

Workshop on Game Theory and Mechanism Design

Computational Social Choice

An Introduction

January 2016, IISc

Social Choice

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Achieve social objectives
that account for
individual preferences.

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that account for
individual preferences.

No money involved.

Computational Social Choice
F. Brandt, V. Conitzer and U. Endriss
Multiagent Systems, MIT Press, 2012

This talk is based on the material in the
following book chapter.

Computational Social Choice
F. Brandt, V. Conitzer and U. Endriss
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A typical voting scenario involves
a set of alternatives, and
a set of voters.

Voters have preferences over the alternatives,
which can be modelled in various ways.

In this talk, we'll think of preferences as
linear orders over the rankings.

A voting rule is a mechanism for collecting preferences and producing a winning alternative, or a consensus ranking over all alternatives that reflects the collective preferences of the society.

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The following slides demonstrate an example with three alternatives (dinner, concert, coffee) demonstrating three voting rules that seem equally reasonable, but each of which have different outcomes.

A group of friends have an evening to spend.









(Plurality)





(Plurality)



We say that a voter (or a group of voters) can manipulate if they can obtain a more desirable outcome by misreporting their preferences.

We'll see that plurality is vulnerable to this behaviour.



(Plurality)





(Plurality)





(Plurality)





(Plurality)



Manipulation



(Plurality)



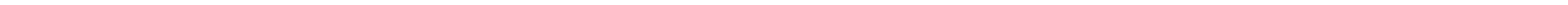
This scheme is intended only for honest men.



















An alternative that beats all the others
in pairwise comparisons.



An alternative that beats all the others
in pairwise comparisons.



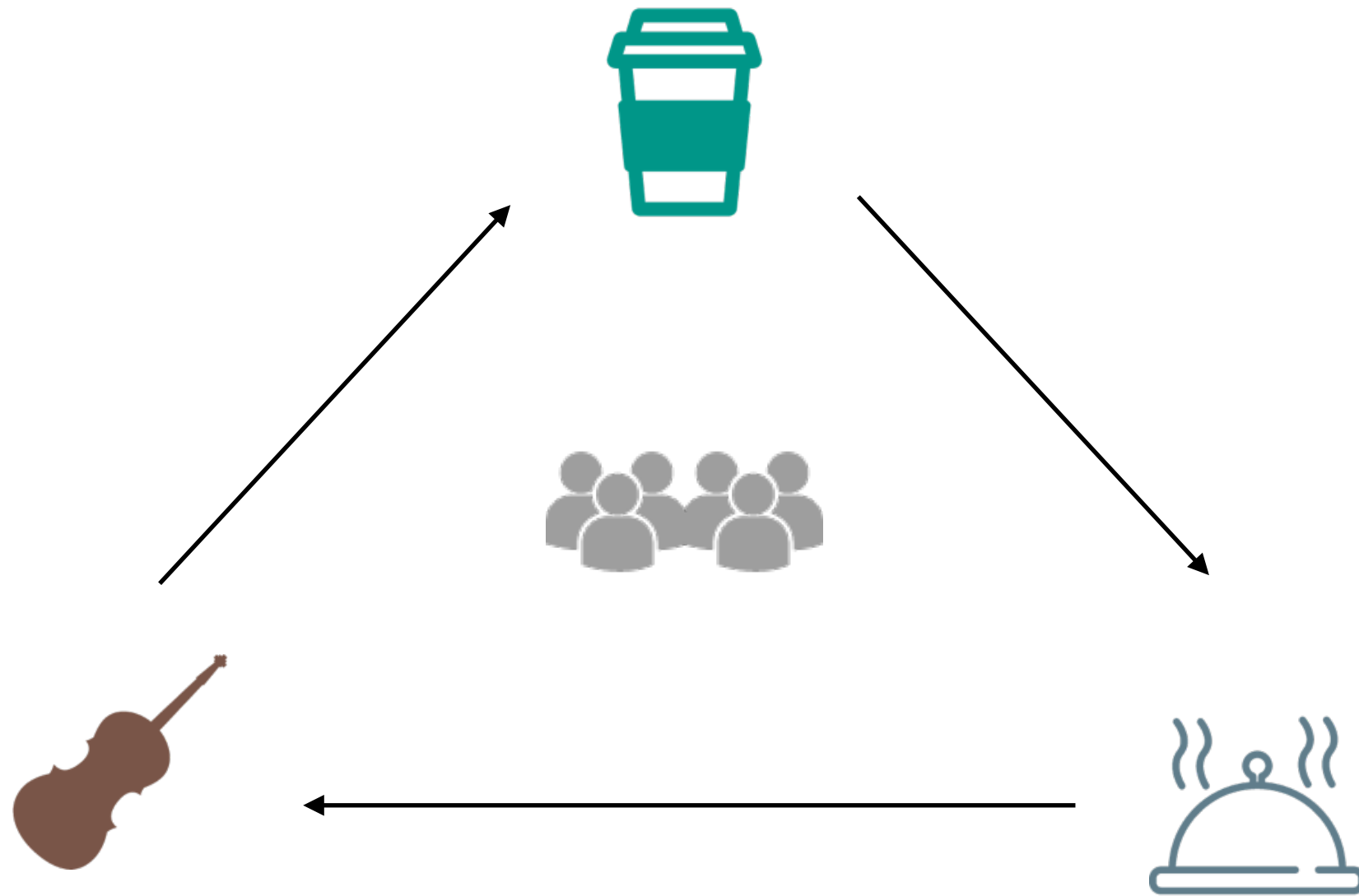
(Condorcet)



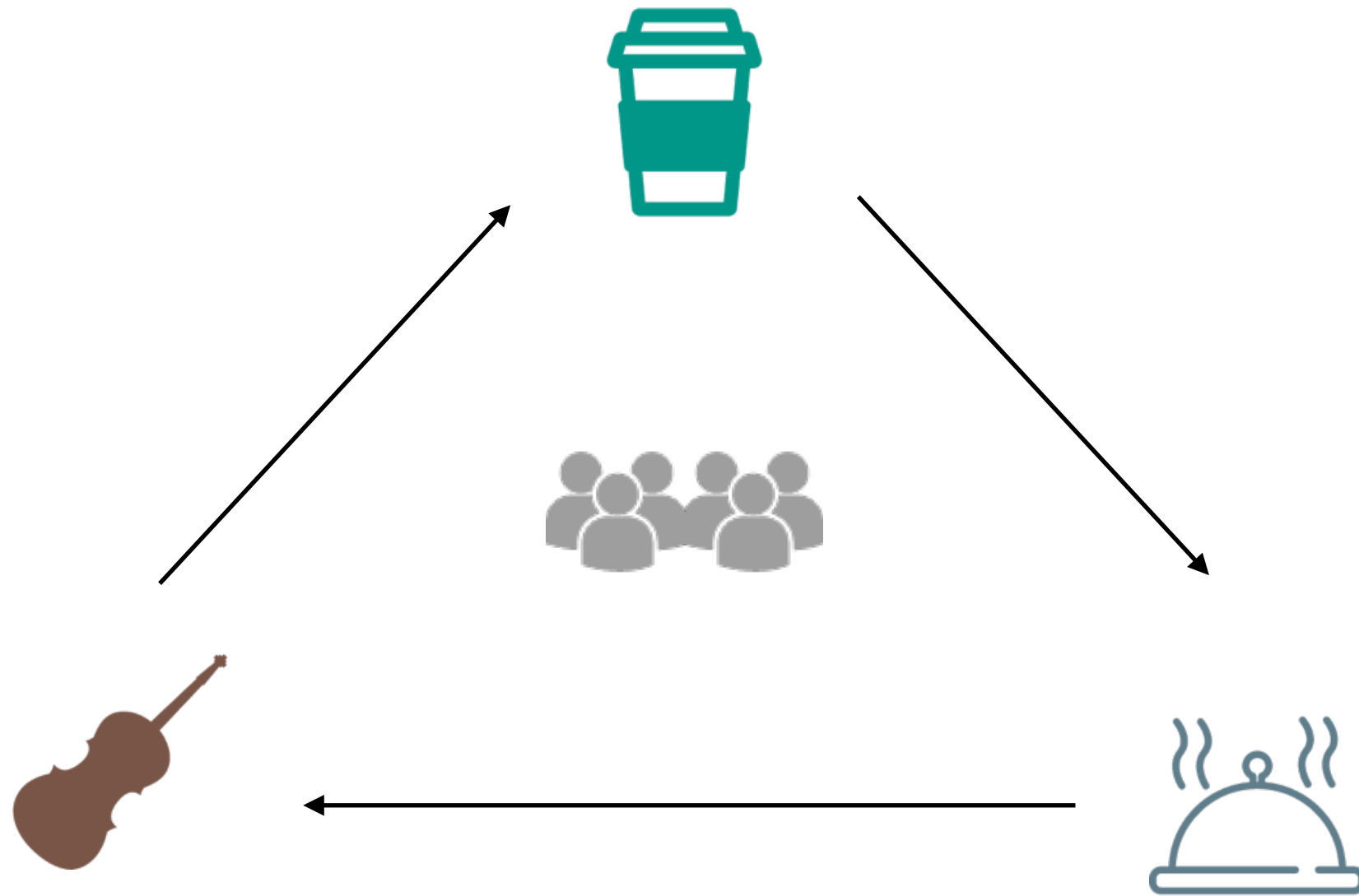
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An alternative that beats all the others
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may not exist!

