

Algorithmic aspects of Firefighting with Critical Nodes

INAE WEBINAR SERIES

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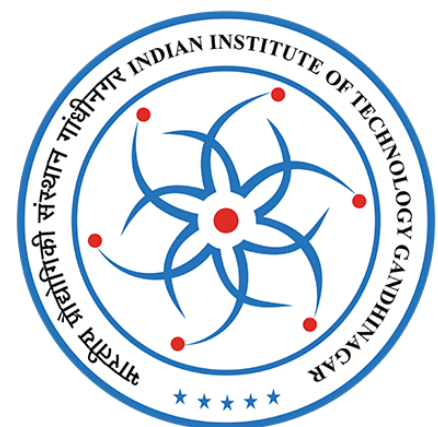
भारतीय राष्ट्रीय अभियांत्रिकी अकादमी
Indian National Academy of Engineering

8nd January, 2021

Neeldhara Misra, IIT Gandhinagar

Based on joint work with:

Jayesh Choudhari, Anirban Dasgupta, M. S. Ramanujan



Talk Outline

- 1** **DEFINING THE GAME & AN OVERVIEW OF KNOWN RESULTS**
- 2** **SEPARATORS: A SMALL DETOUR**
- 3** **A QUICK NOTE ON TREES**
- 4** **OTHER VARIANTS & CONCLUDING REMARKS**

