that the red vertices were doing.

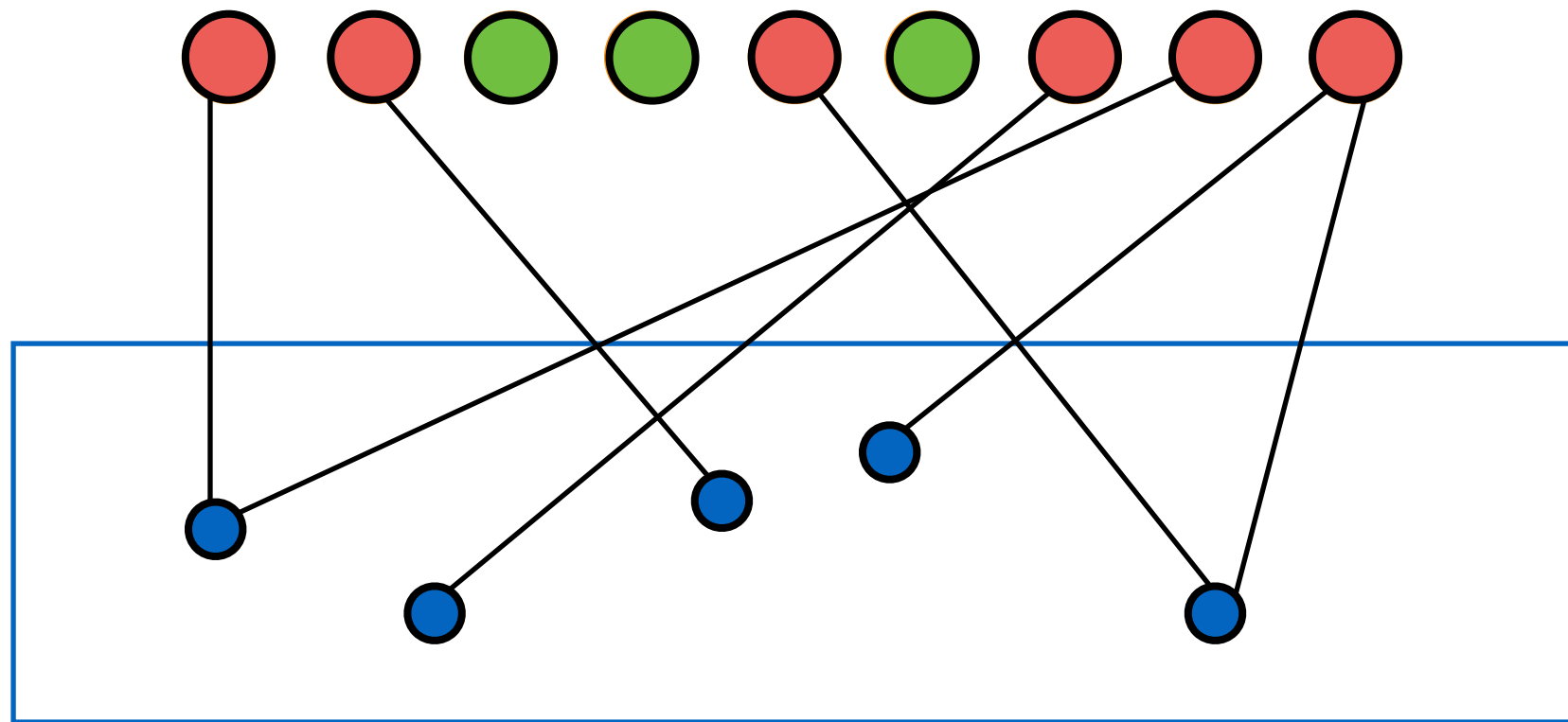
VERTEX COVER

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A graph G with a vertex cover of size $k+1$.

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Question



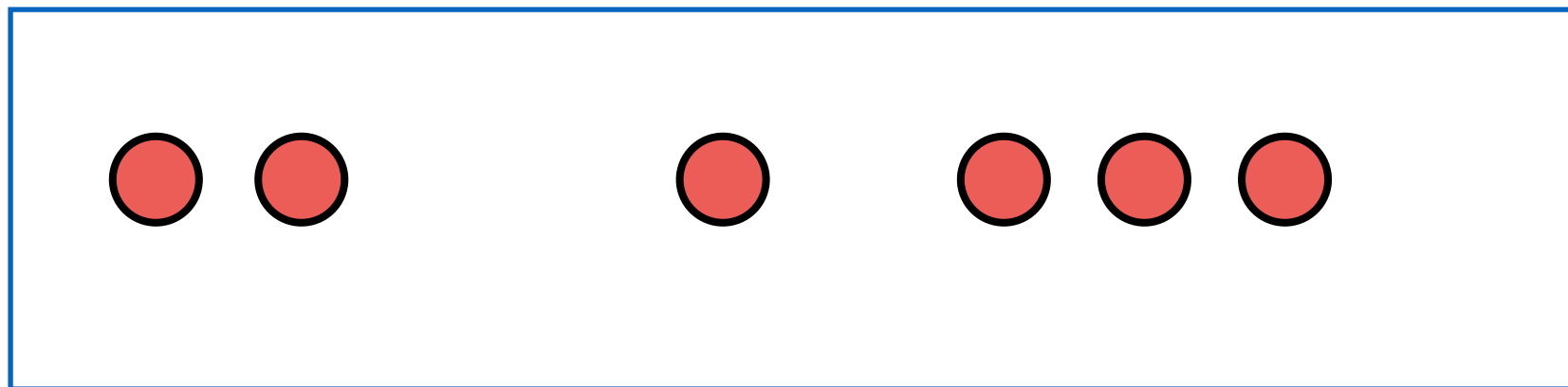
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Input

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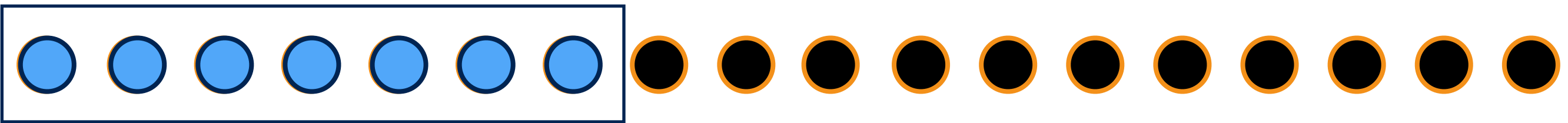
VERTEX COVER

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The first $k+2$ vertices in G .

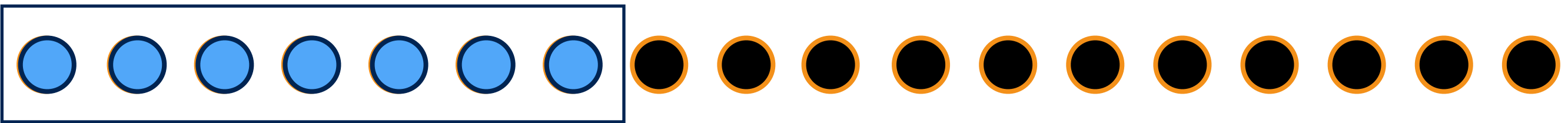
VERTEX COVER

Input

A graph $G = (V, E)$ with n vertices, m edges, and k .

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Question



The first $k+2$ vertices in G . It has an easy vertex cover of size $k+1$.