Desirable properties of voting rules.

We'll now pause to formally define
social choice functions and
social welfare functions.





Social Welfare Functions (SWF)





Social Choice Functions (SWF)





Unanimity



Unanimity



Unanimity

If everyone prefers A to B then
the consensus ranking also prefers
A over B.





Independence of Irrelevant Alternatives



Independence of Irrelevant Alternatives



Independence of Irrelevant Alternatives



Independence of Irrelevant Alternatives

Suppose a SWF prefers A over B in the consensus ranking for a profile P.

Let Q be a profile that is the same as P when projected on the candidates A and B.

Then the SWF must prefer A over B in the consensus ranking that it determines for Q as well.



Impossibility Results



Arrow (1949)

Arrow (1949)

When voters have three or more alternatives,
any social welfare function which respects
unanimity and
independence of irrelevant alternatives
is a dictatorship.









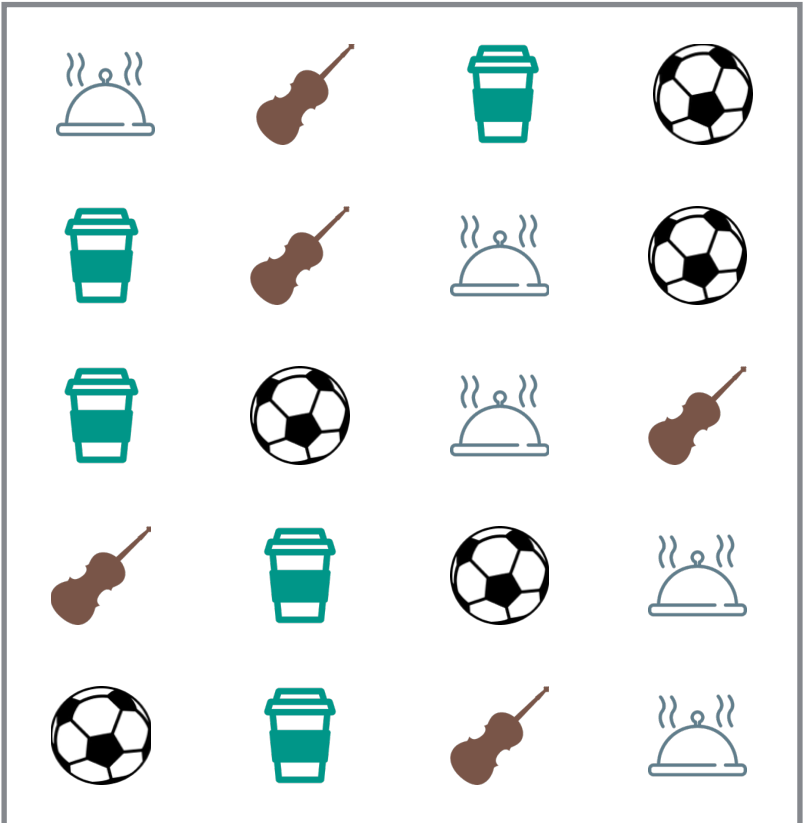
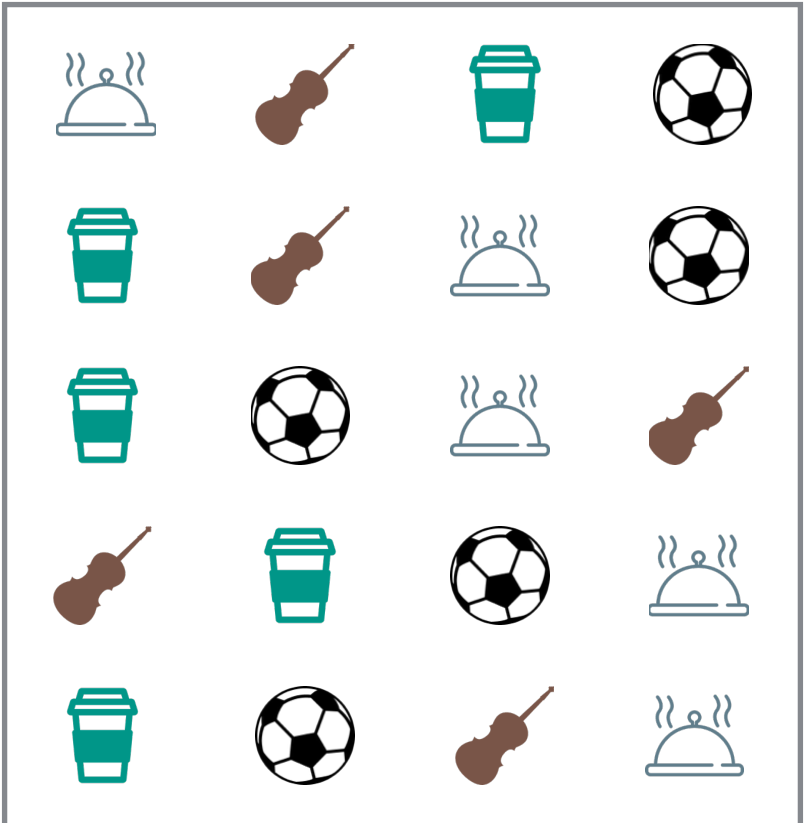
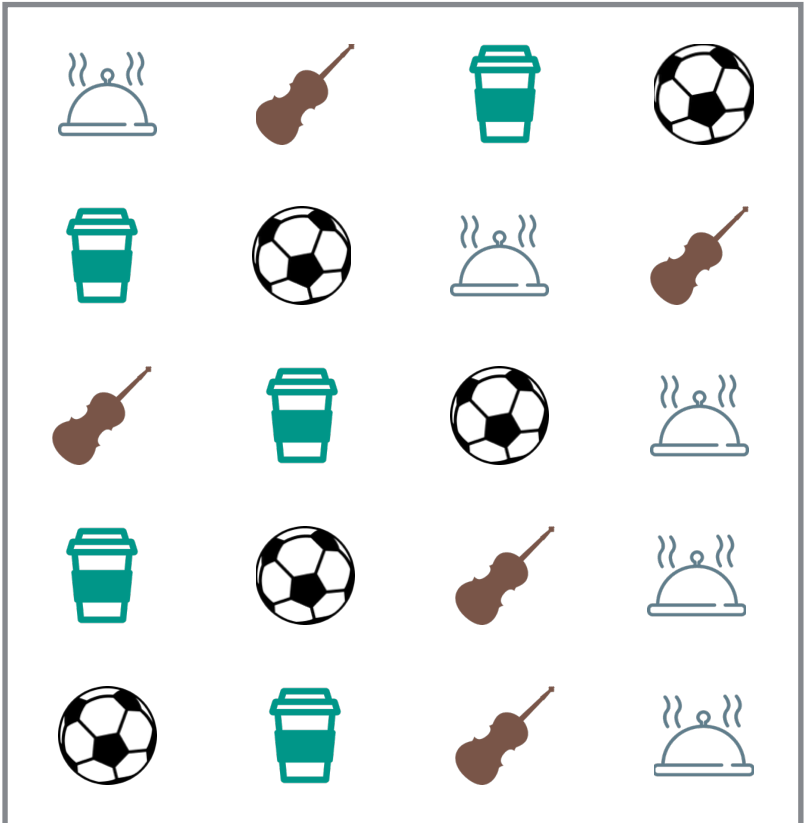
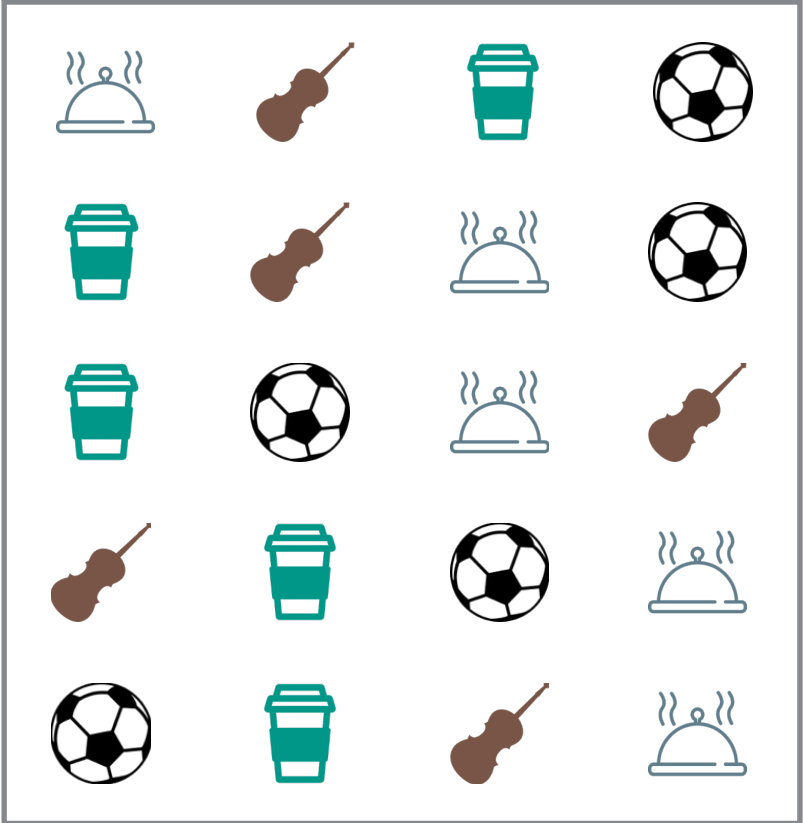
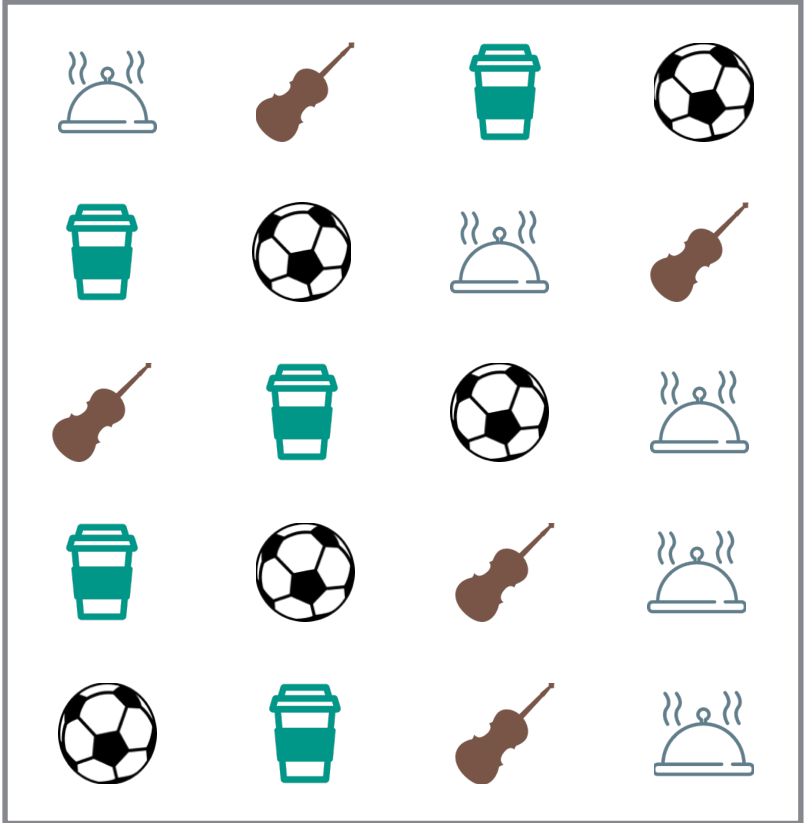