Talk Outline

1 DEFINING THE GAME & AN OVERVIEW OF KNOWN RESULTS

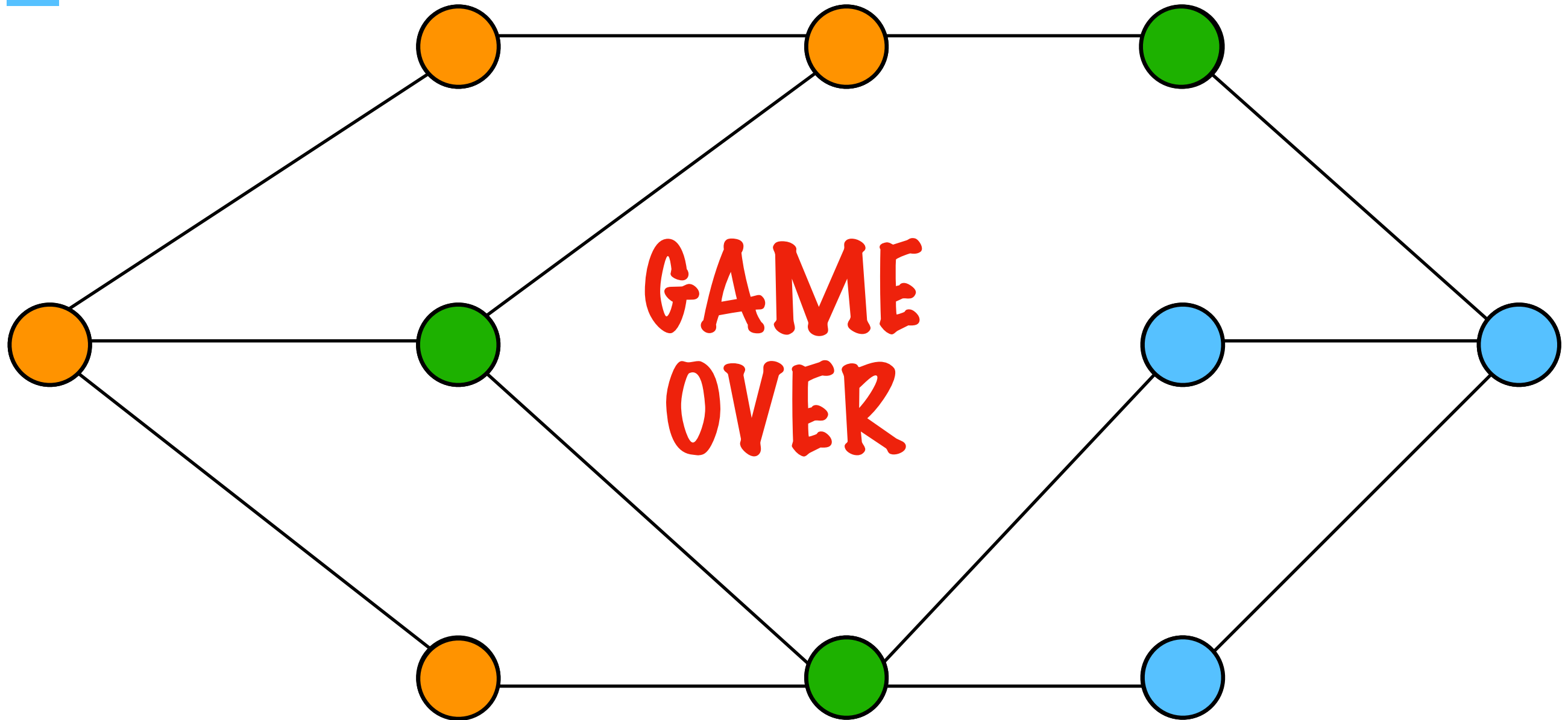
2 SEPARATORS: A SMALL DETOUR

3 A QUICK NOTE ON TREES

4 OTHER VARIANTS & CONCLUDING REMARKS

-  BURNT
-  PROTECTED
-  SAVED

Firefighting Rules



0 1 2 3 4 5

Firefighting Goals

maximizing
saved vertices



Firefighting Goals

minimizing
burned vertices



Firefighting Goals

minimizing

firefighters^{*}



Firefighting Goals

minimizing
number of rounds



Firefighting Goals

saving
specific vertices



Firefighting Known

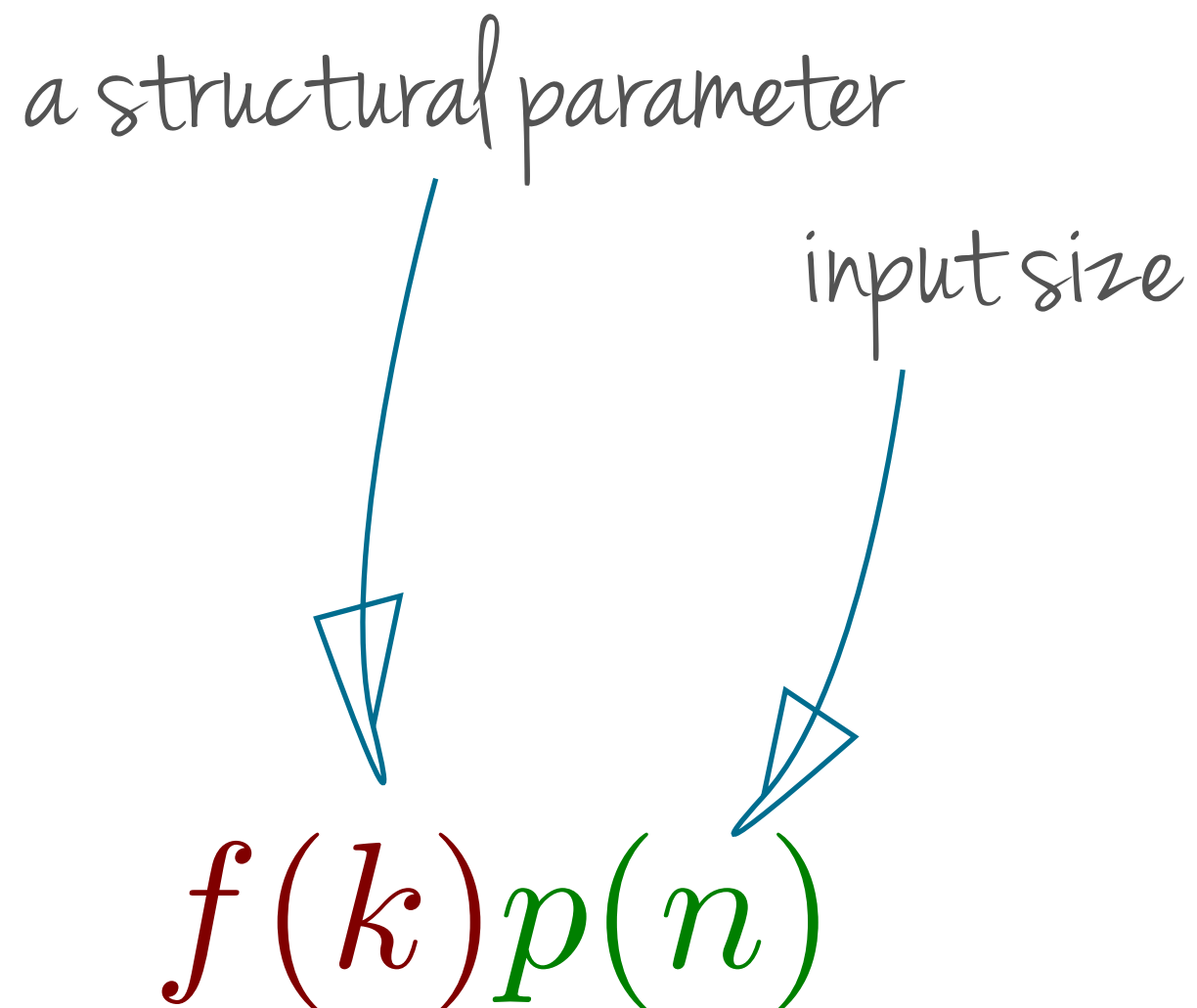
King et.al. showed that for a tree with maximum degree at most 3, **it is NP-hard to save a critical set** with budget of one firefighter per round, but is polynomial time when the fire starts from a vertex of degree at most 2.

Chopin extended the hardness result to a per-round budget $b > 1$ and to trees with maximum degree $b + 2$.

Chalermsook et.al. gave **an approximation to the number of firefighters per round** when trying to protect a critical set.

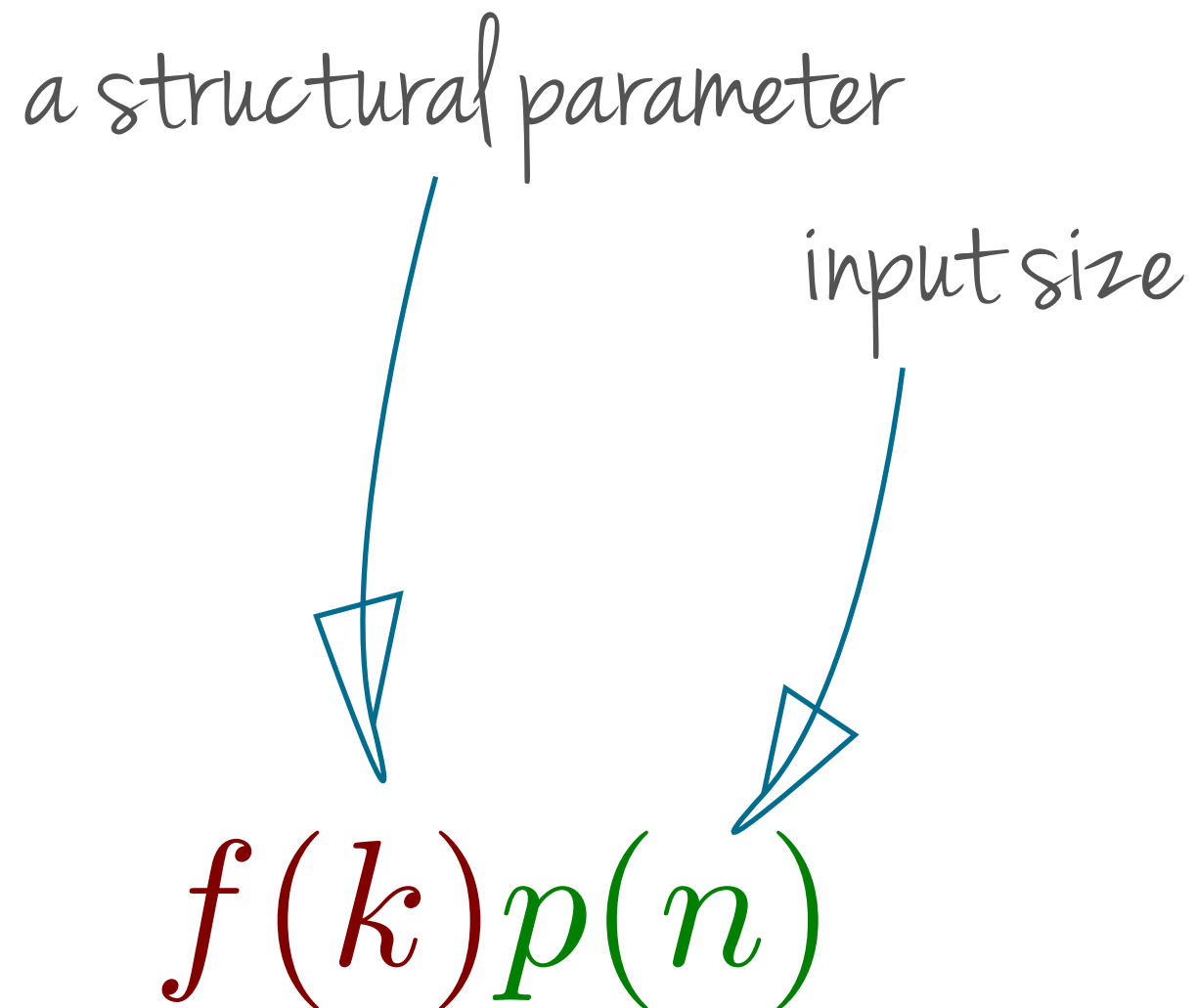
Firefighting Framework

Evaluate running times of algorithms as a function of the size of input **and** additional structural parameters.



Firefighting Framework

Problems that admit algorithms with such a running time are “fixed-parameter tractable”.



