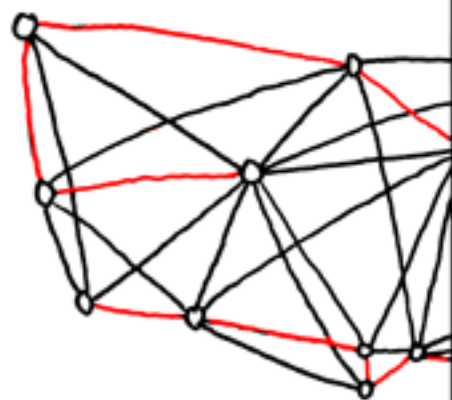
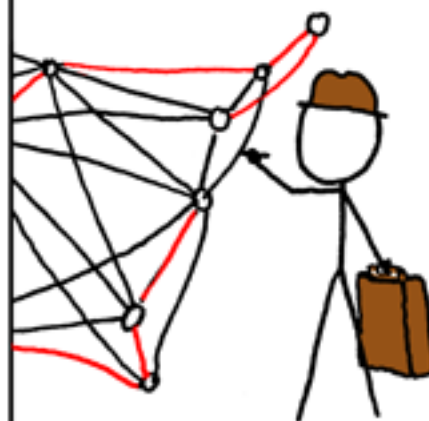


BRUTE-FORCE
SOLUTION:
 $O(n!)$



DYNAMIC
PROGRAMMING
ALGORITHMS:
 $O(n^2 2^n)$



SELLING ON EBAY:
 $O(1)$

STILL WORKING
ON YOUR ROUTE?

SHUT THE
HELL UP.



Cheat Sheets for Hard Problems

Some problems tend to be **harder** than others.