Gibbard-Satterthwaite (1973/75)

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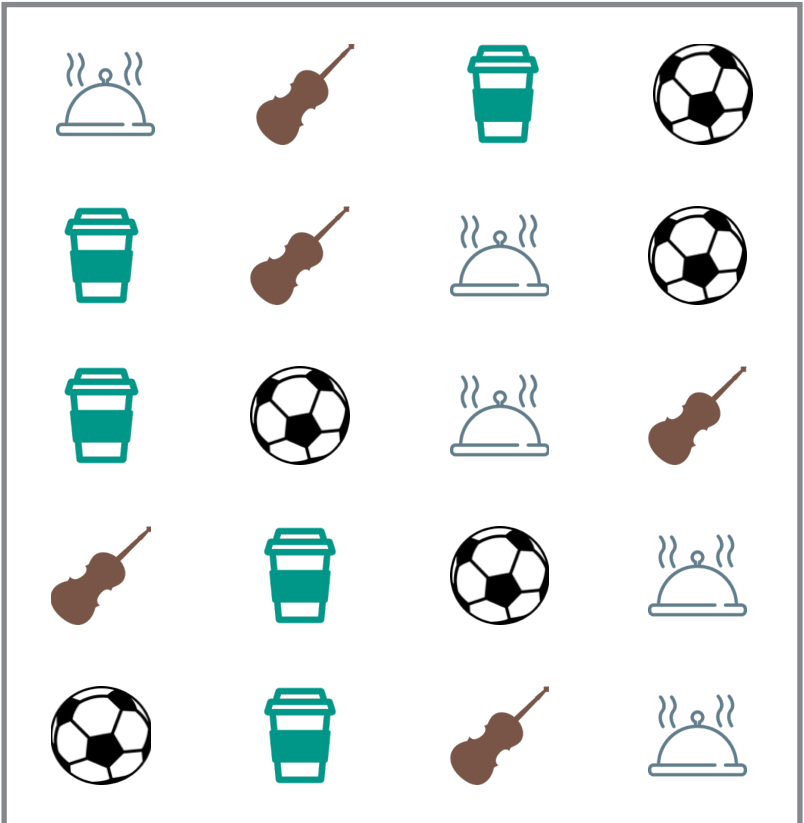
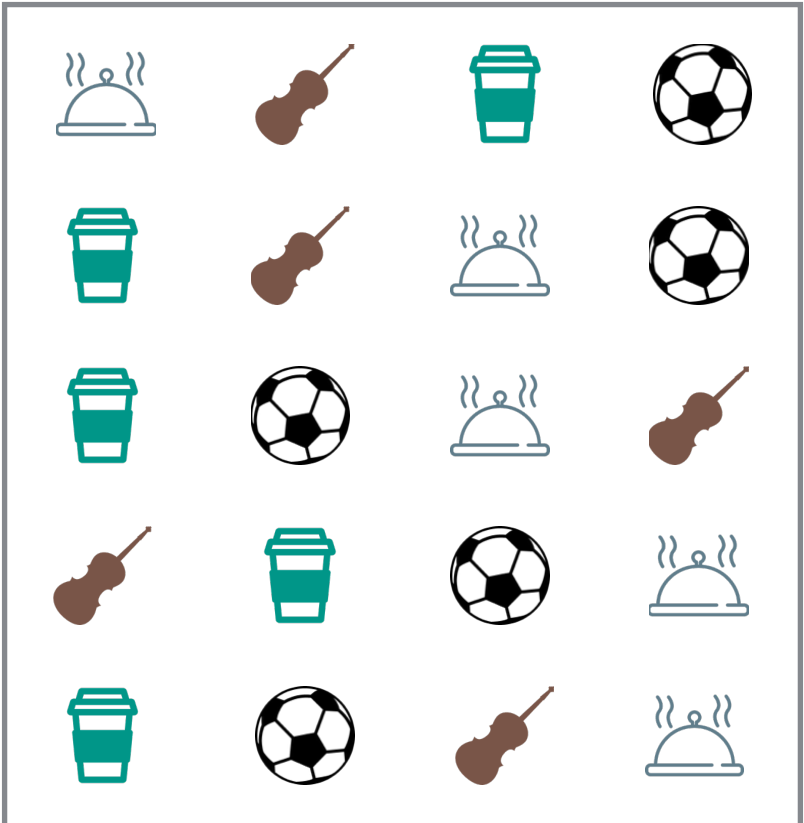
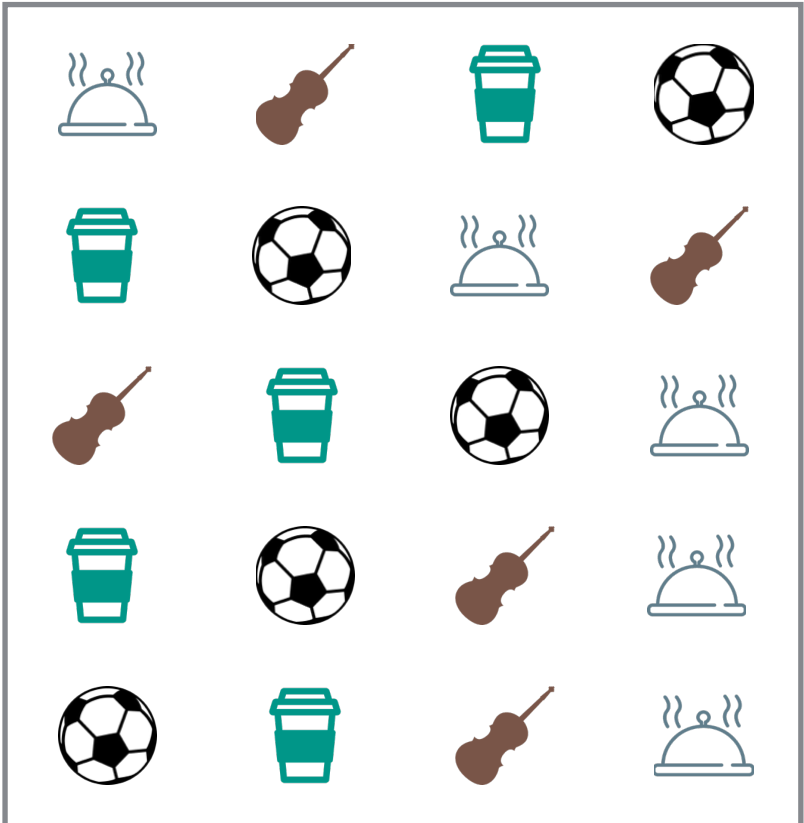
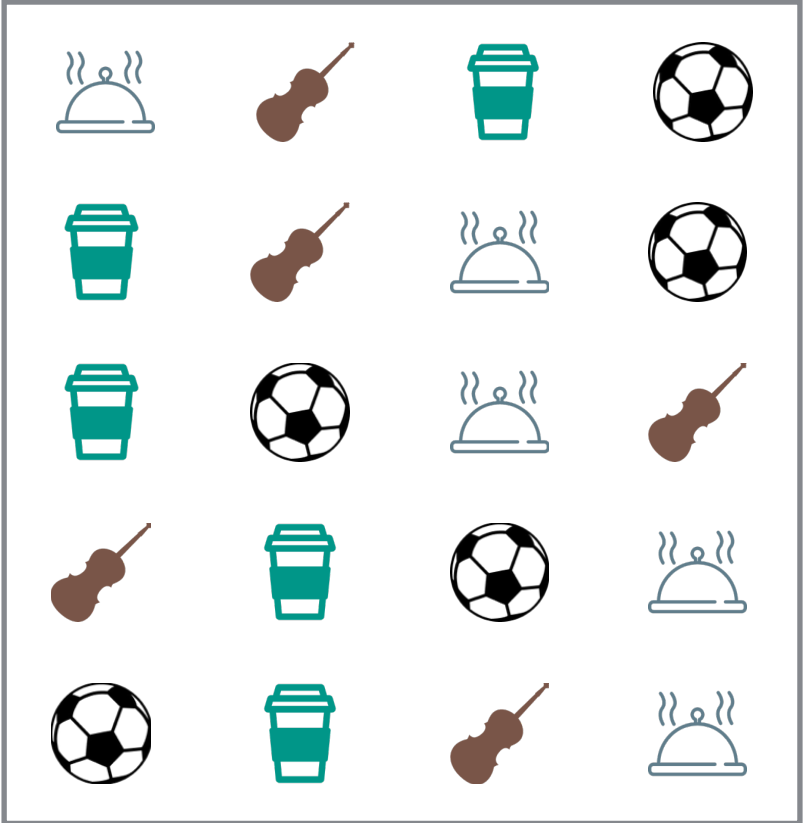
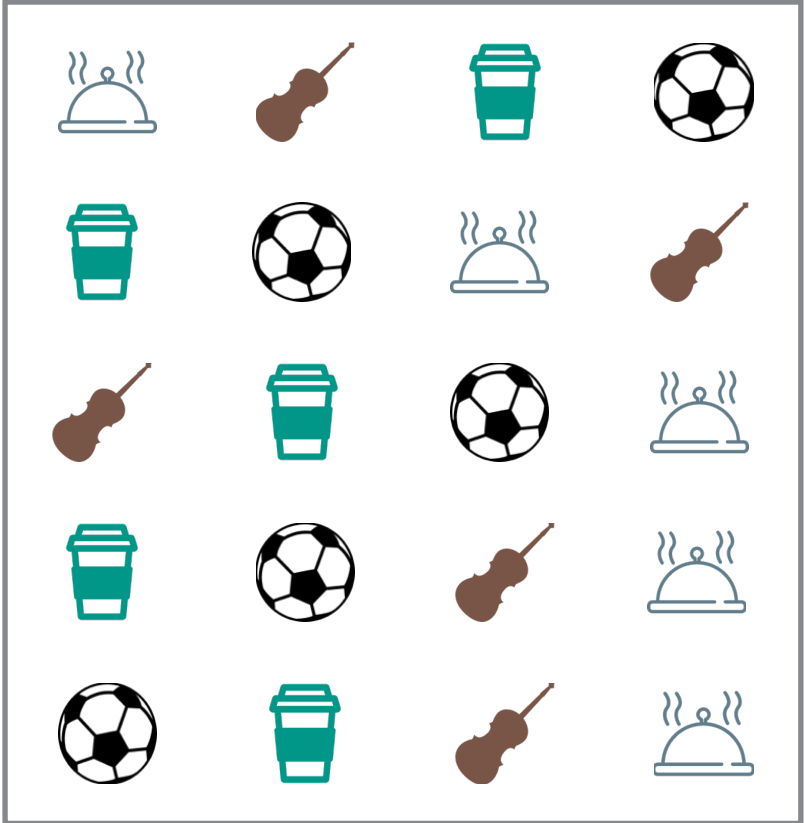
Any **strategy-proof** SCF where at least three candidates have some chance of being selected must be a **dictatorship**.



Gibbard-Satterthwaite: Generalizations

Gibbard-Satterthwaite: Generalizations

The further a SCF is from being a
dictatorship,
the greater the chances that it is
manipulable.





Workarounds

Workarounds

Domain Restrictions

Workarounds

Domain Restrictions

Randomised Voting Rules

Workarounds

Domain Restrictions

Randomised Voting Rules

Computational Hardness

Nice Voting Rules



Dodgson



Dodgson



Dodgson

Dodgson score of c

Smallest #of swaps
needed to **make c a Condorcet winner.**

Kemeny



Kemeny

Kemeny score of a ranking

Sum of pairwise agreements
across all votes.