Firefighting Parameters

(as hard as CLIQUE)

#of critical nodes

(already hard for TREES)

the treewidth of the graph

(things work out)

#of firefighters

(open, well-studied without critical nodes)

Other Parameters

Talk Outline

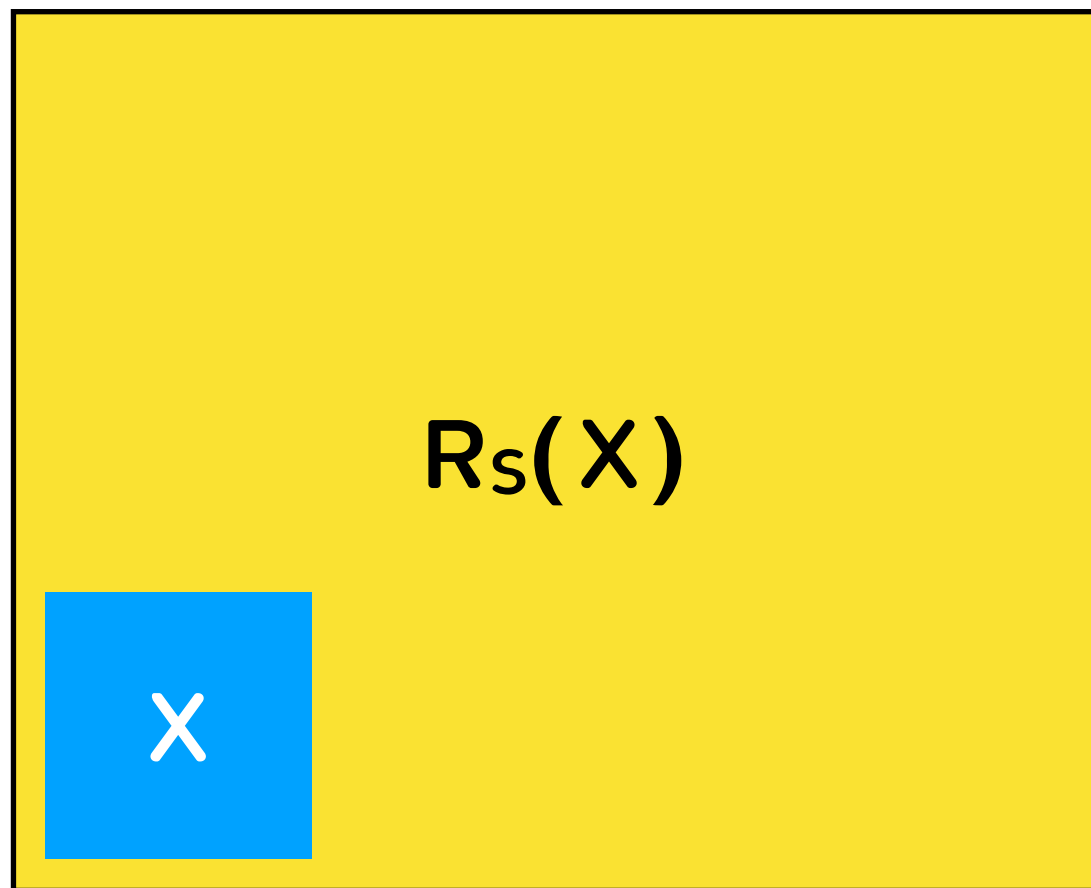
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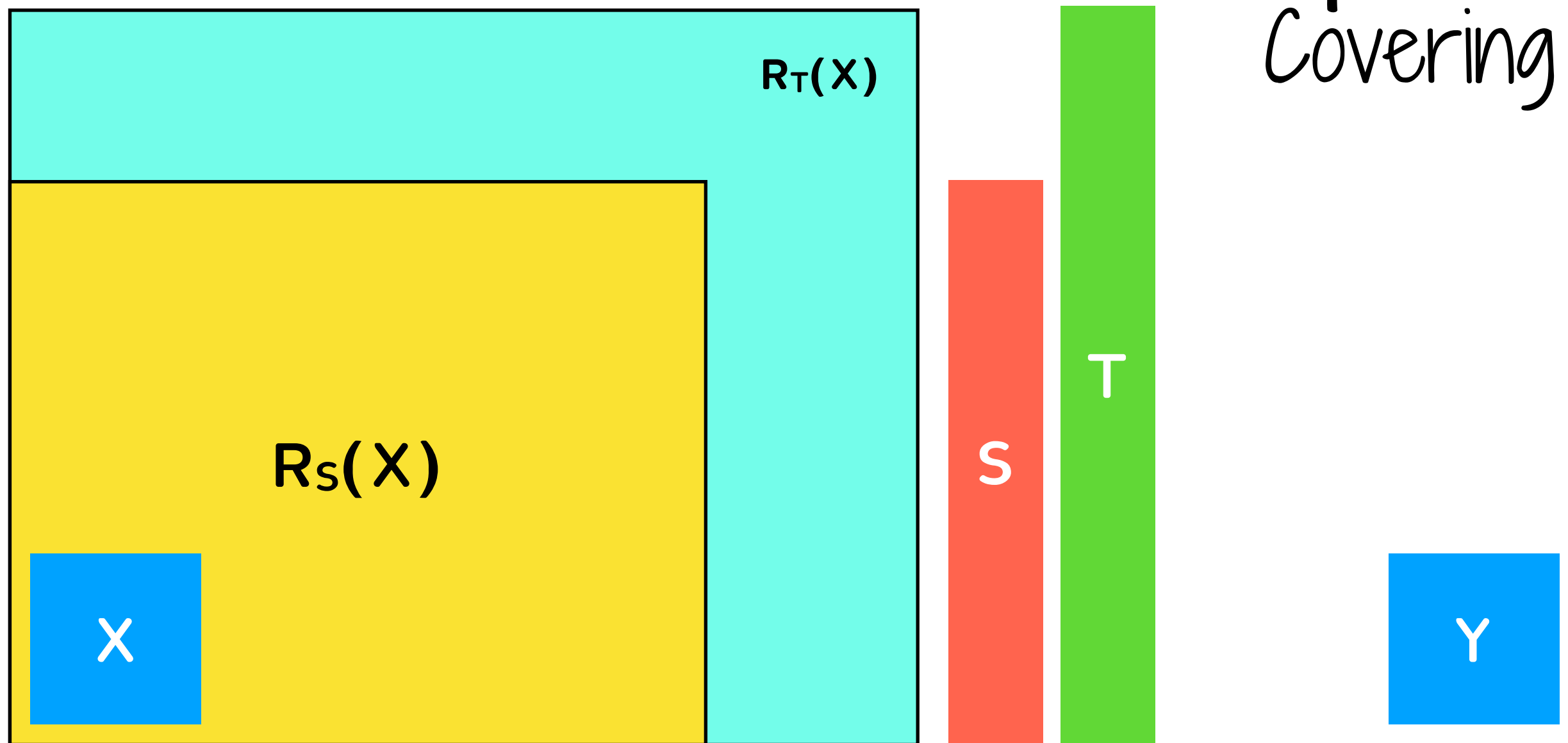
4 OTHER VARIANTS & CONCLUDING REMARKS