NP

NP

P



NP

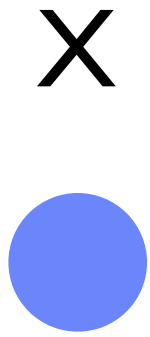
P



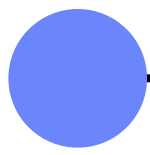
NP-COMPLETE

P

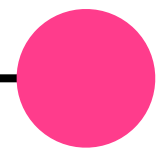




X



TSP



solveTSP{

blah

blah



blah

blah

blah

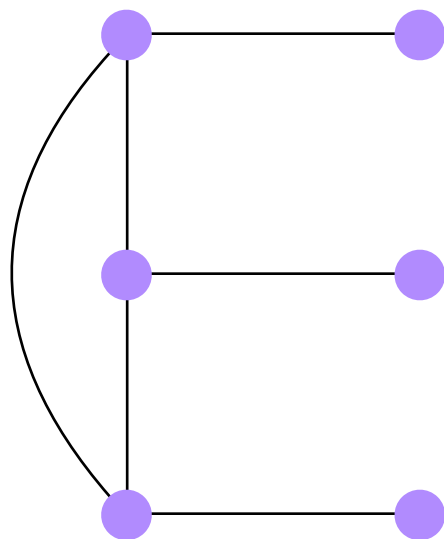
}

X

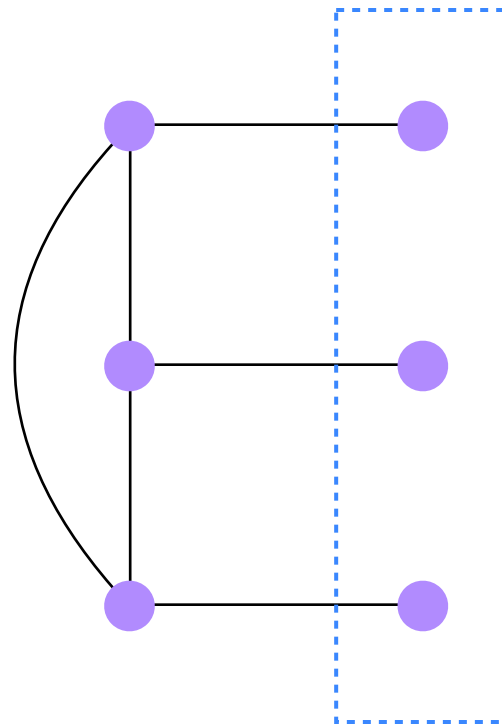


TSP

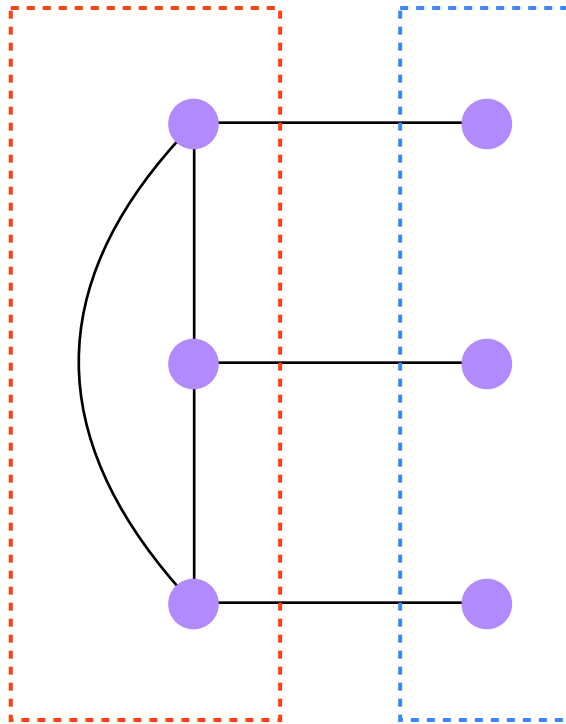




Independent Set

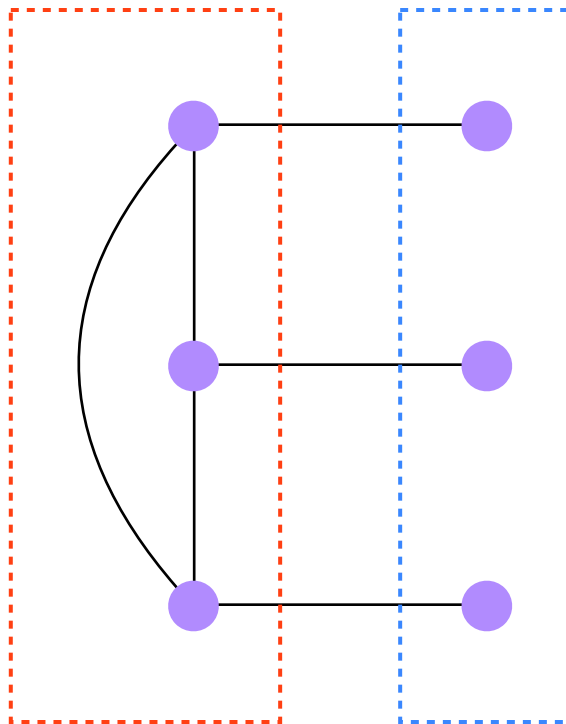


Independent Set



Clique

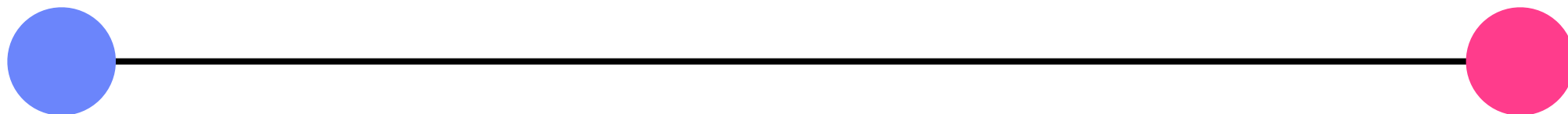
Independent Set