Social Choice

Computational Social Choice



**Winner
Determination**

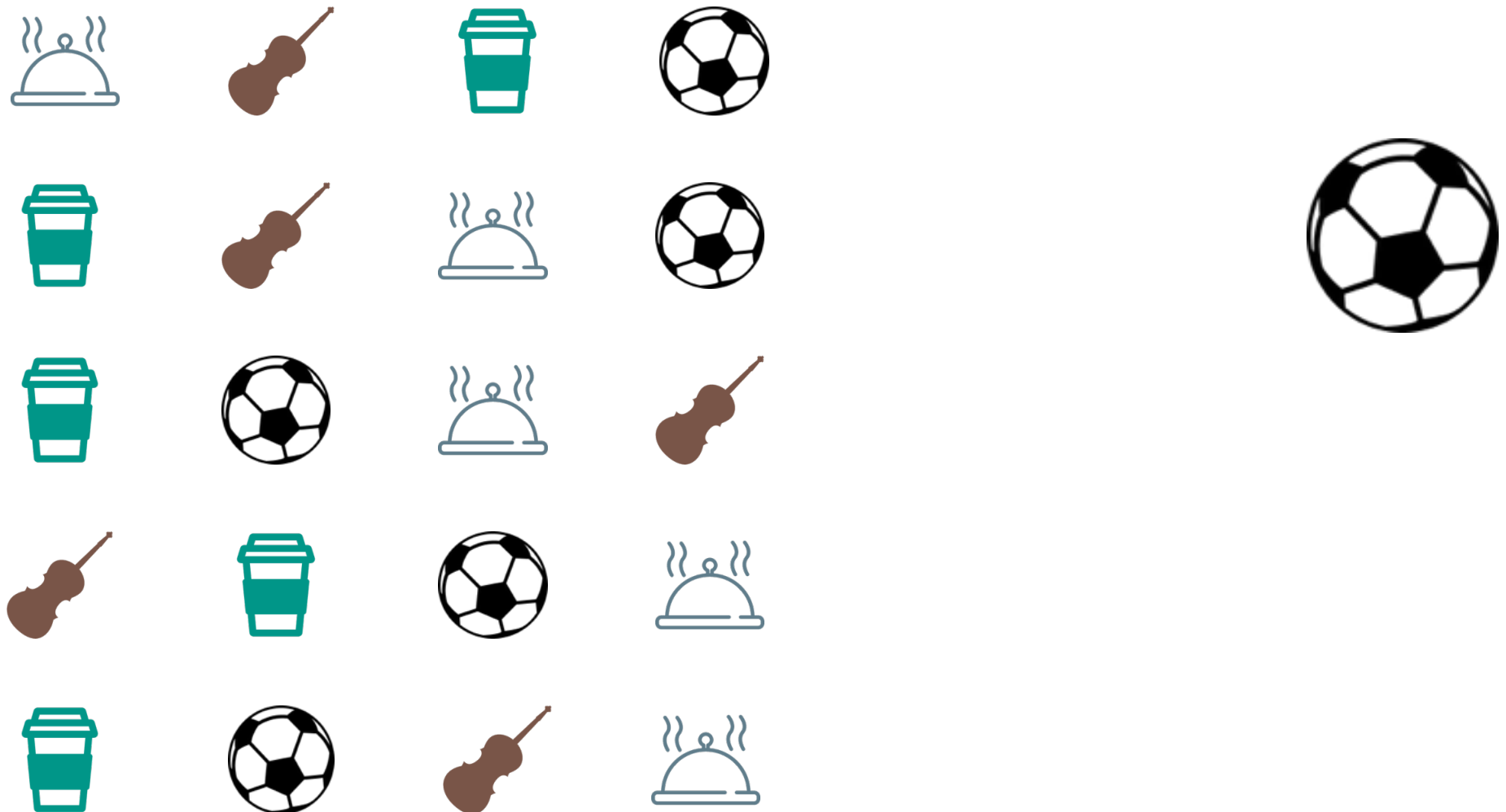


Manipulation



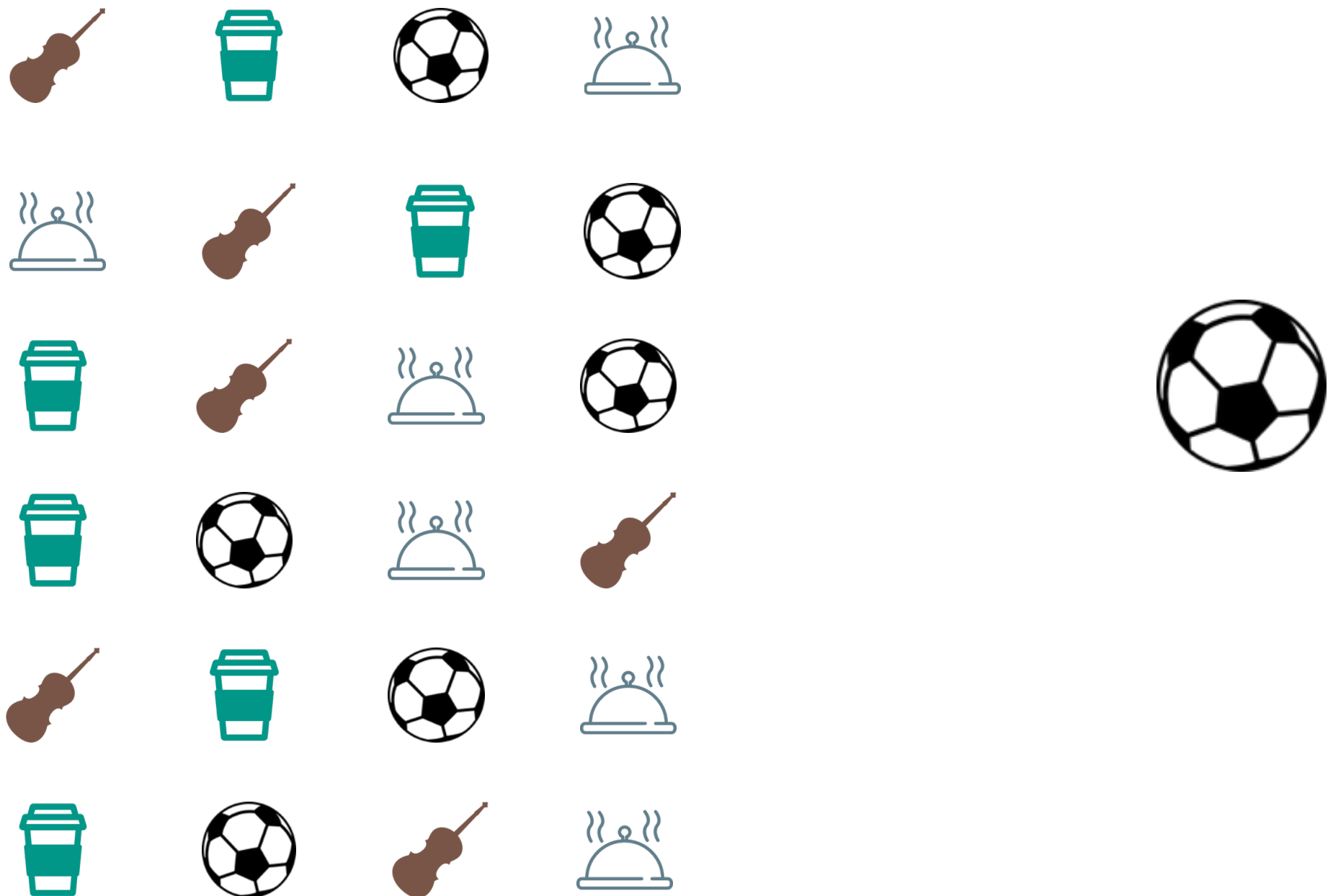
Manipulation: The Computational Problem

Input: Voting rule, profile, & a favourite candidate.