Separators Reachability



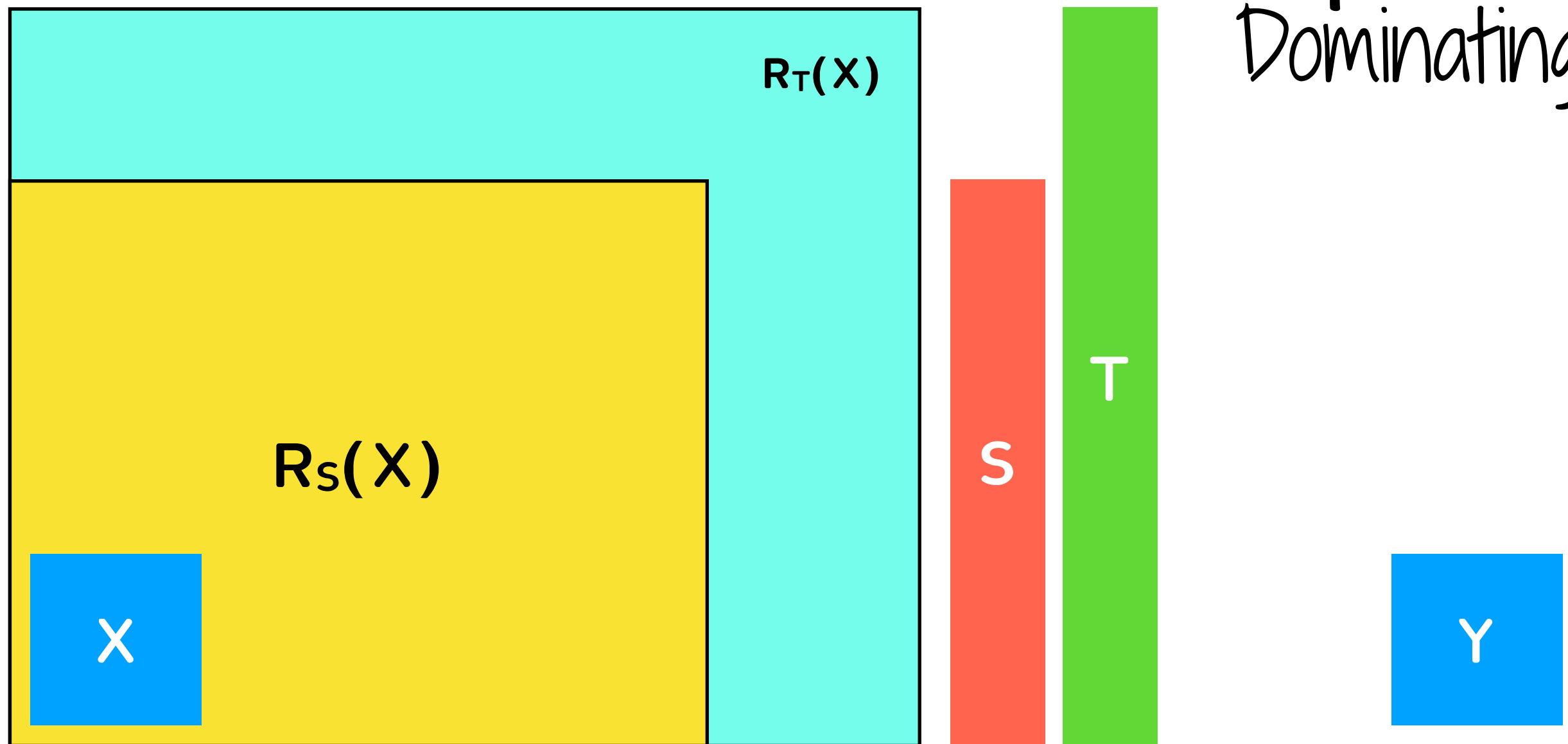
Reachability Set of X with respect to S :
Everyone reachable from X in $G \setminus S$.

Separators Covering



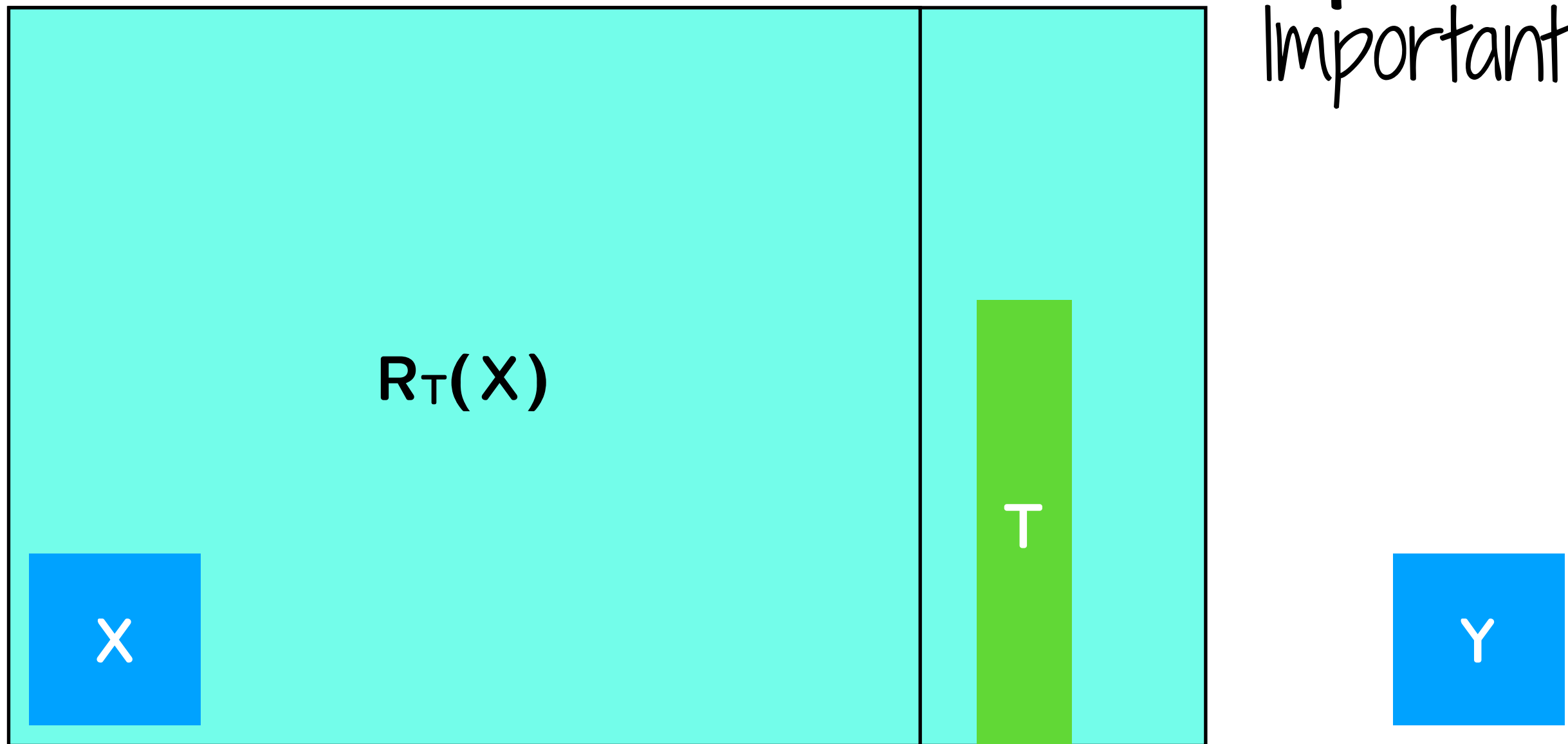
T “**covers**” S if $R_T(X)$ contains $R_S(X)$.

*Separators
Dominating*



T “**dominates**” S if $|T|$ is at most $|S|$
and T covers S .

*Separators
Important*



“Important”: minimal &
not dominated by anyone.