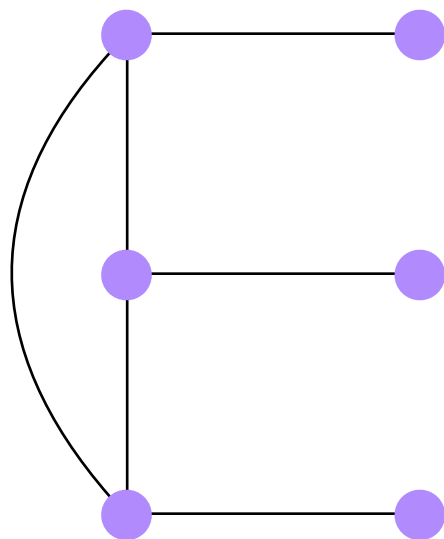


Clique

Clique

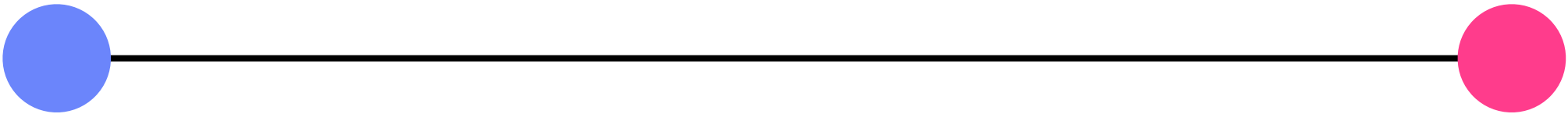
Independent Set

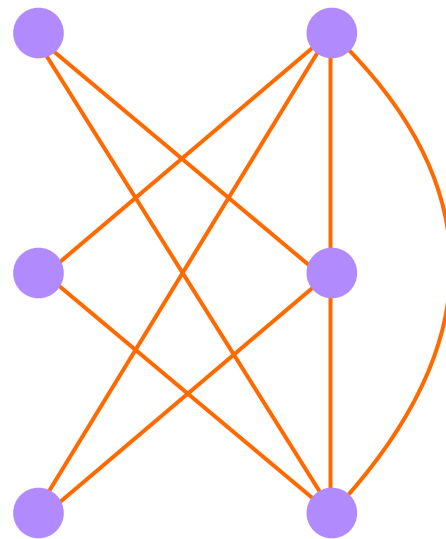




Clique

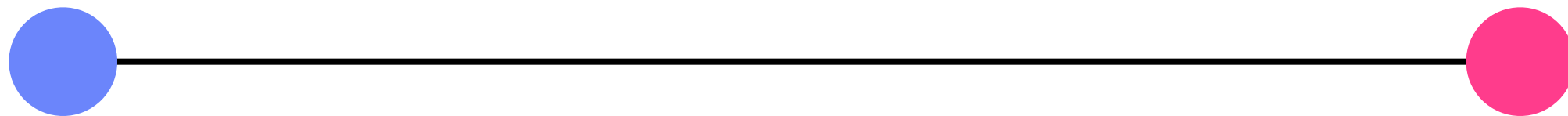
Independent Set



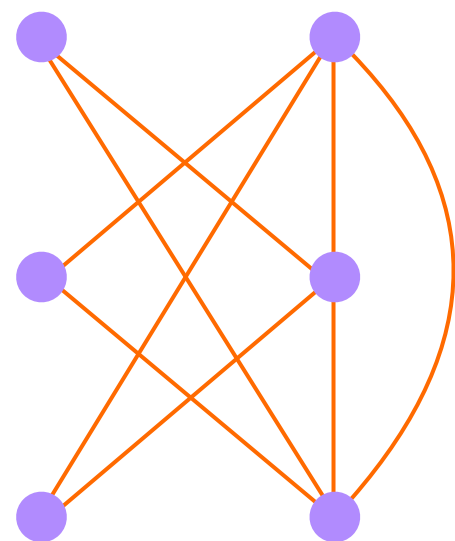


Clique

Independent Set

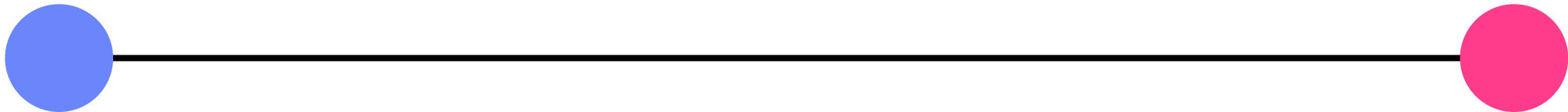


SolveIndSet { Return Clique($\overline{G}$); }



Clique

Independent Set



solveTSP{

blah

blah



blah

blah

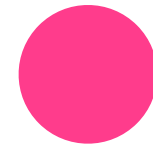
blah

}

X



TSP



DID YOU SAY NP-COMPLETE?