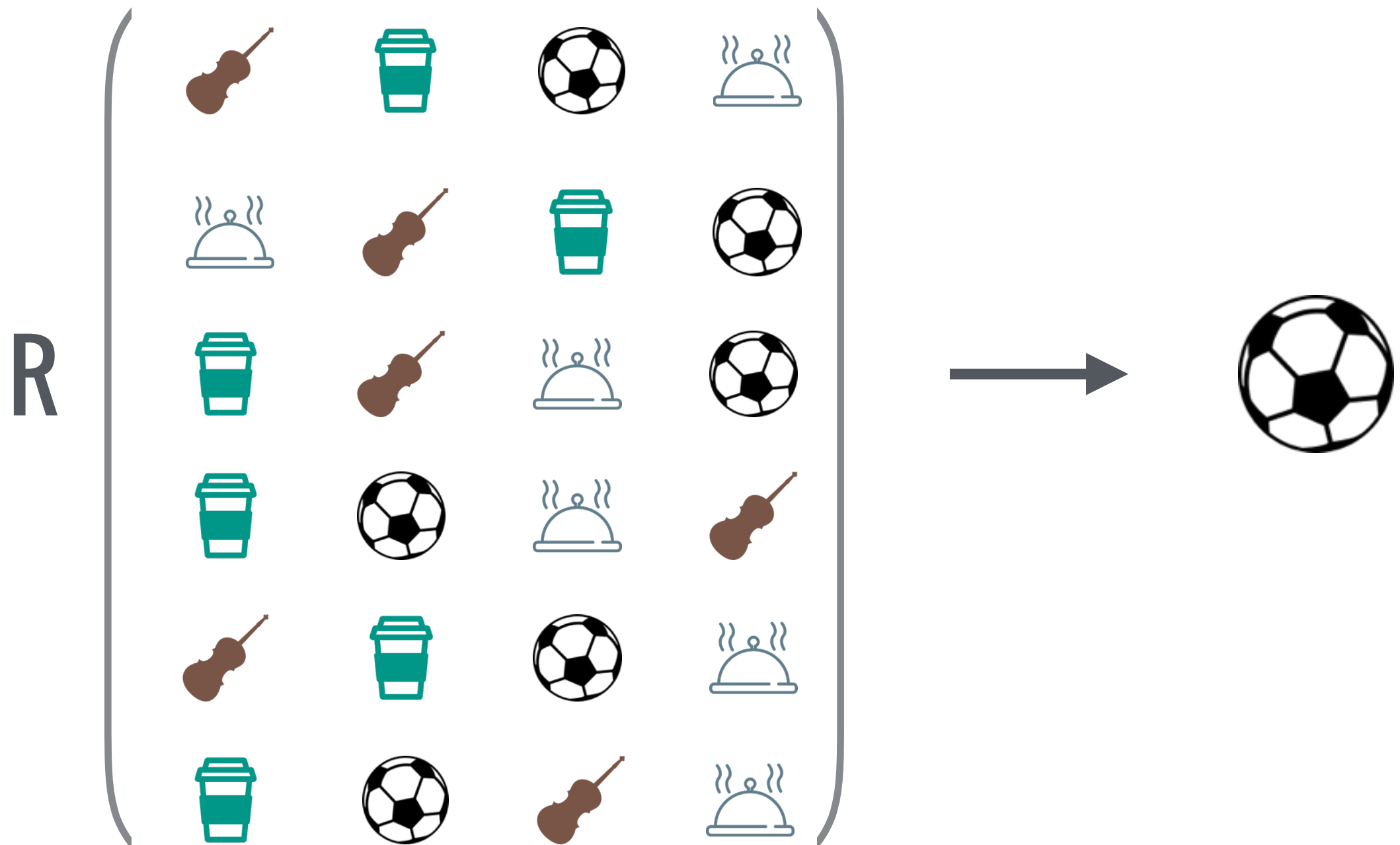
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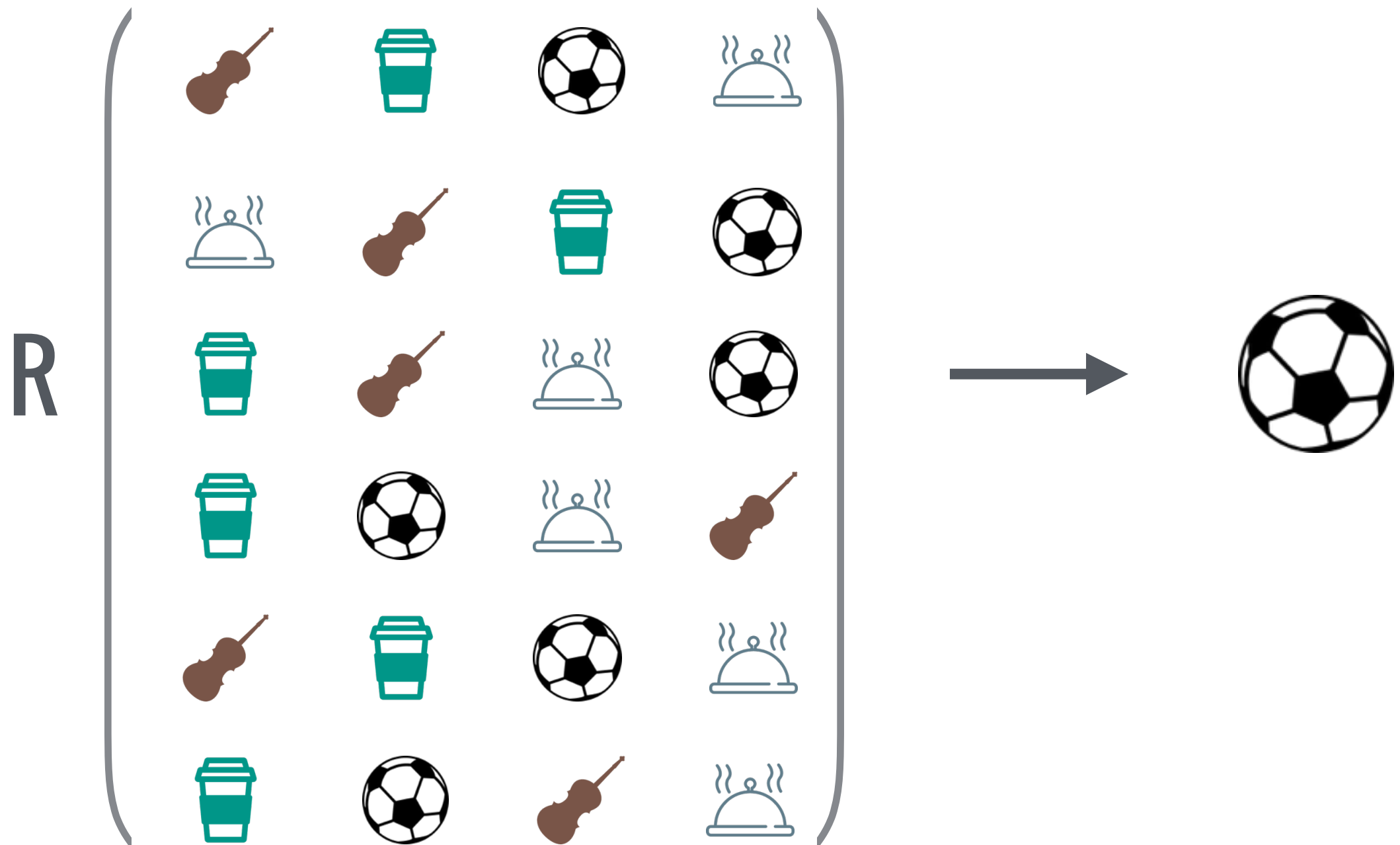
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Manipulation: The Computational Problem

Output: A vote that makes the favourite win wrt the rule.



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Output: A vote that makes the favourite win wrt the rule.



The Computational Difficulty of Manipulating an Election*

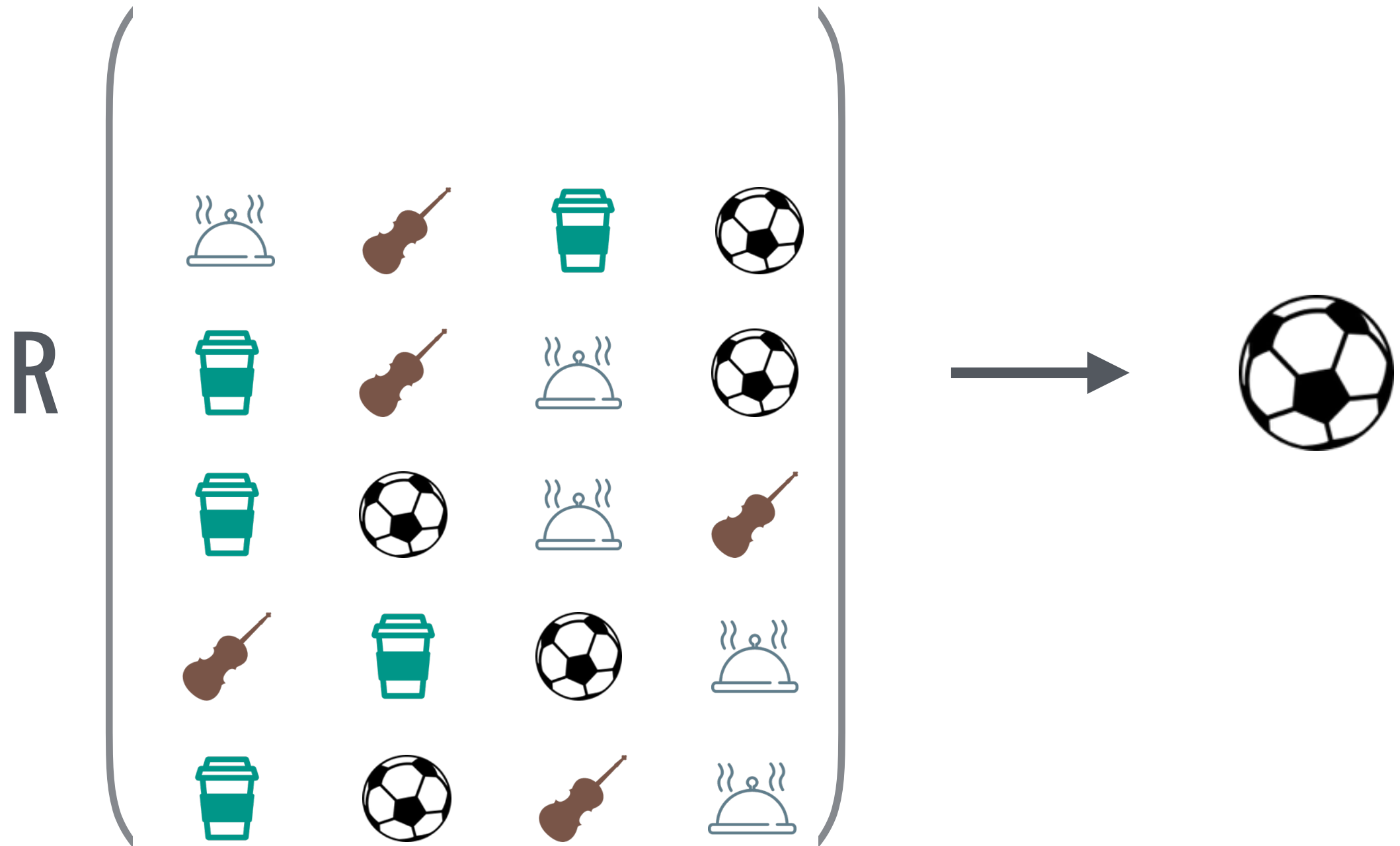
J. J. Bartholdi III, C. A. Tovey, and M. A. Trick**