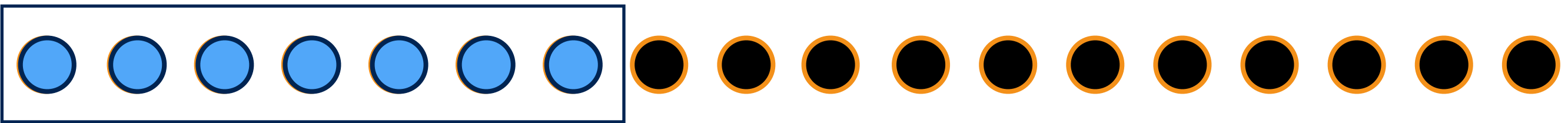
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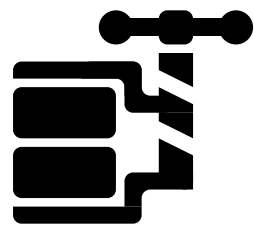
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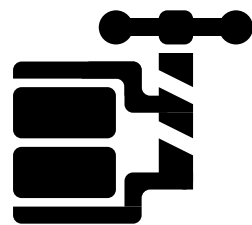
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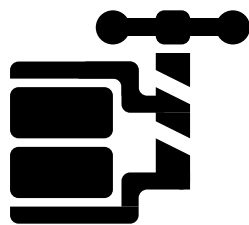
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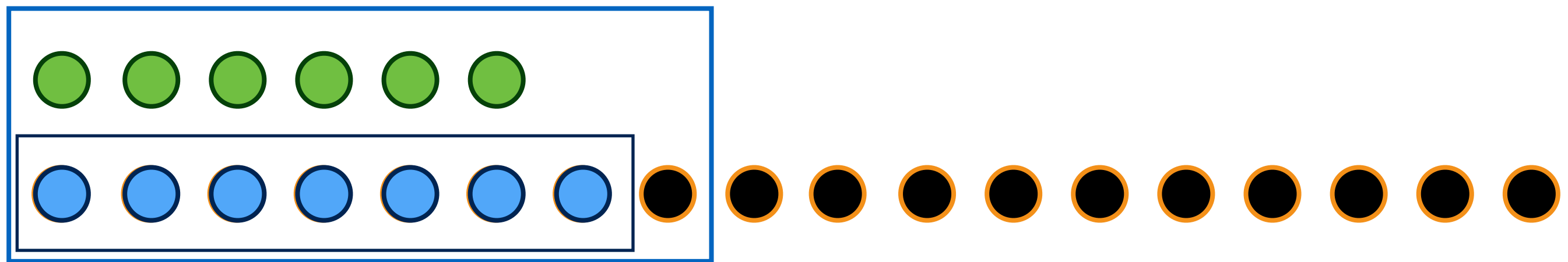
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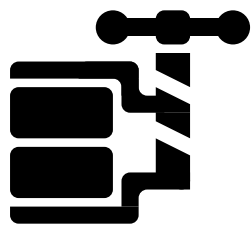
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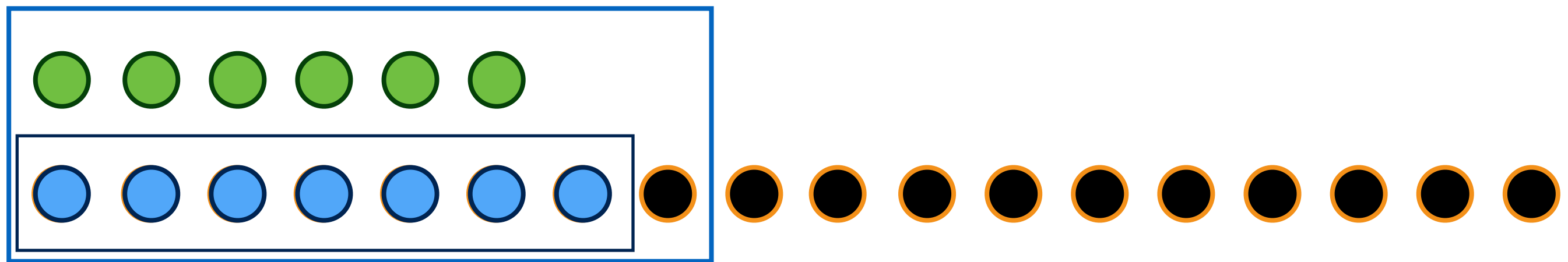
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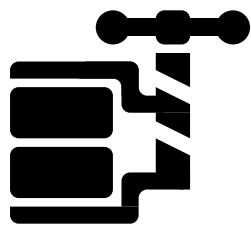
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The first $k+2$ vertices in G . It has an easy vertex cover of size $k+1$.



Compress again... rinse, repeat.

VERTEX COVER

Input

A graph $G = (V, E)$ with n vertices, m edges, and k .

Is there a subset of vertices S of size at most k that intersects all the edges?

Question

We have a $O^*(2^k)$ algorithm for Vertex Cover.

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We have a $O^*(2^k)$ algorithm for Vertex Cover.



VERTEX COVER

Input

A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

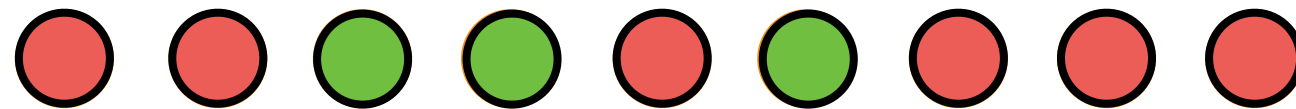
FEEDBACK VERTEX SET

Input

A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question



A Feedback Vertex Set of size $(k+1)$.

Let us “guess” how a FVS of size at most k interacts with this one.