DID YOU SAY NP-COMPLETE?









Heuristics



Heuristics

Formal analysis?



You have Polynomial Time.



You have Polynomial Time.
WORK BACKWARDS!



Approximation & Randomized Algorithms



A no-compromise situation?

A no-compromise situation?



A no-compromise situation?
Exploit additional structure in the input.



Parameterized & Exact Analysis



Parameterized & Exact Analysis

Chromatic Number is easy on Interval Graphs.



Parameterized & Exact Analysis

Chromatic Number is easy on **Planar*** Graphs.

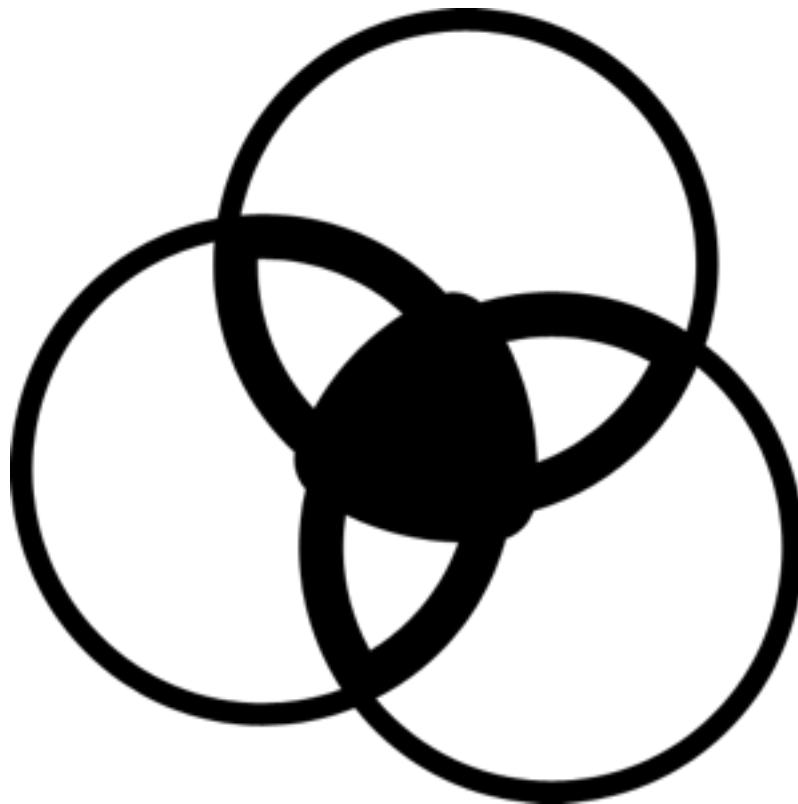


Parameterized & Exact Analysis

Chromatic Number is easy on **Bipartite** Graphs.