School of Industrial and Systems Engineering, Georgia Institute of Technology,
Atlanta, GA 30332, USA

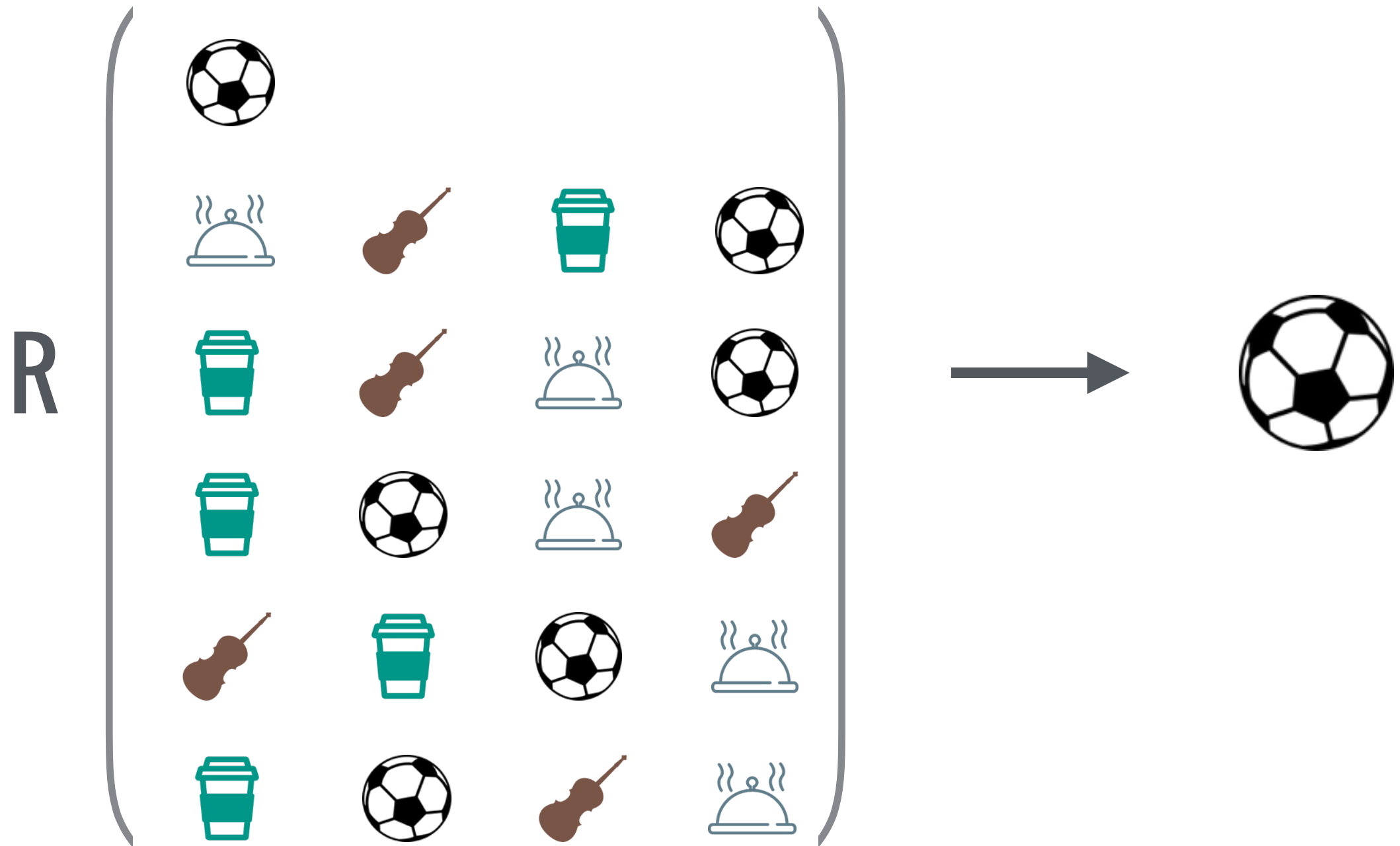
Received June 9, 1987 / Accepted July 29, 1988

Abstract. We show how computational complexity might protect the integrity of social choice. We exhibit a voting rule that efficiently computes winners but is computationally resistant to strategic manipulation. It is *NP*-complete for a manipulative voter to determine how to exploit knowledge of the preferences of others. In contrast, many standard voting schemes can be manipulated with only polynomial computational effort.