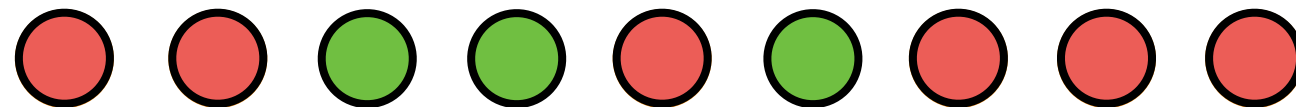
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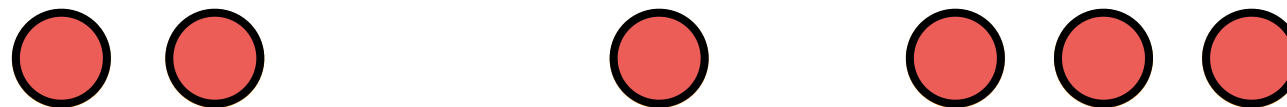
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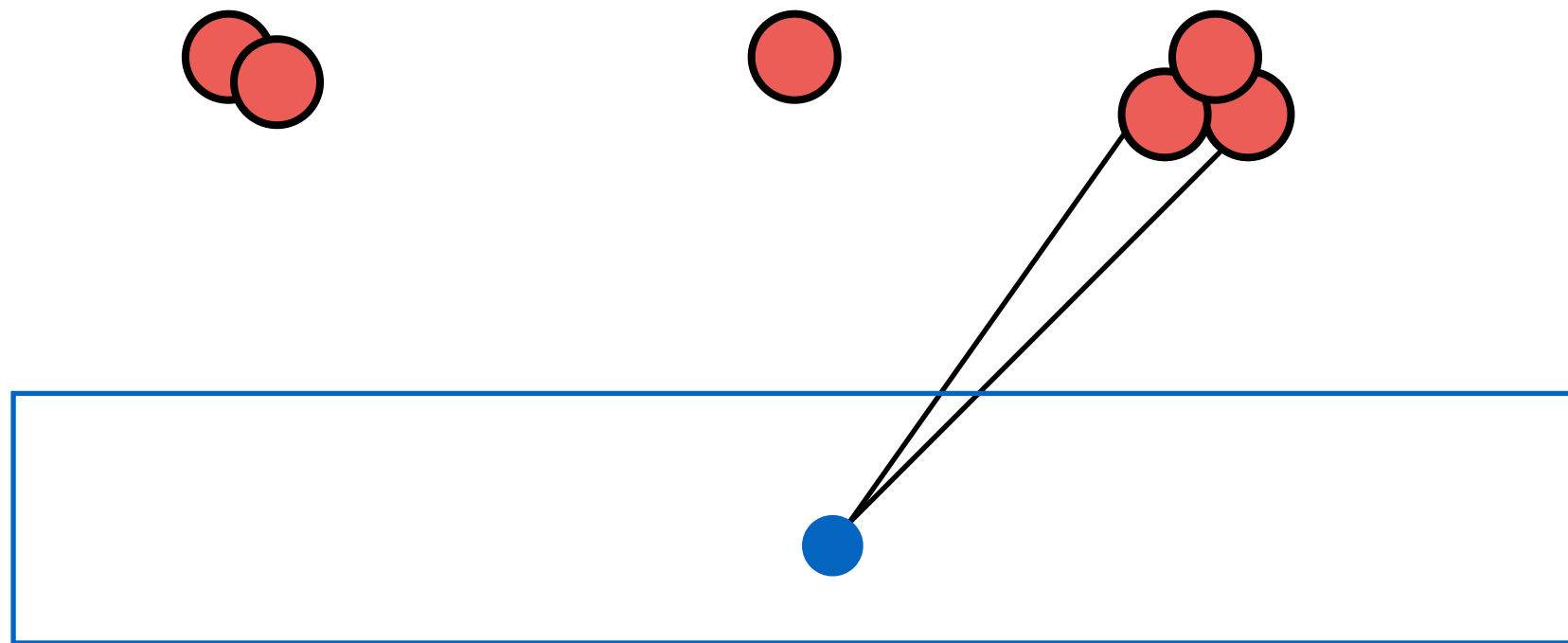
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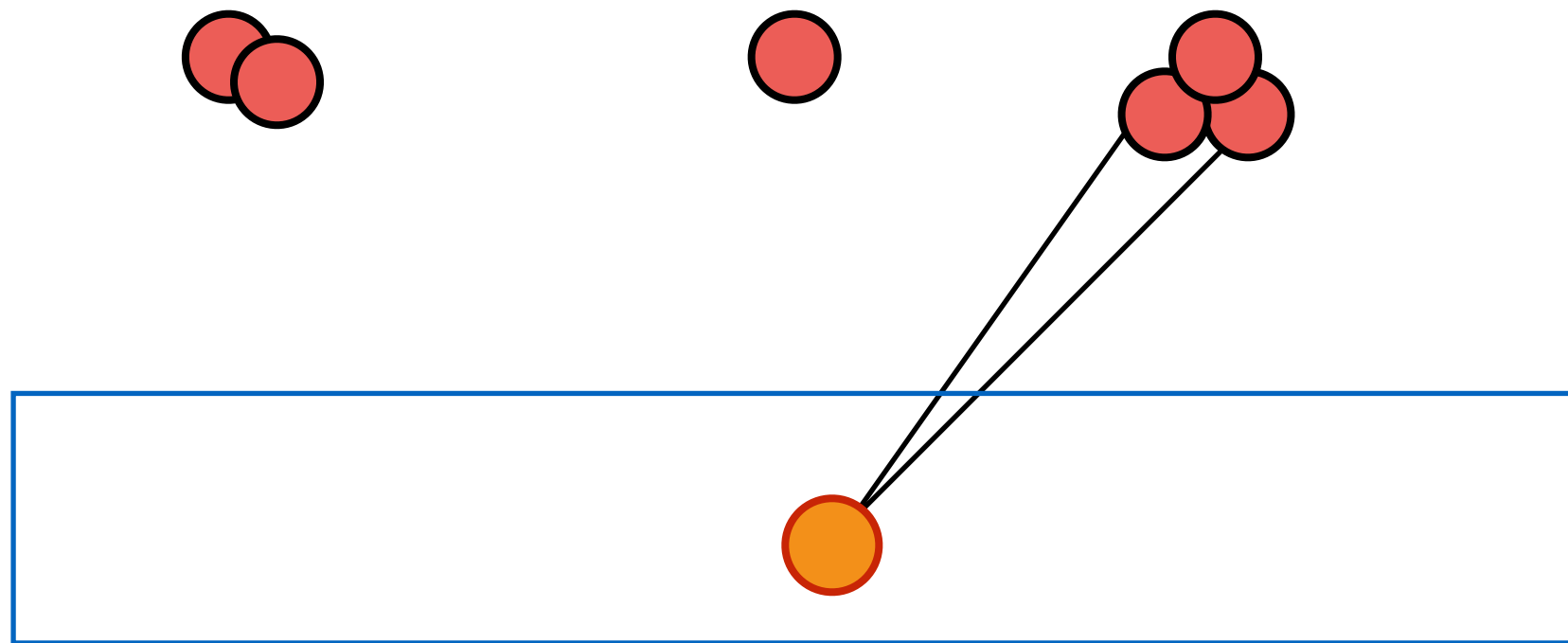
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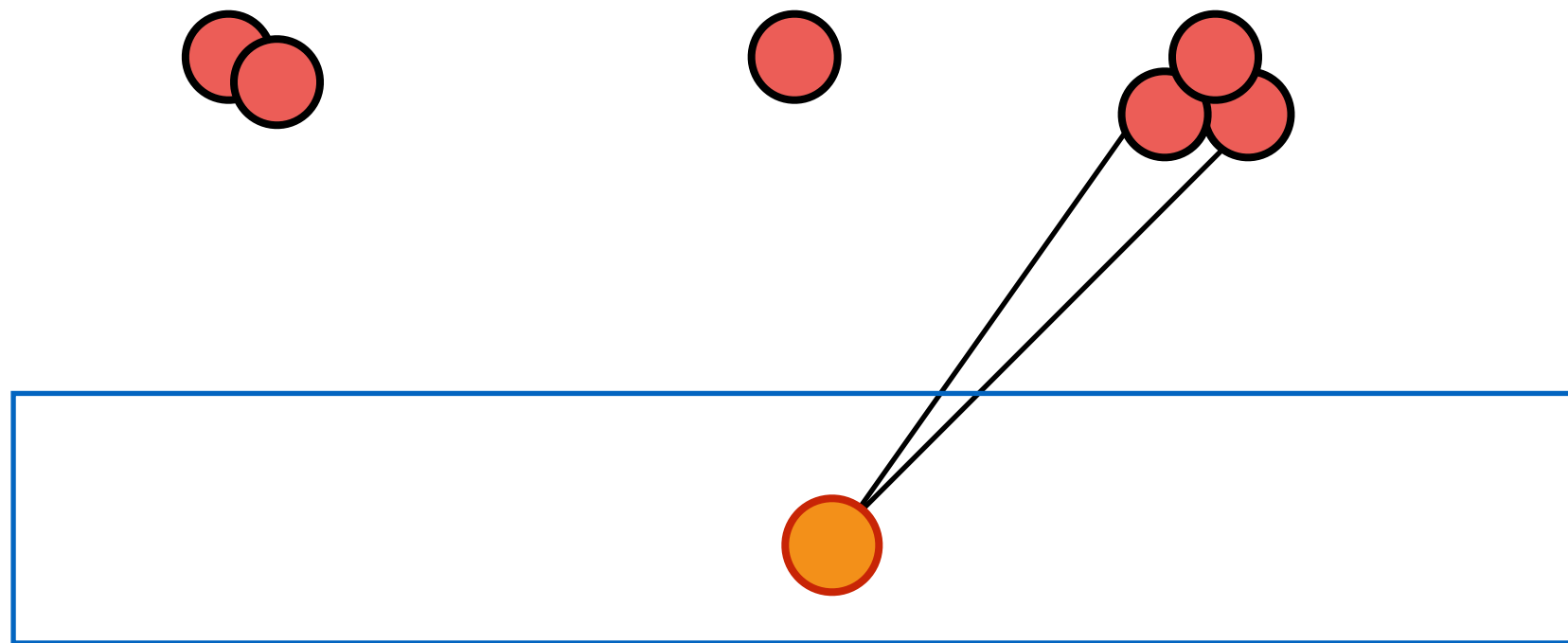
FEEDBACK VERTEX SET

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A vertex with two neighbors in the same component is forced.