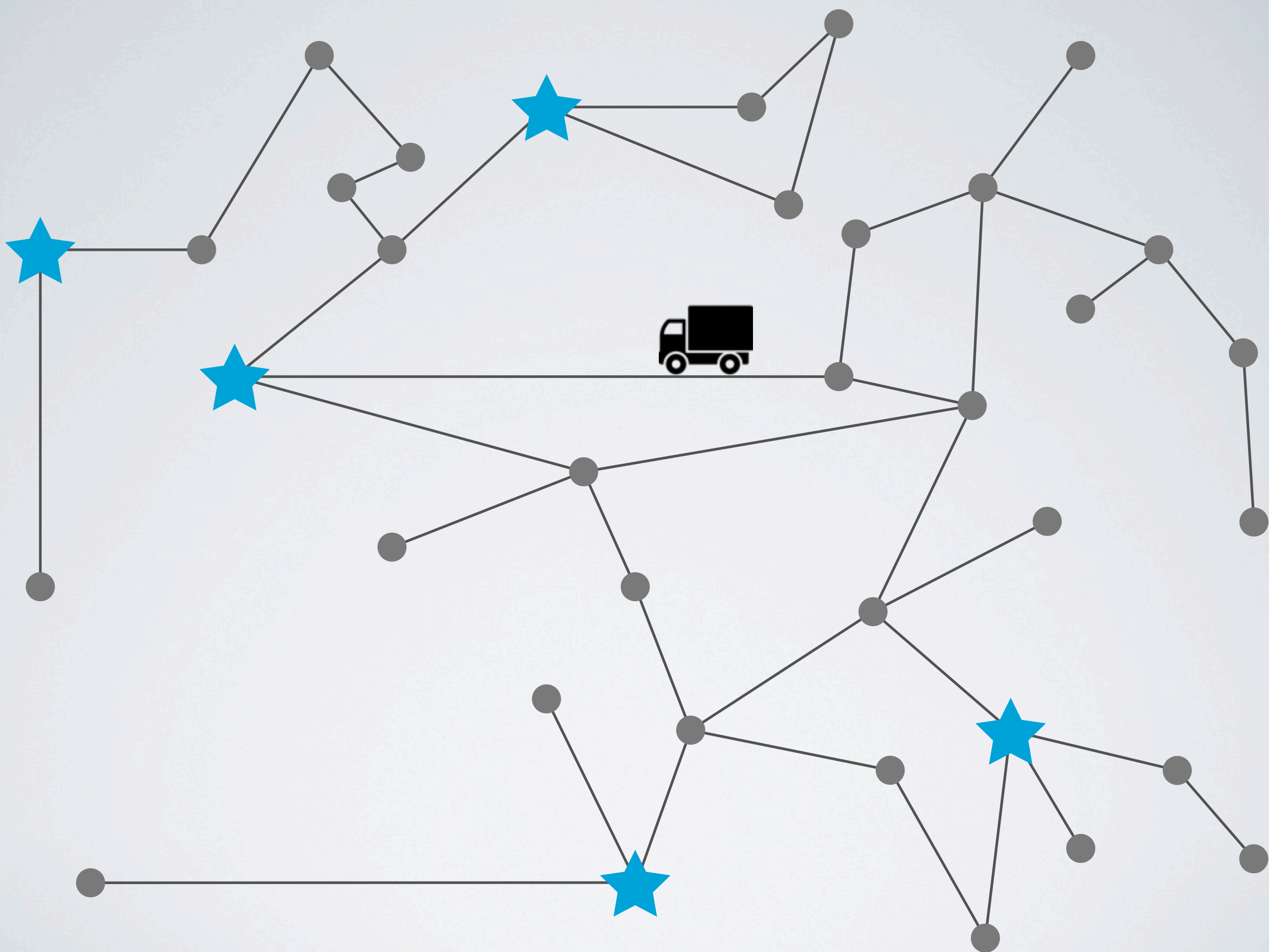
Good solutions tend to involve a combination of several techniques.