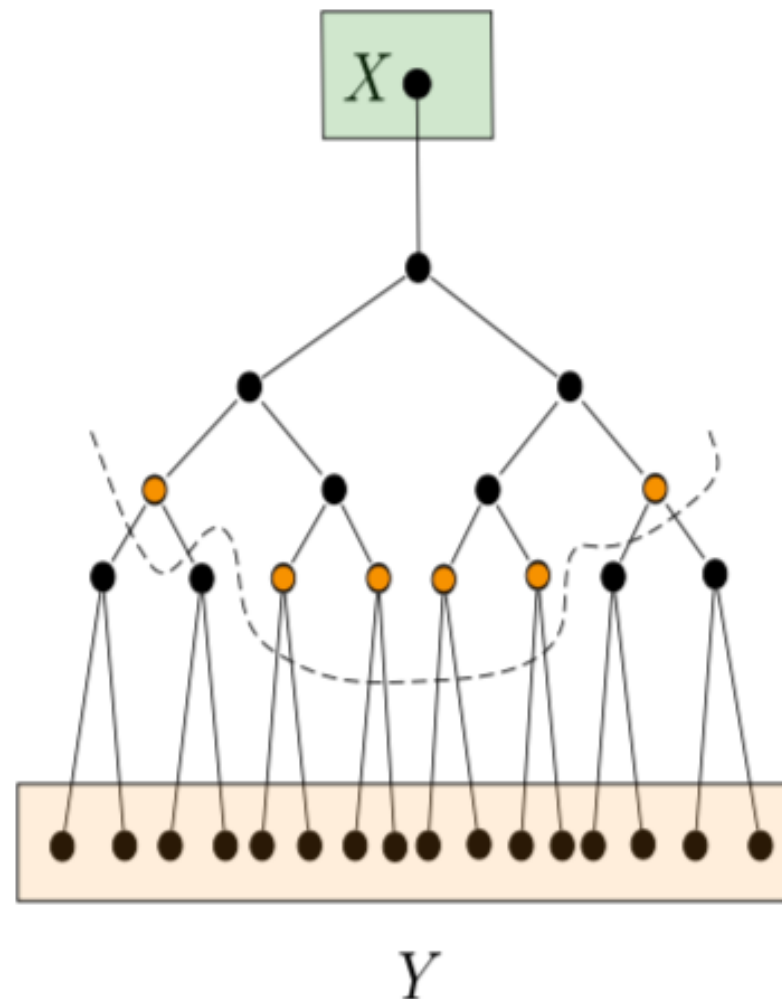
Separators Important

For every $k \geq 0$ there are at most 4^k important X-Y separators of size at most k .



Separators Important

For every $k \geq 0$ there are at most 4^k important X - Y separators of size at most k .



*Picture and example from Marx's slides titled "The bound is also tight up to polynomial factors. Important separators and spiders."

Separators Intuition

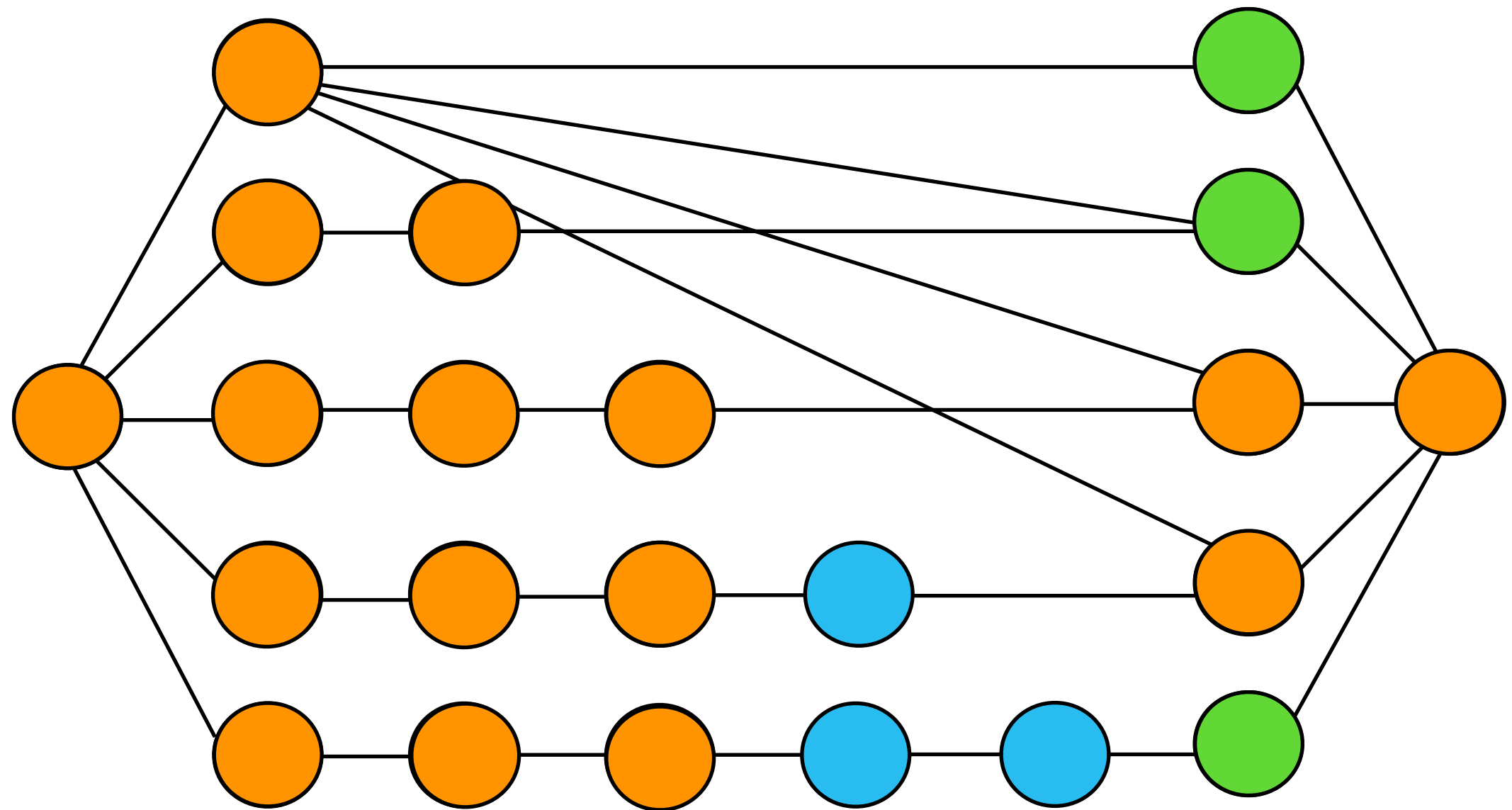
Pros of Important Separators

Since an important separator is far away from the source of the fire, it “buys us time” to safely position our firefighters.

Cons of Important Separators

By choosing to place firefighters on an important separator, we might fail to “block a leak” early even when there is an opportunity.