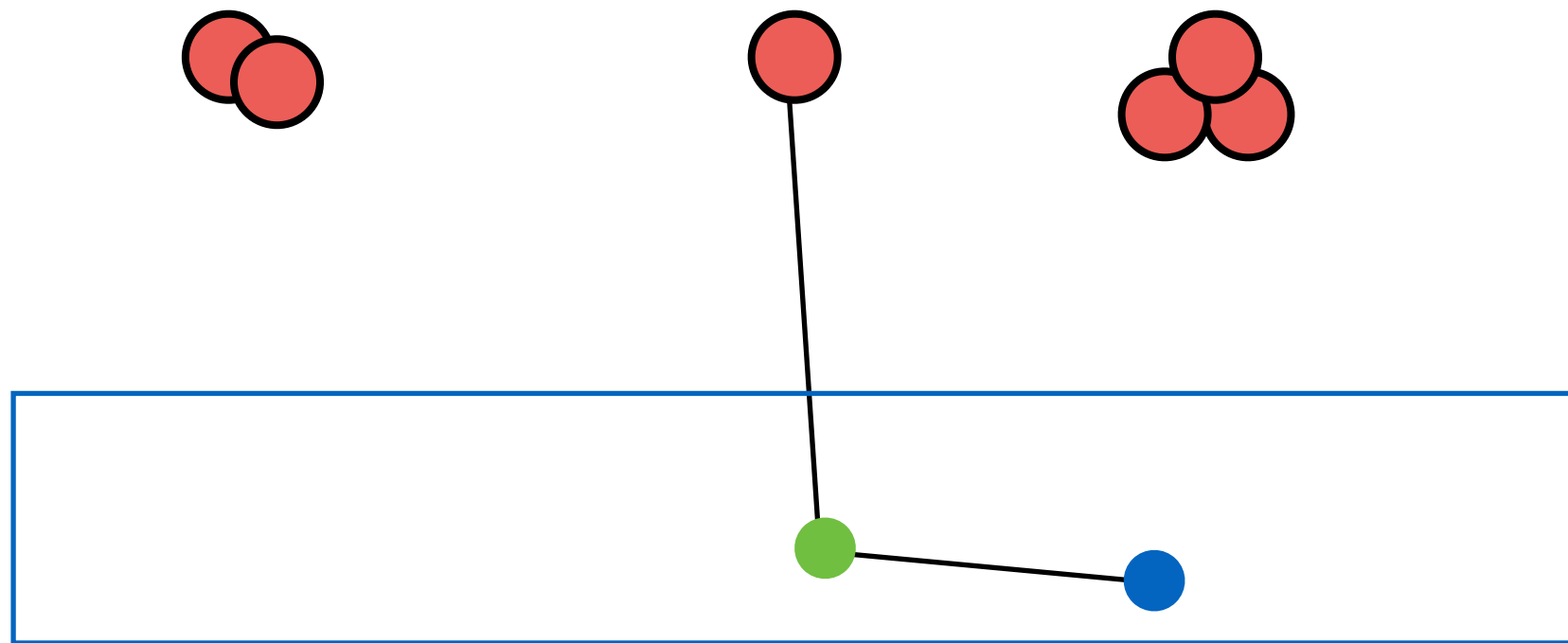
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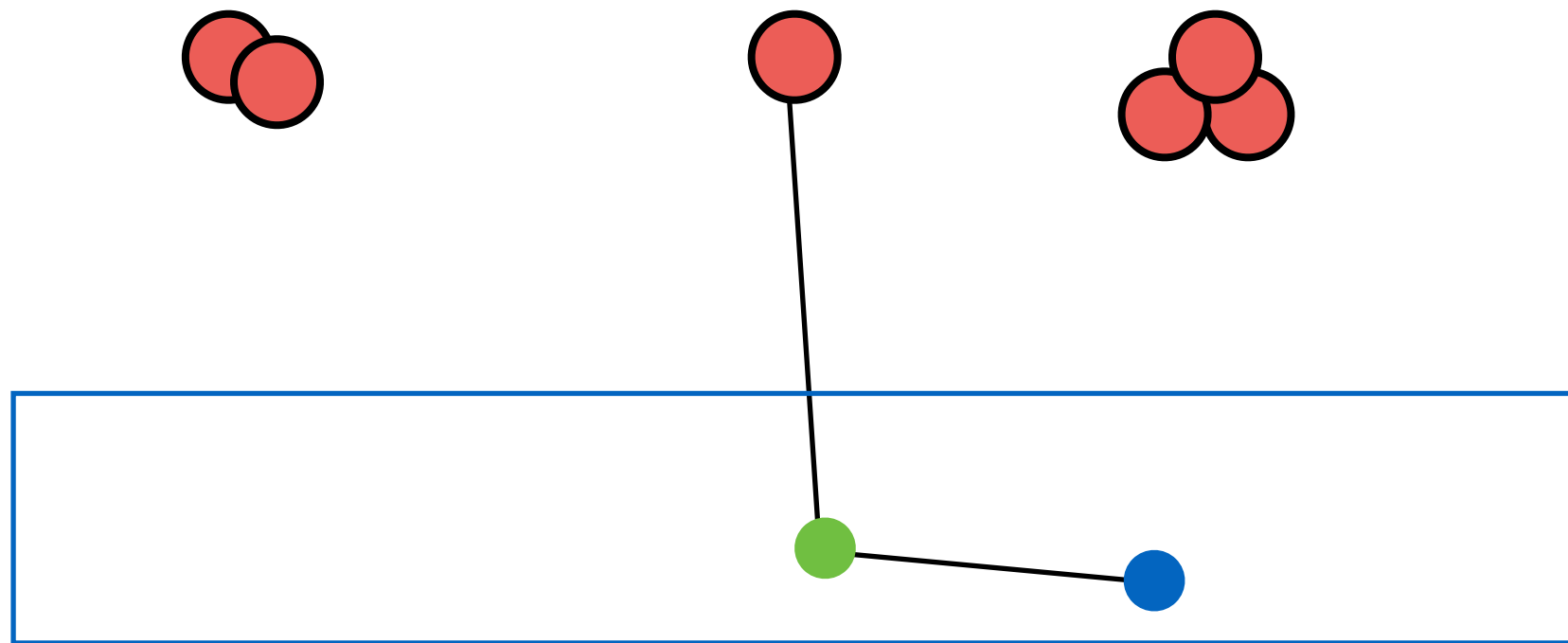
FEEDBACK VERTEX SET

Input

A graph on n vertices, m edges and an integer k .

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Question



A leaf that has exactly one neighbor above can be preprocessed.