Vertex Cover

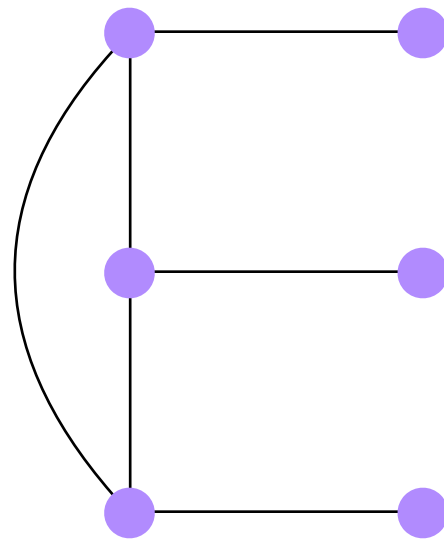






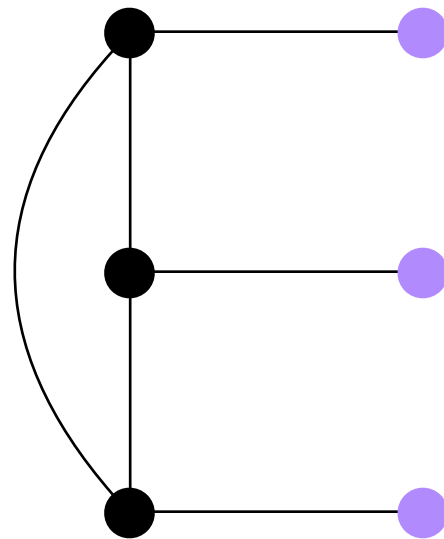


Every edge has at least one end point in a vertex cover.