Separators Intuition



0 1 2 3 4 5 6

Important
Separator

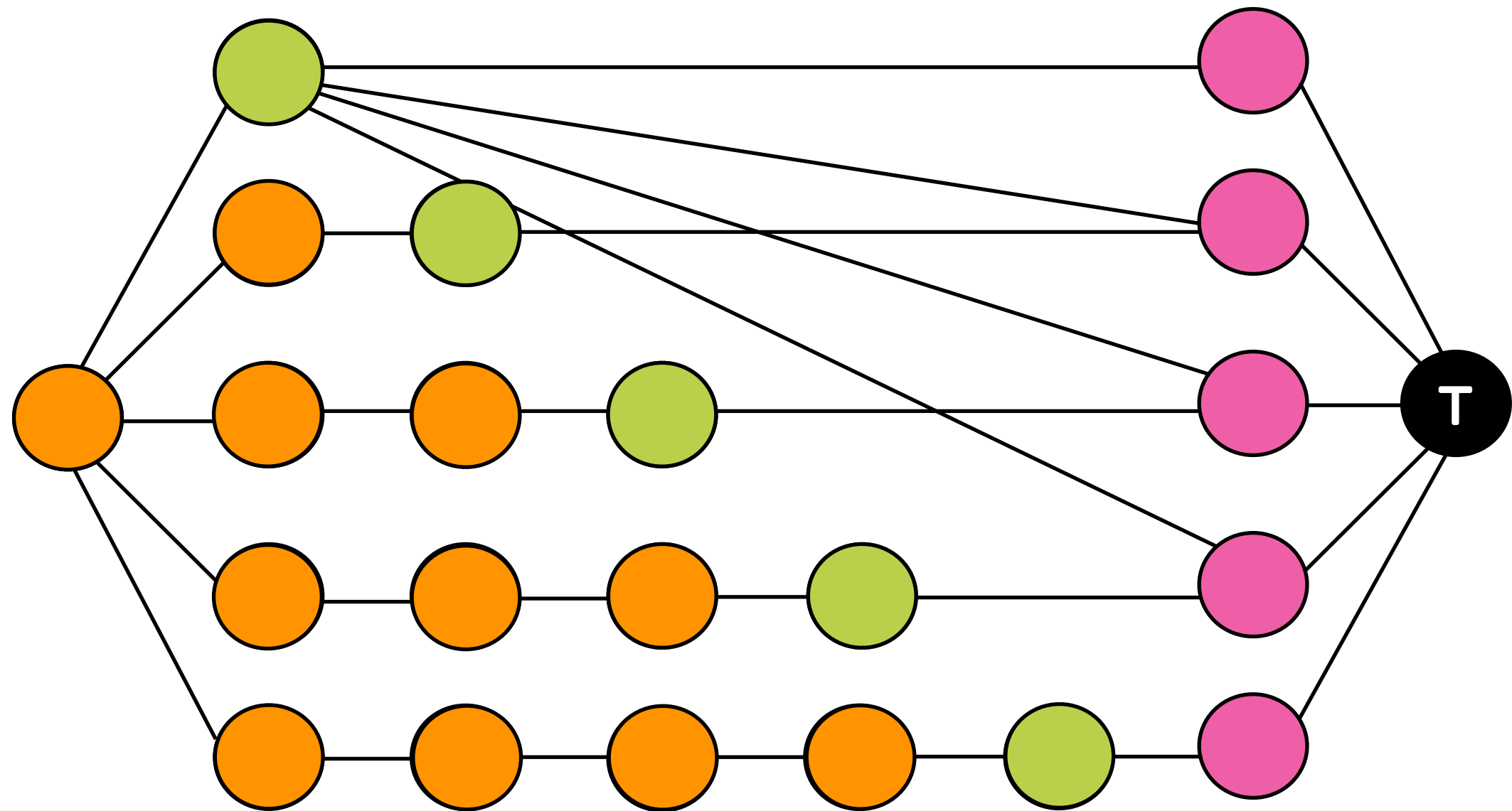
Firefighting
Solution



Important
Separator



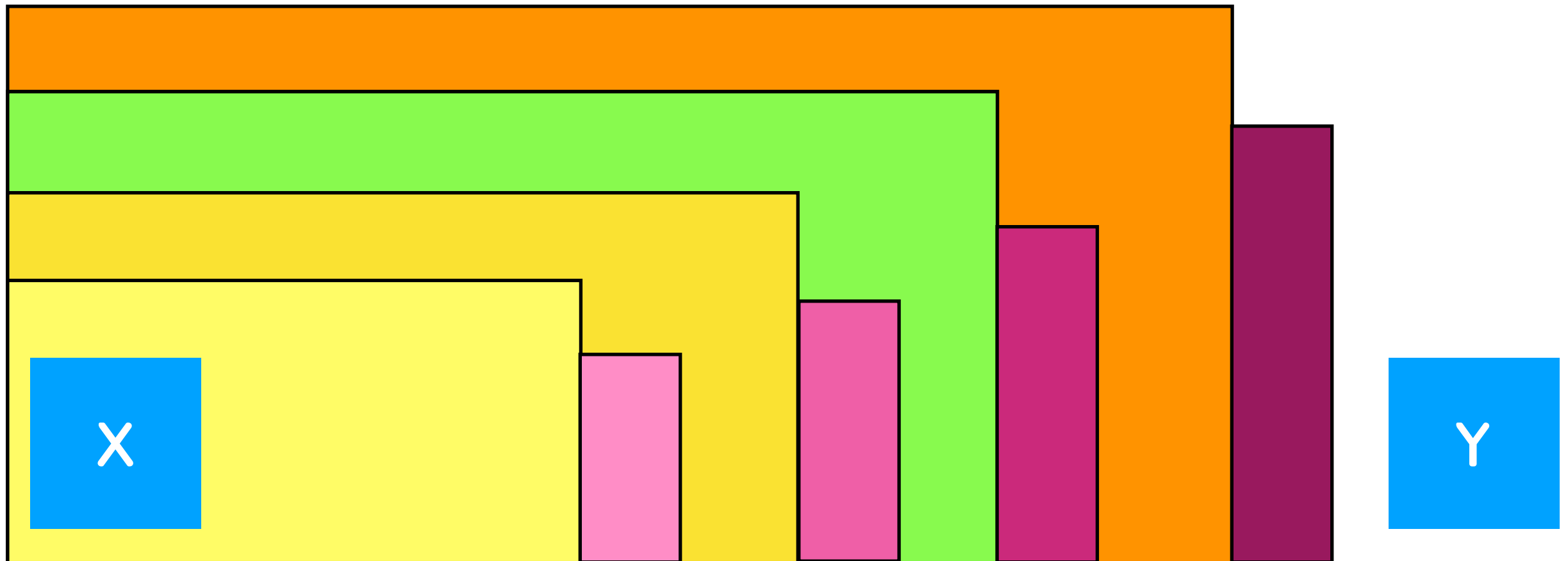
*Separators
Intuition*



0 1 2 3 4 5 6 7 8 9

Separators

Tight Sequences



The pink sets are minimal X,Y-separators.

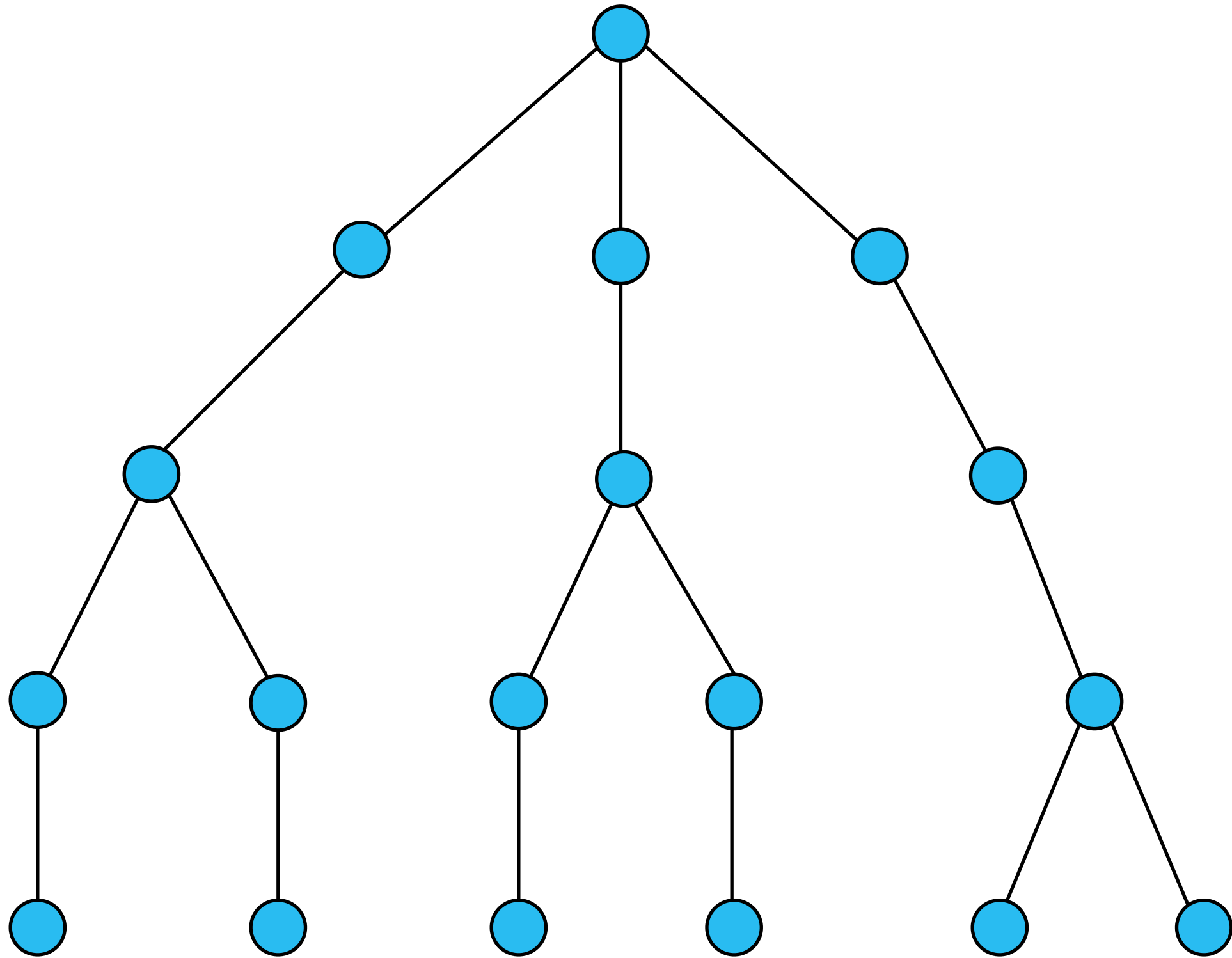
No minimal separator lies entirely in the green region.