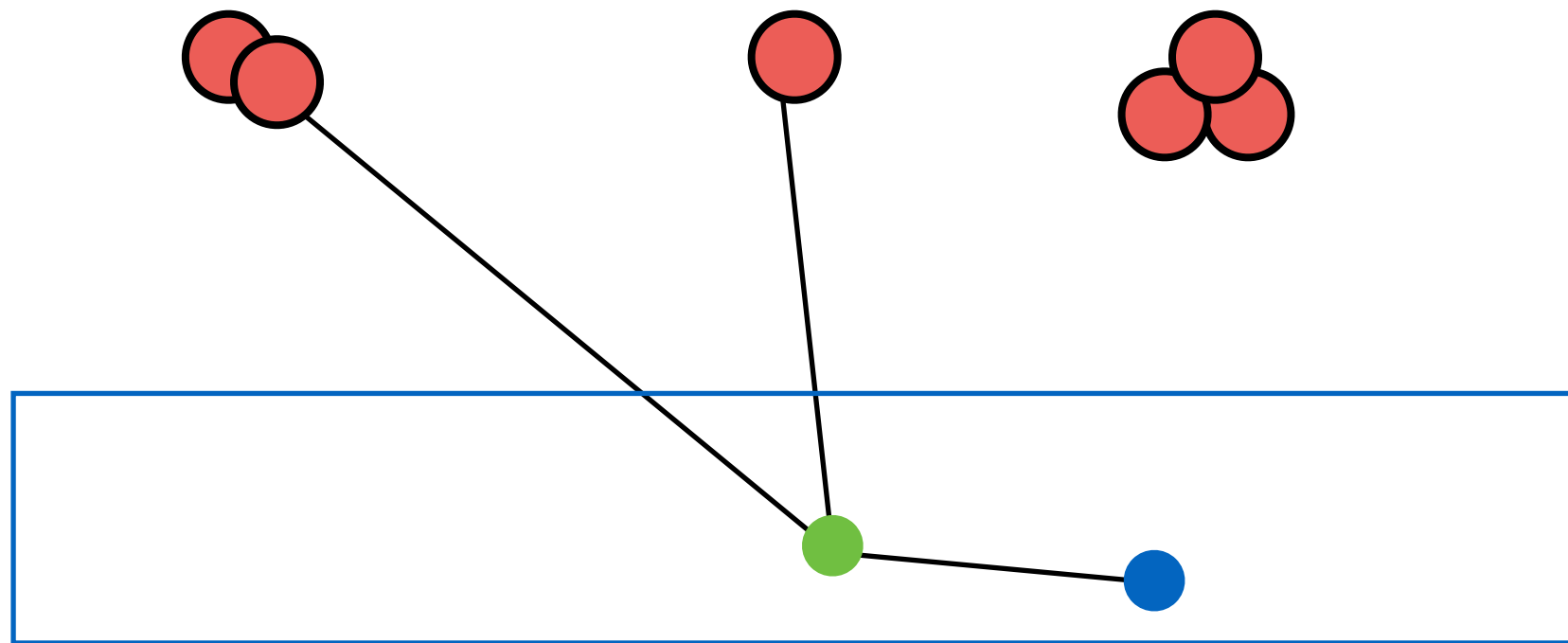
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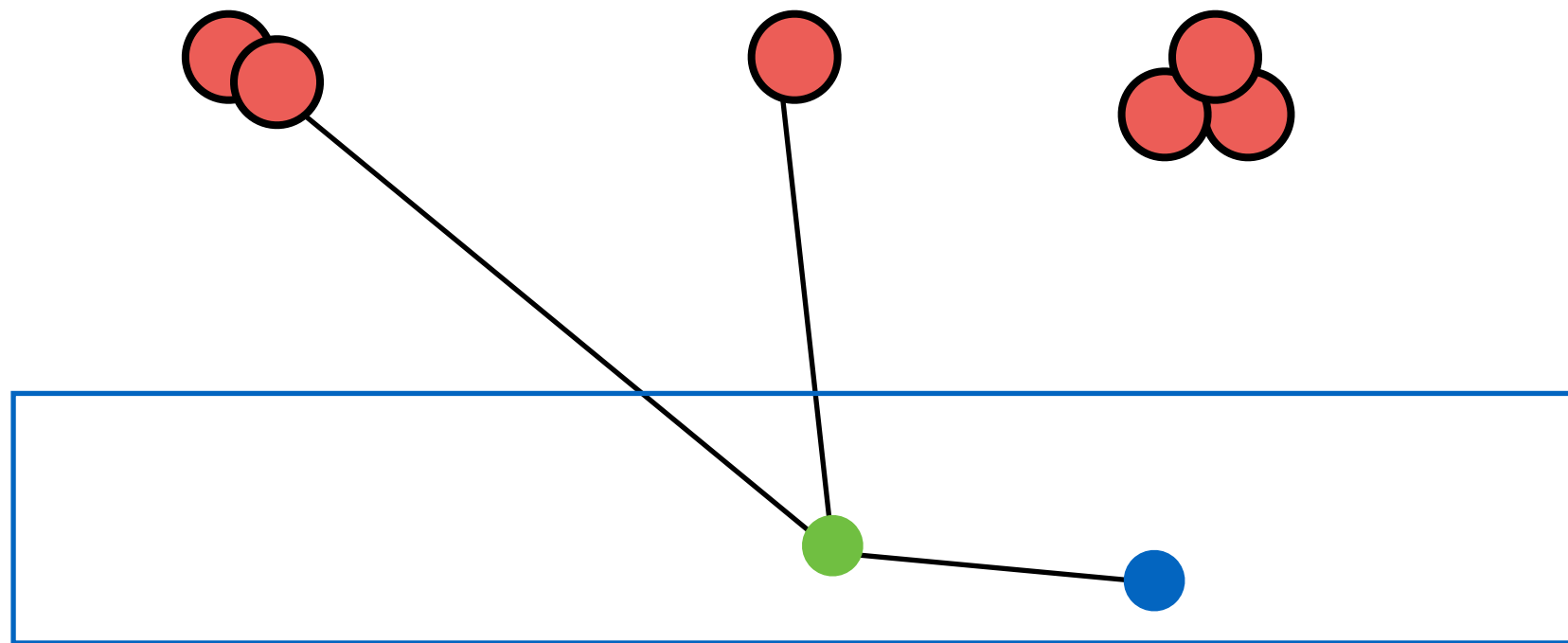
FEEDBACK VERTEX SET

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Question



...a leaf with at least two neighbors in different components.

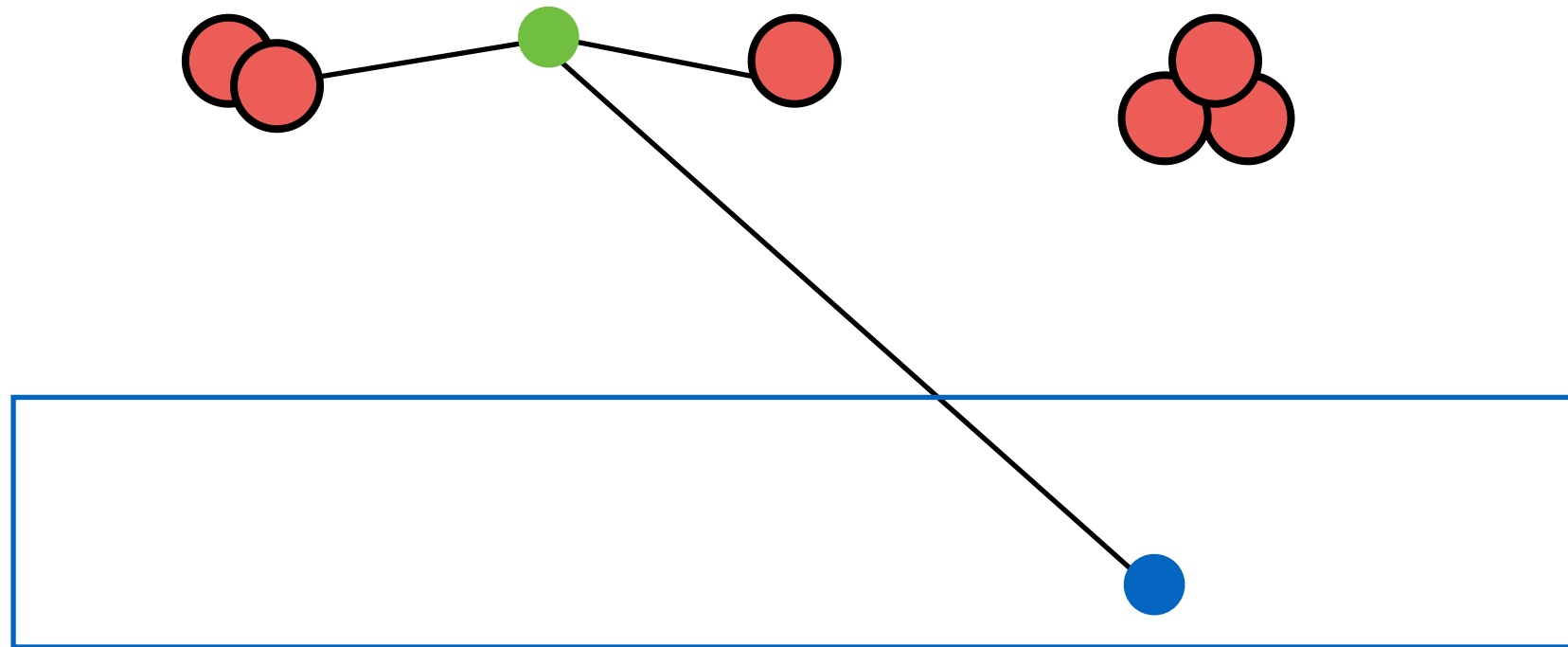
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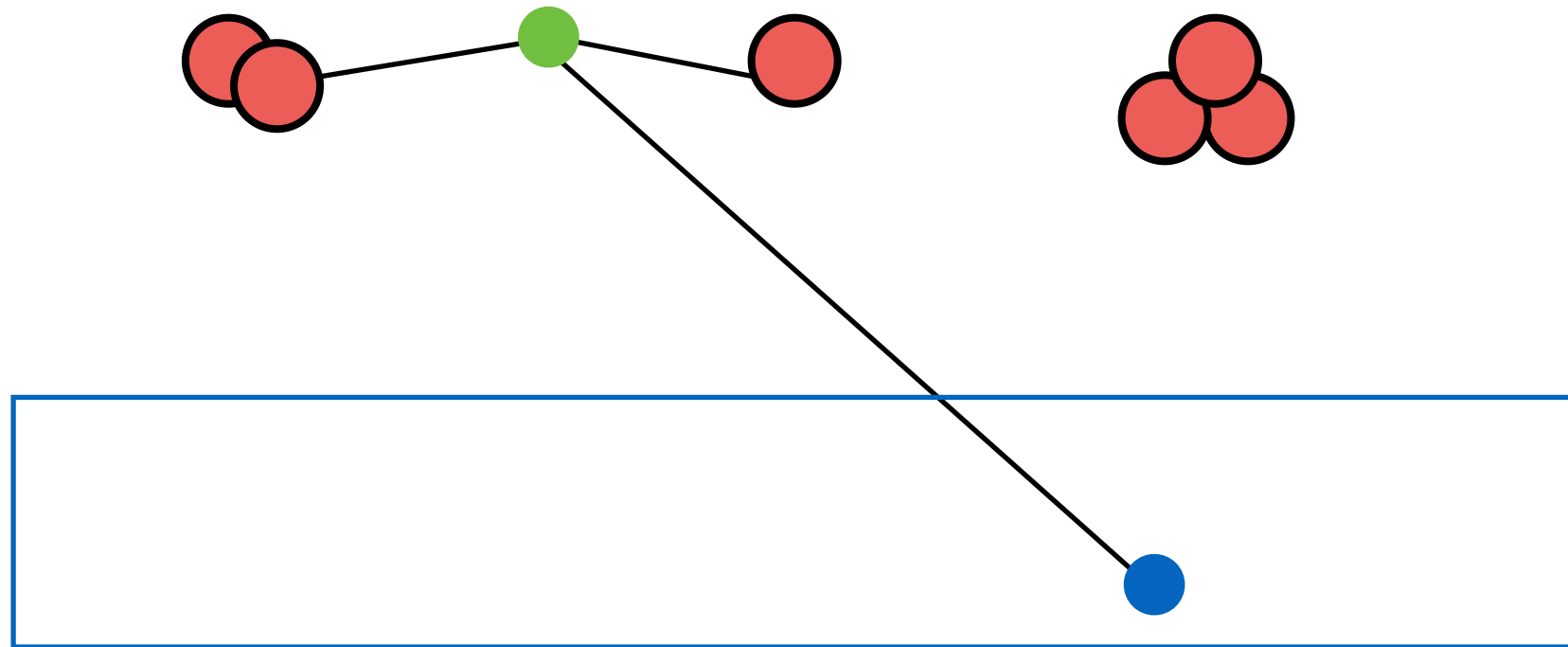
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The leaf merges two components when we don't include it.