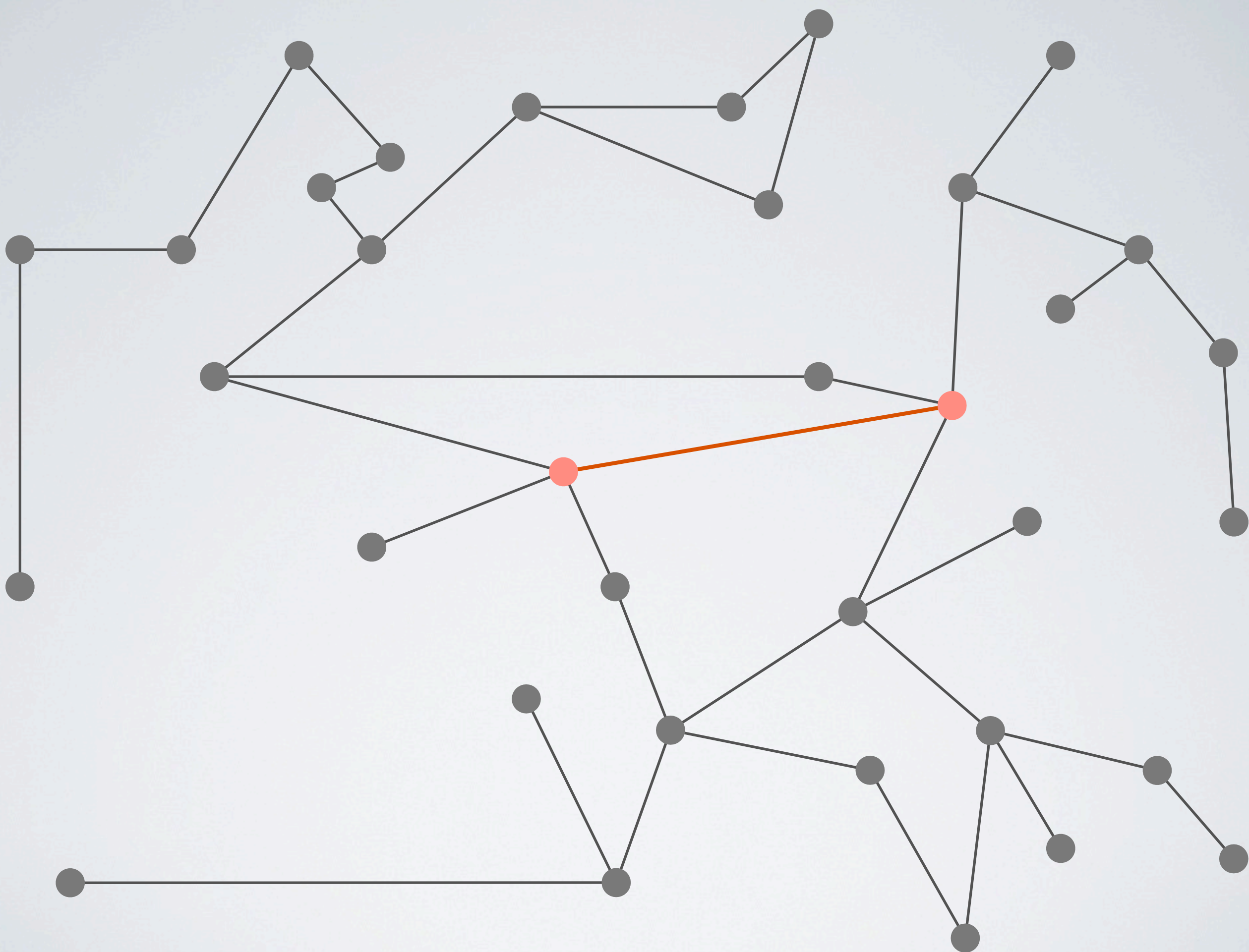
Vertex Cover

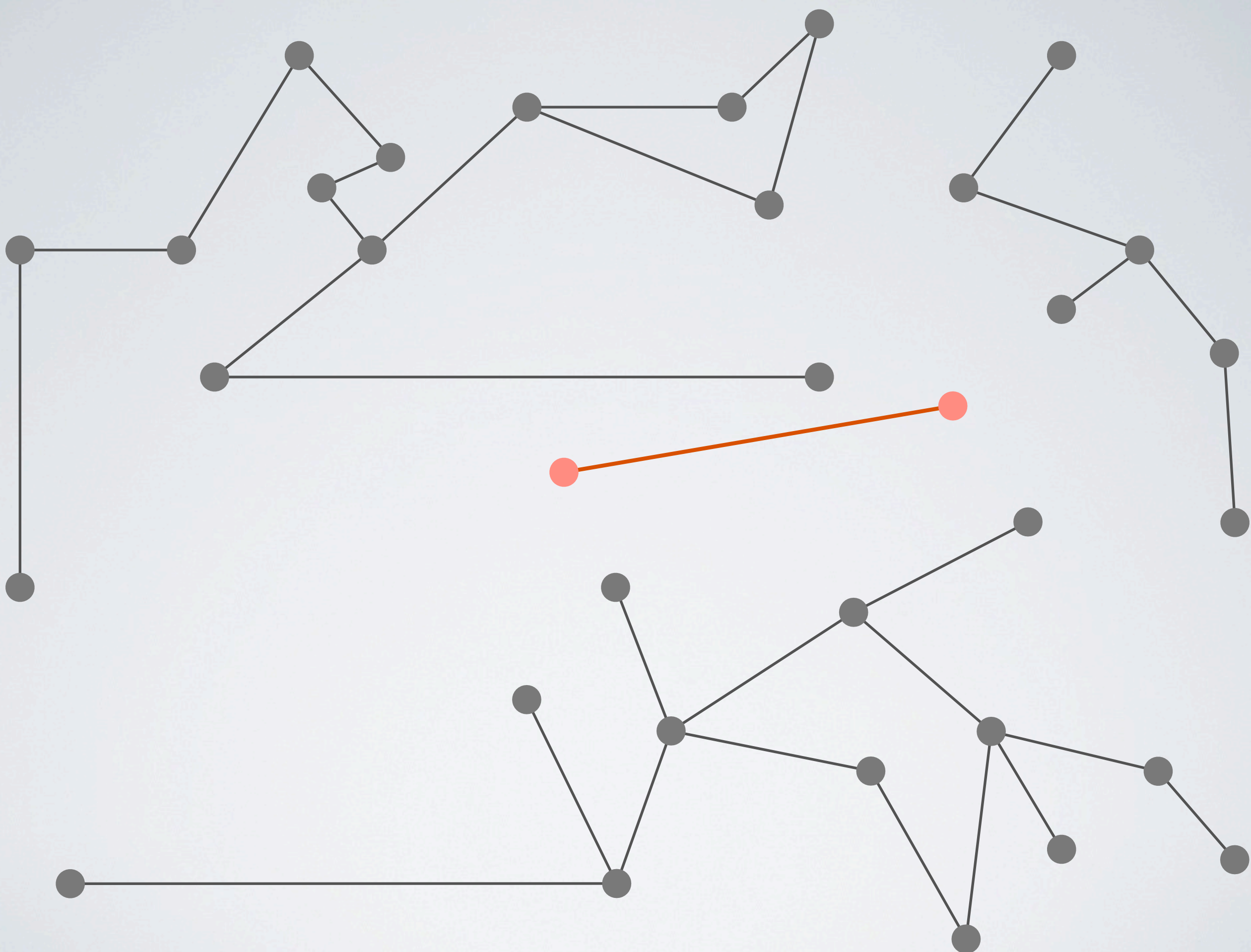
Every edge has at least one end point in a vertex cover.