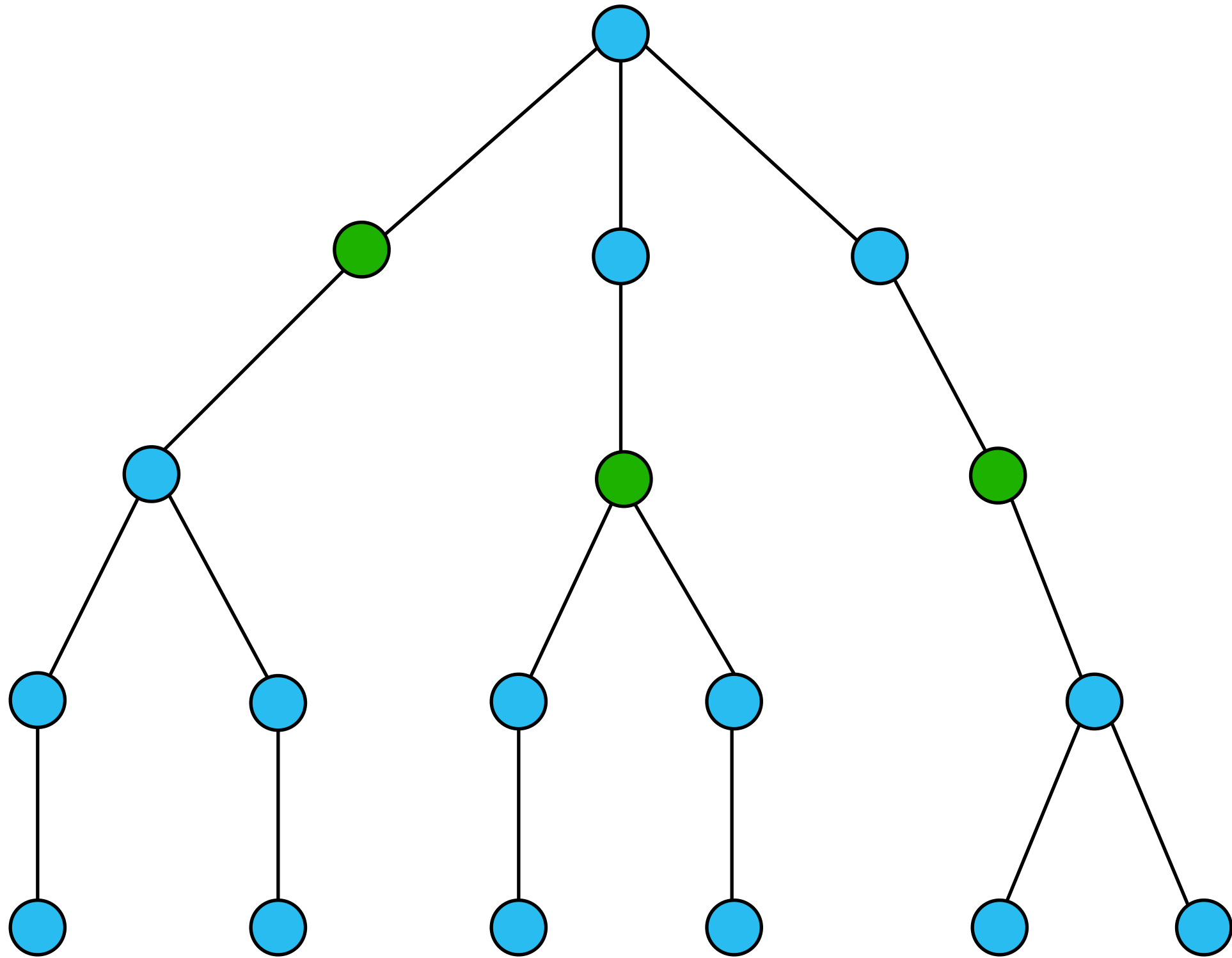
Talk Outline

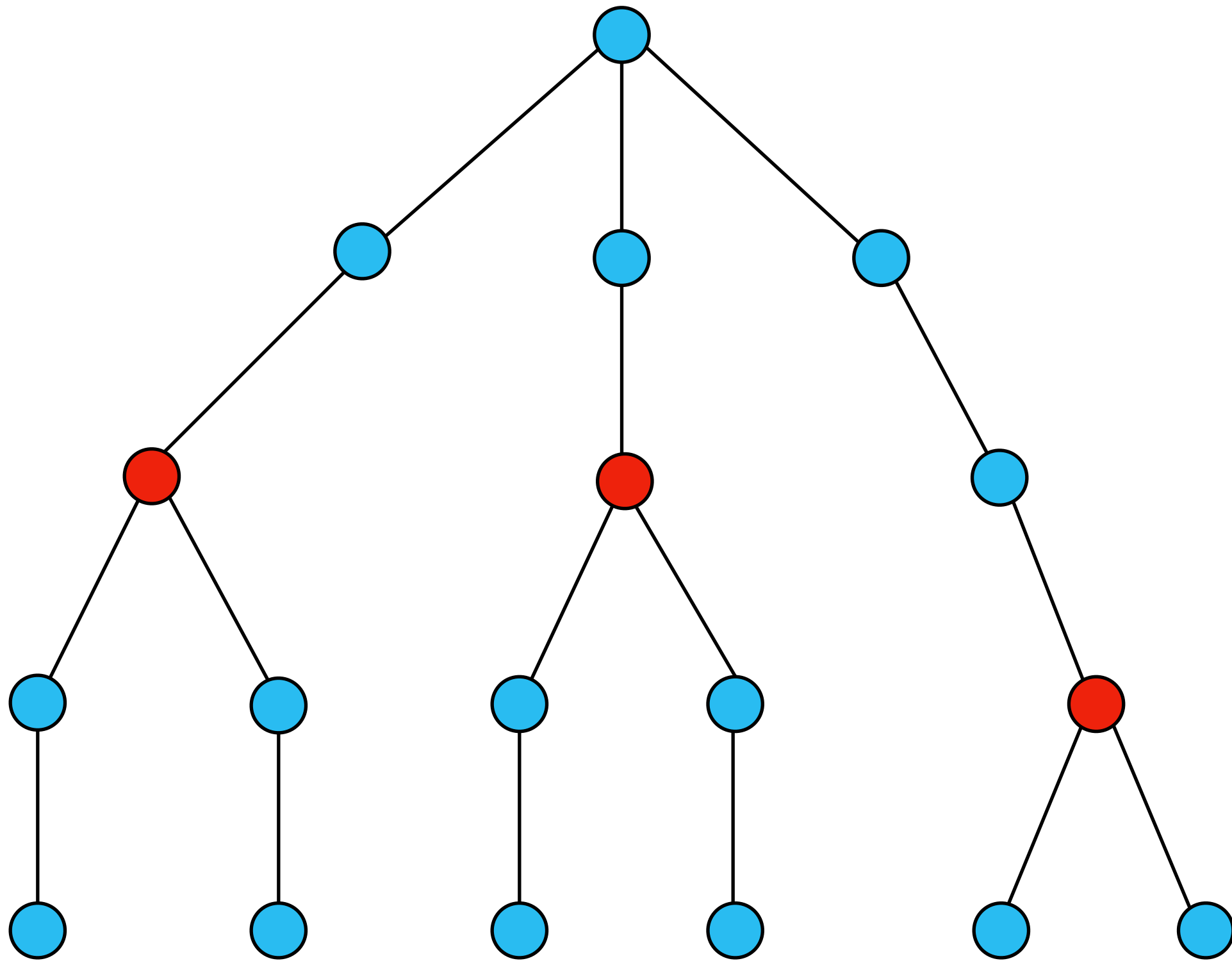
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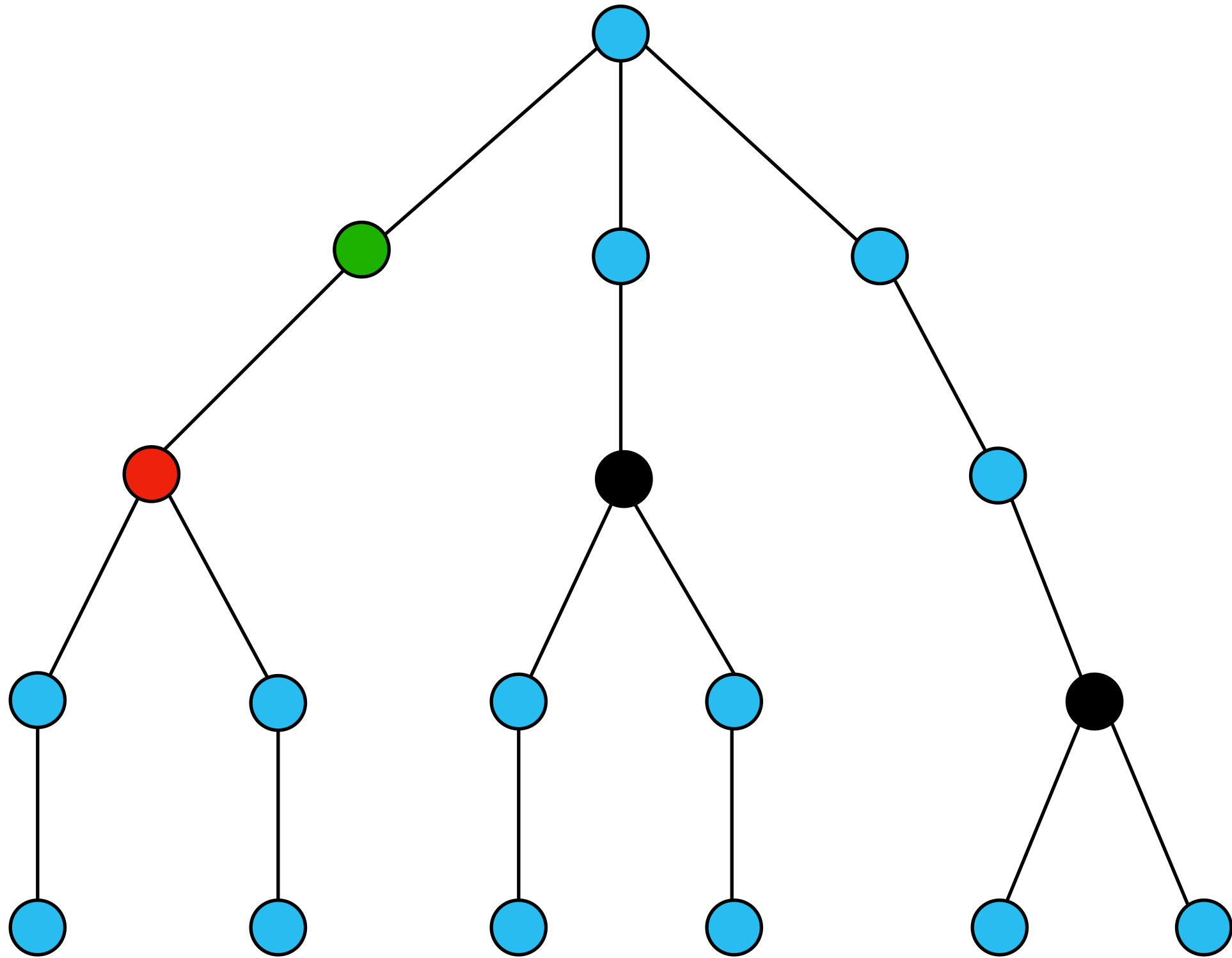
On trees, it is enough to
restrict our attention to just
the important separators.

Trees









Trees

Kernelization

(x, k)



(x', k')

$|x|^{O(1)}$ **FAST** time

$|x'| = f(k)$ **SMALL** and $k \leq k'$

$(x, k) \equiv (x', k')$ **SAME**



$f(k)$ -kernel

Trees **do not** admit a polynomial-sized kernel when parameterized by the number of firefighters.

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The “spreading” model.

The protection spreads BFS-style just like the fire.

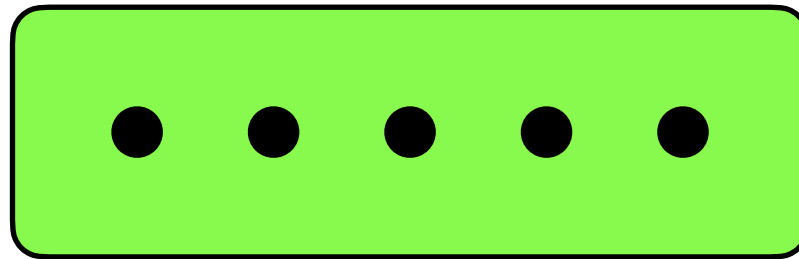
(as hard as DOMINATING SET)

Accounting for thresholds.

Accounting for multiple optimization objectives.

Optimizing the number of firefighters/round.

Other Parameters



Tractable class of instances.

“Making lives easier for firefighters.”

Fedor V. Fomin, Pinar Hegghernes, and Erik Jan van Leeuwen

Influence Maximization

Flipping the goal:

Reaching out rather than throttling.

Thank you!