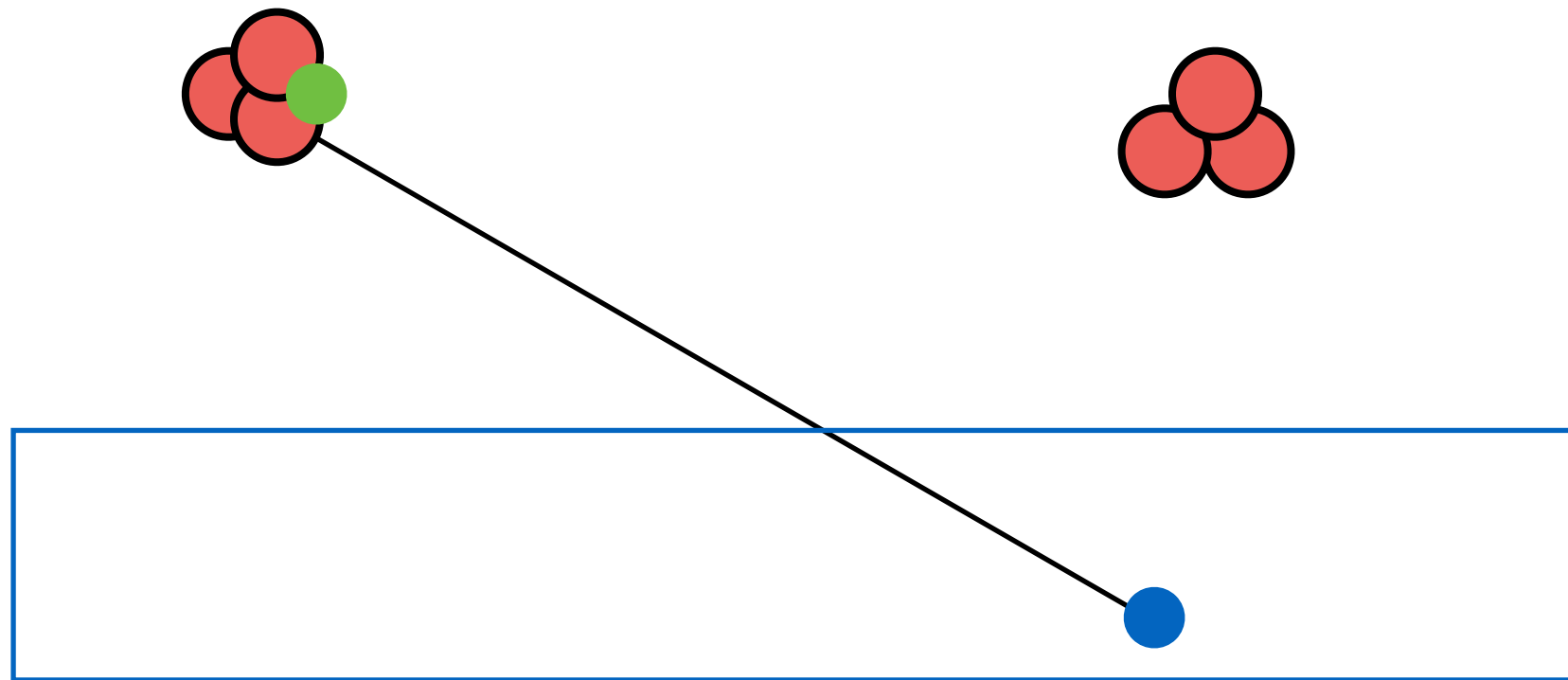
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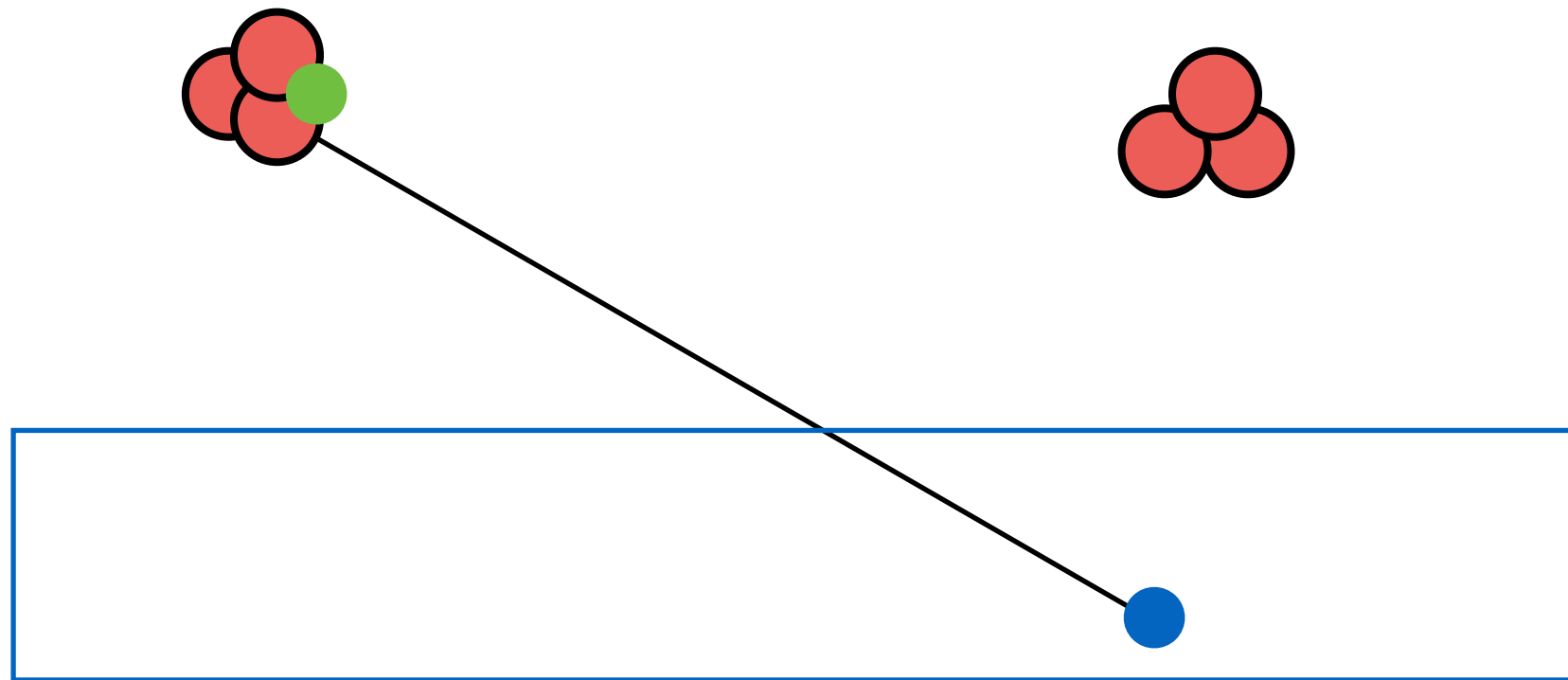
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Question