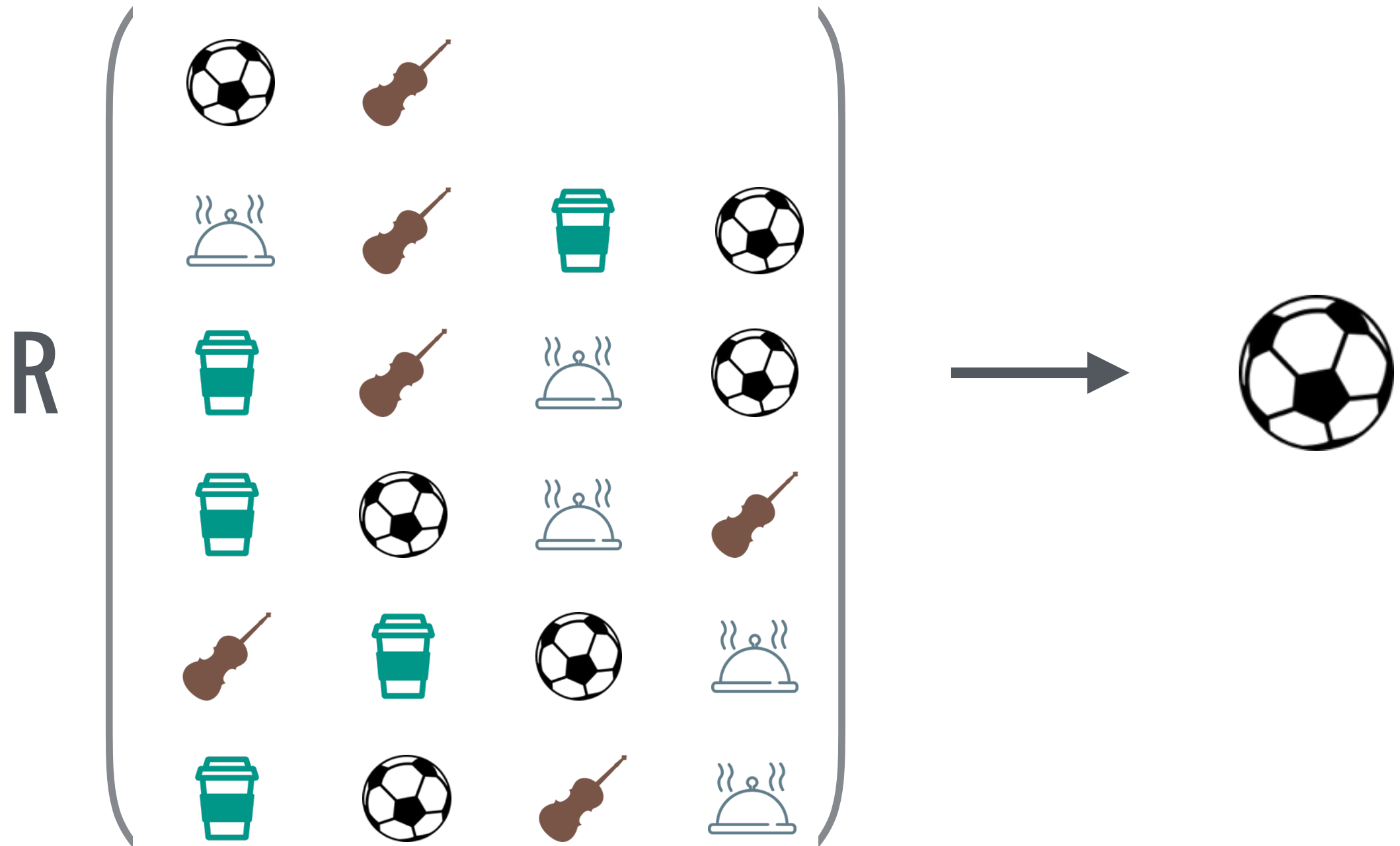
Manipulation: A Greedy Algorithm



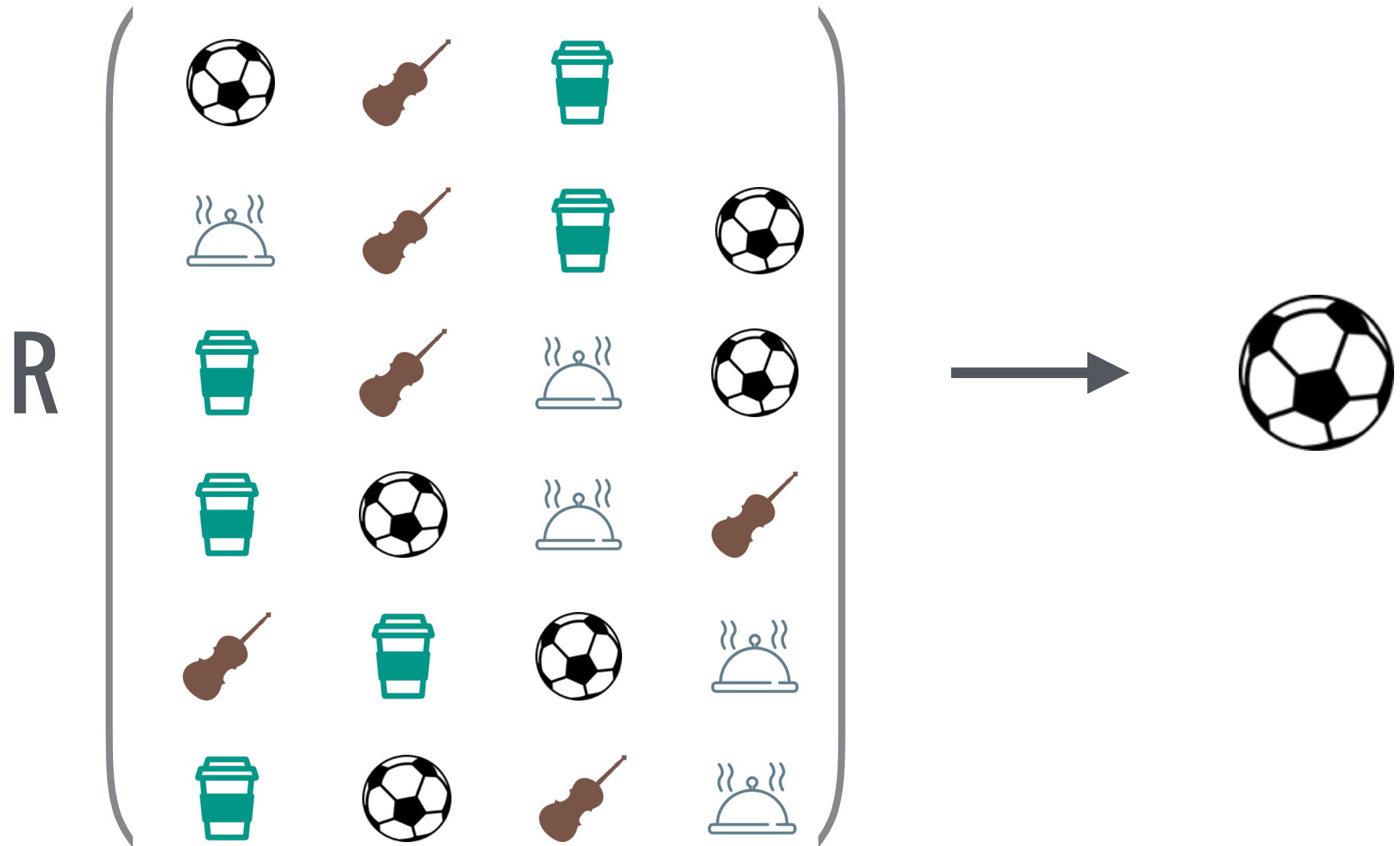
Manipulation: A Greedy Algorithm



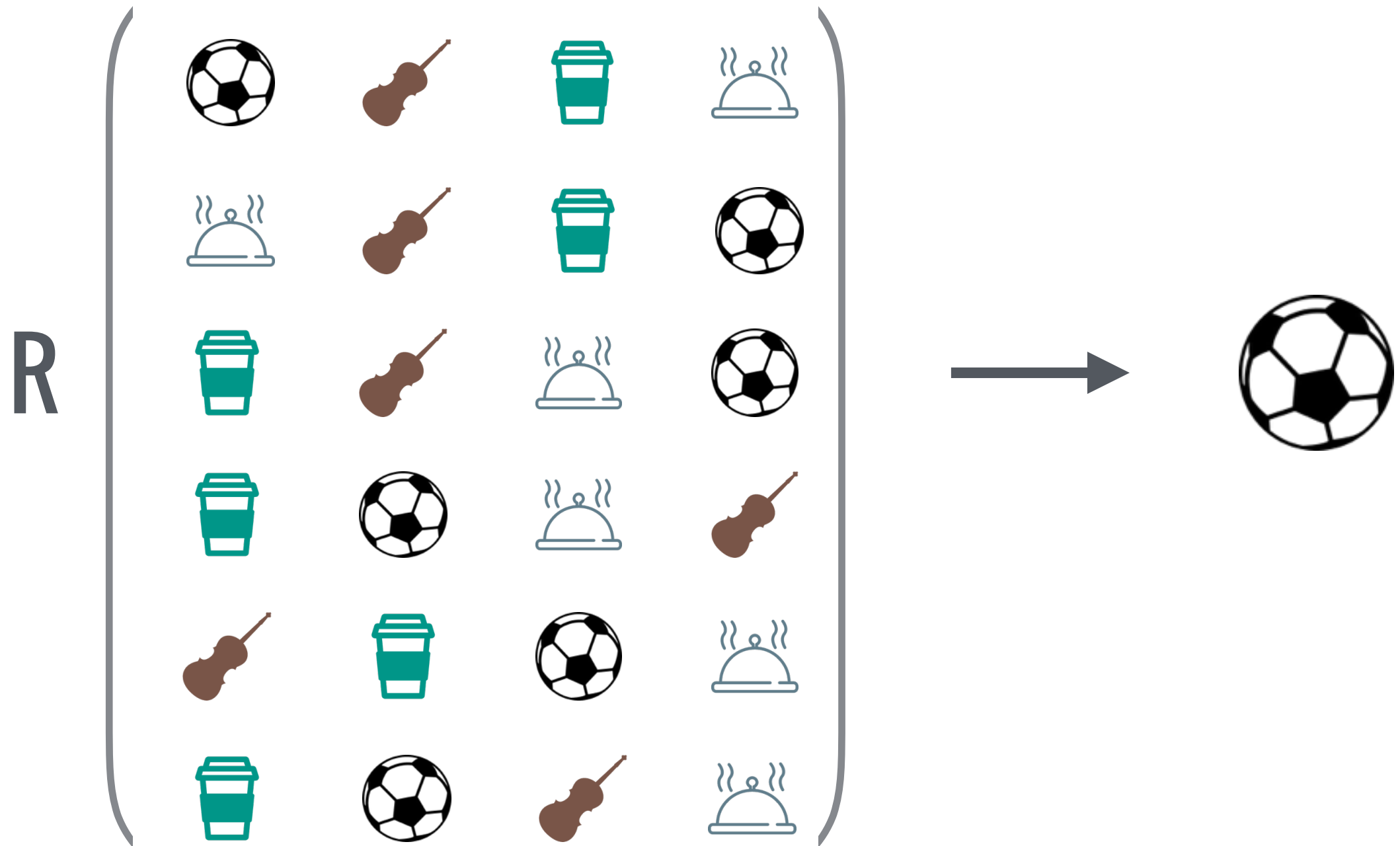
Manipulation: A Greedy Algorithm



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Manipulation: Hardness Results

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Manipulating some voting rules (STV, Second Order Copeland) is NP-hard even for a single manipulator.

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For many common voting rules, manipulation is NP-hard when there are **two or more manipulators** involved.

Manipulation: Hardness Results

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For many common voting rules, manipulation is NP-hard when there are **two or more manipulators** involved.

For **weighted elections**, the manipulation problem is frequently NP-hard, even when there are a constant number of candidates.

Manipulation: Objections to NP-hardness

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It is only worst-case hardness, and random choices of manipulative votes have a high probability of success.

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AI's War on Manipulation: Are We Winning?

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References