The number of components “on top” decreases.

FEEDBACK VERTEX SET

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A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Start with a leaf.

Two neighbors in one component: **forced**.

At most one neighbor above: **preprocess**.

At least two neighbors, all in different components above: **branch**.

FEEDBACK VERTEX SET

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Question

FEEDBACK VERTEX SET

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A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Let t denote the number of components among the red vertices. Let $w = (k+t)$.

FEEDBACK VERTEX SET

Input

A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Let t denote the number of components among the red vertices. Let $w = (k+t)$.

Include $v... k$ drops by 1
Exclude $v... t$ drops by at least 1

FEEDBACK VERTEX SET

Input

A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

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Let t denote the number of components among the red vertices. Let $w = (k+t)$.

Include $v \dots k$ drops by 1
Exclude $v \dots t$ drops by at least 1

Either way, w drops by at least one.

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Either way, w drops by at least one.

FEEDBACK VERTEX SET

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Either way, w drops by at least one.

Running time: $2^w = 2^{(k+t)}$

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Input

A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Either way, w drops by at least one.

Running time: $2^w = 2^{(k+t)} \leq 2^{k+k} \leq 4^k$

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Either way, w drops by at least one.

Running time: $2^w = 2^{(k+t)} \leq 2^{k+k} \leq 4^k$

Overall Running Time...

FEEDBACK VERTEX SET

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A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

Either way, w drops by at least one.

Running time: $2^w = 2^{(k+t)} \leq 2^{k+k} \leq 4^k$

Overall Running Time...

$$\sum_{i=1}^k \binom{k}{i} 4^i = 5^k$$

FEEDBACK VERTEX SET

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A graph on n vertices, m edges and an integer k .

Is there a subset of at most k vertices whose removal makes the graph acyclic?

Question

This implies an $O(5^k)$ algorithm for FVS.

FEEDBACK VERTEX SET

Input

A graph on n vertices, m edges and an integer k .

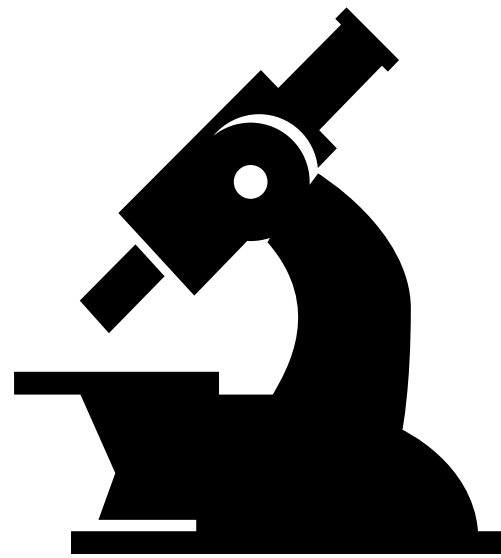
Is there a subset of at most k vertices whose removal makes the graph acyclic?

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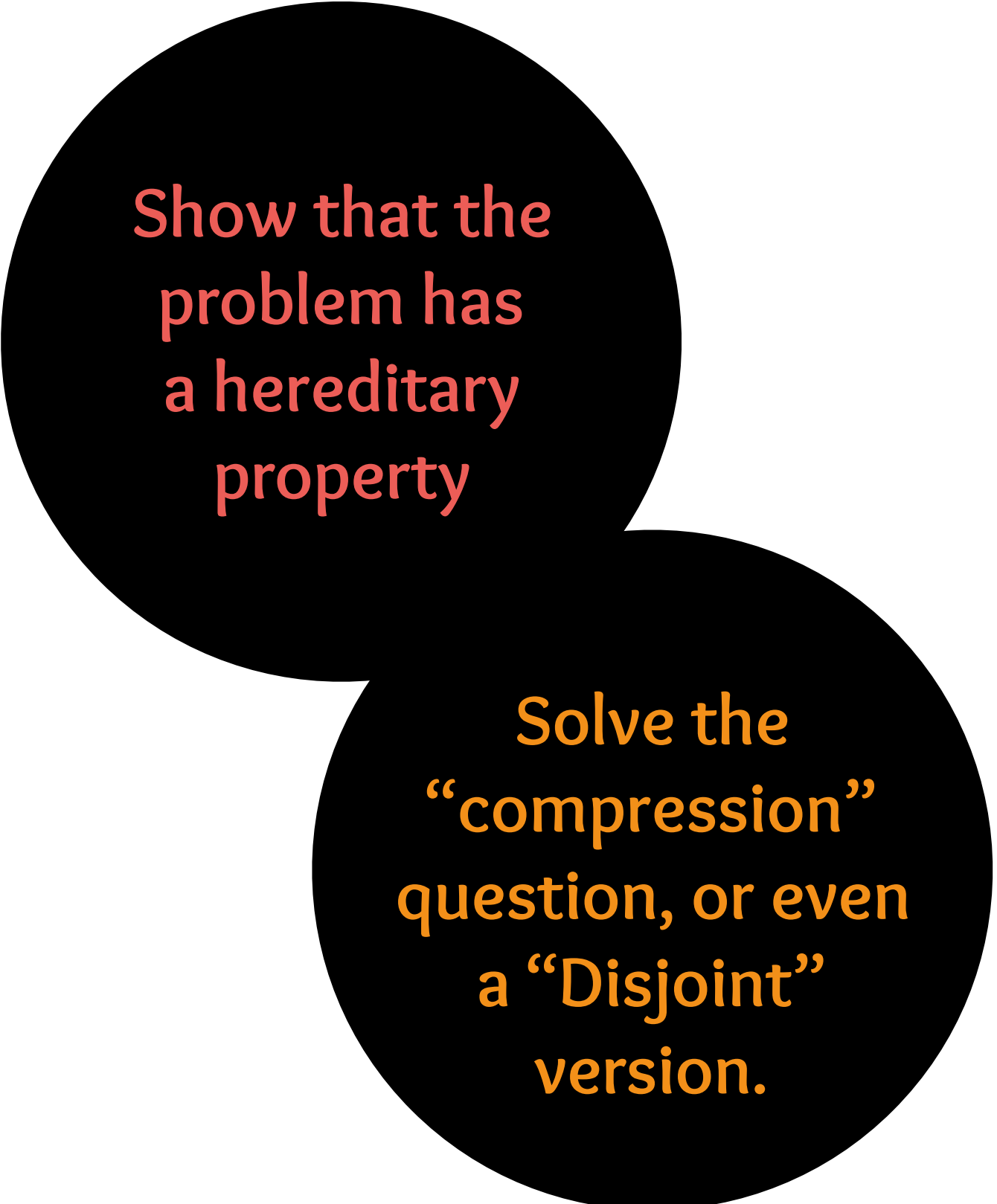
This implies an $O(5^k)$ algorithm for FVS.



FEEDBACK VERTEX SET

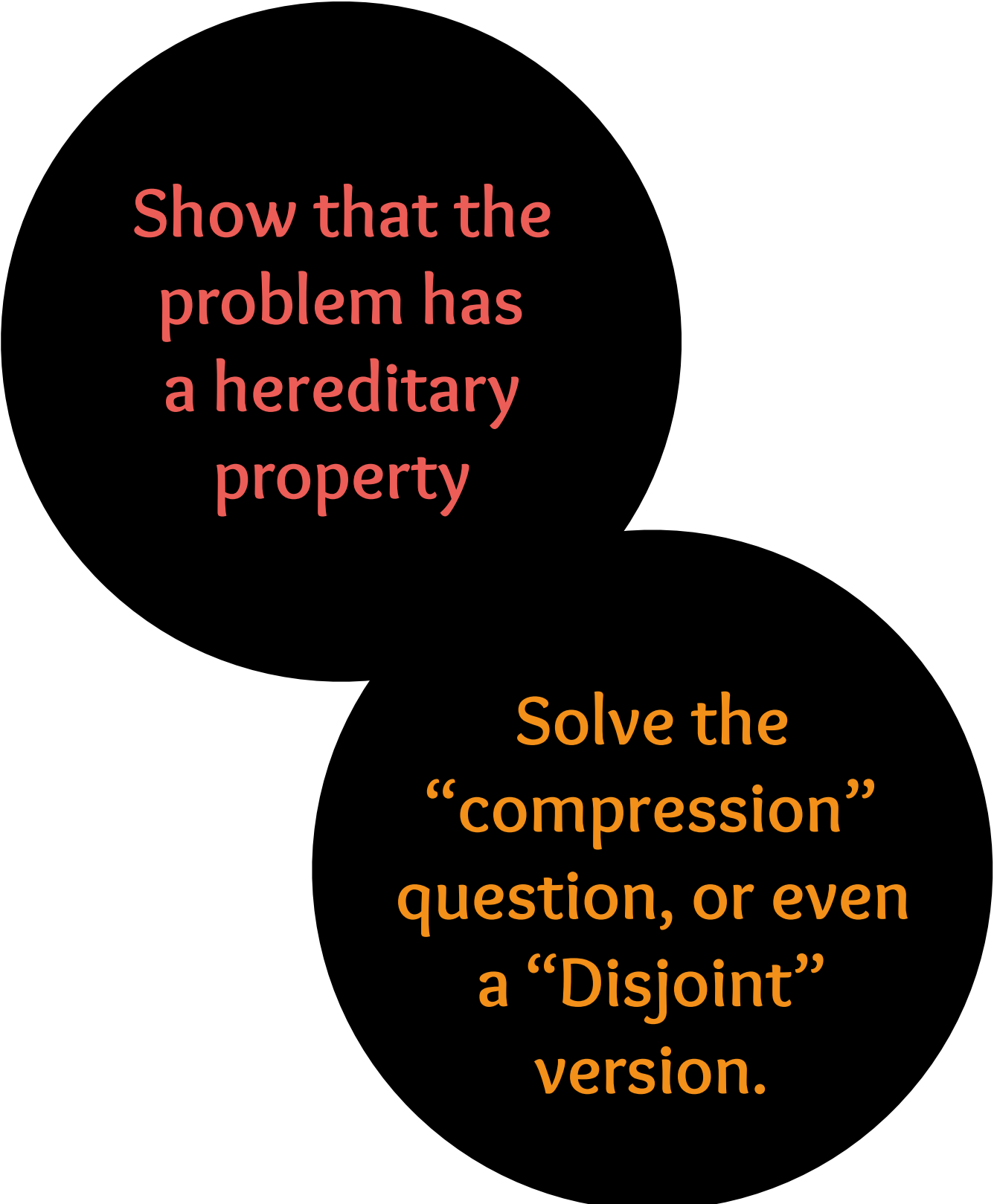


TAKE AWAY...



Show that the
problem has
a hereditary
property

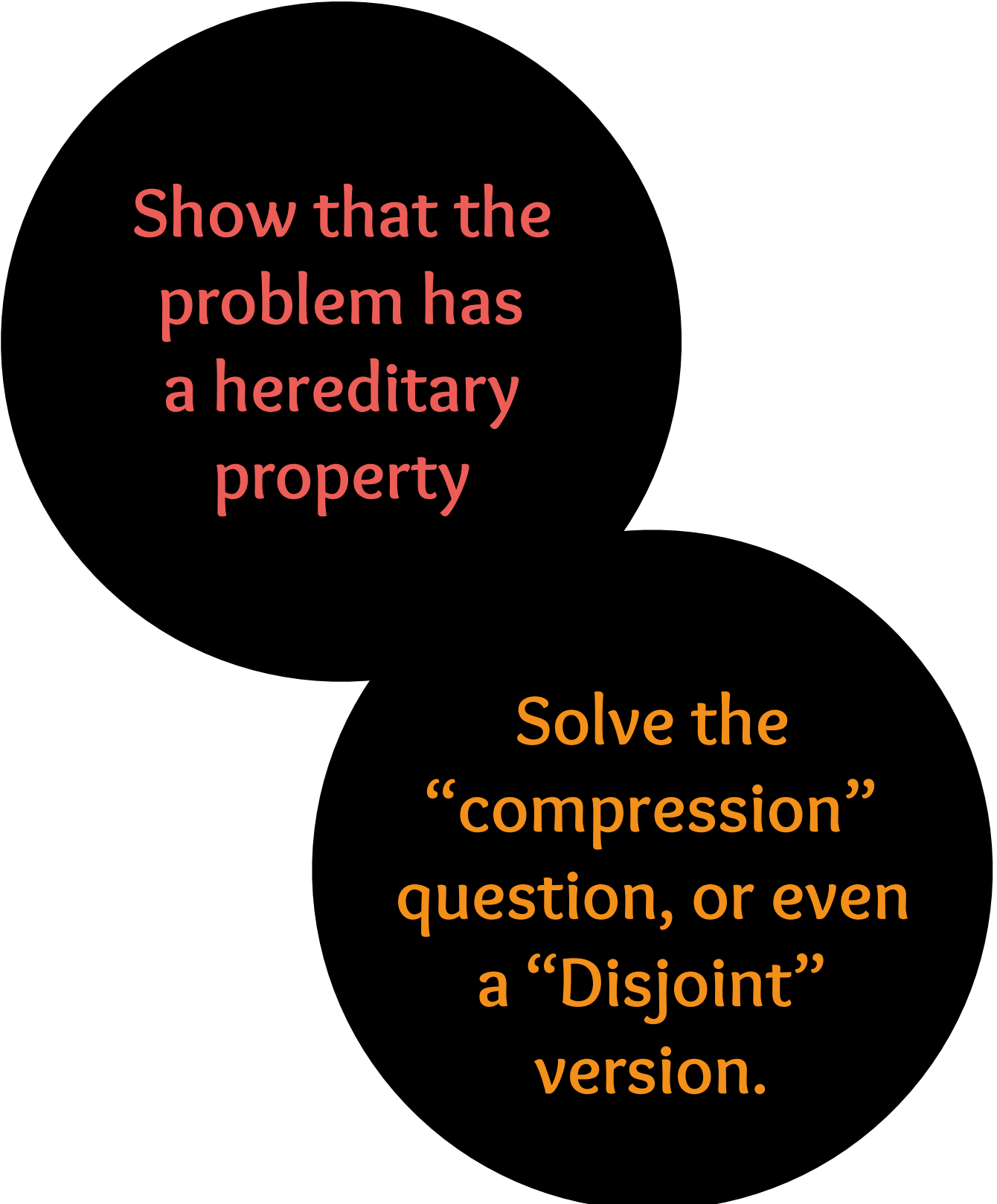
Solve the
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The compression algorithm
can be fairly non-trivial.

THANK YOU!