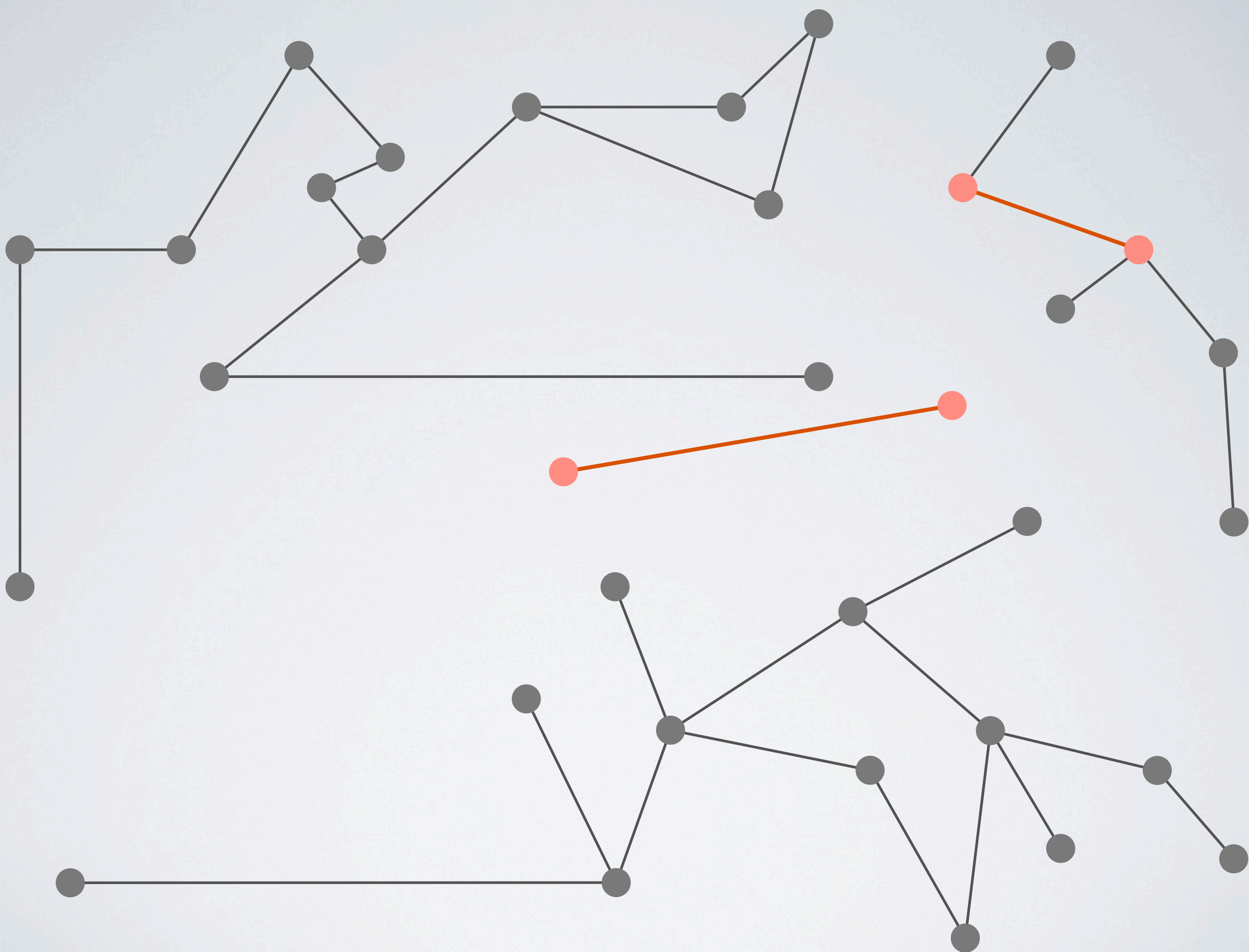


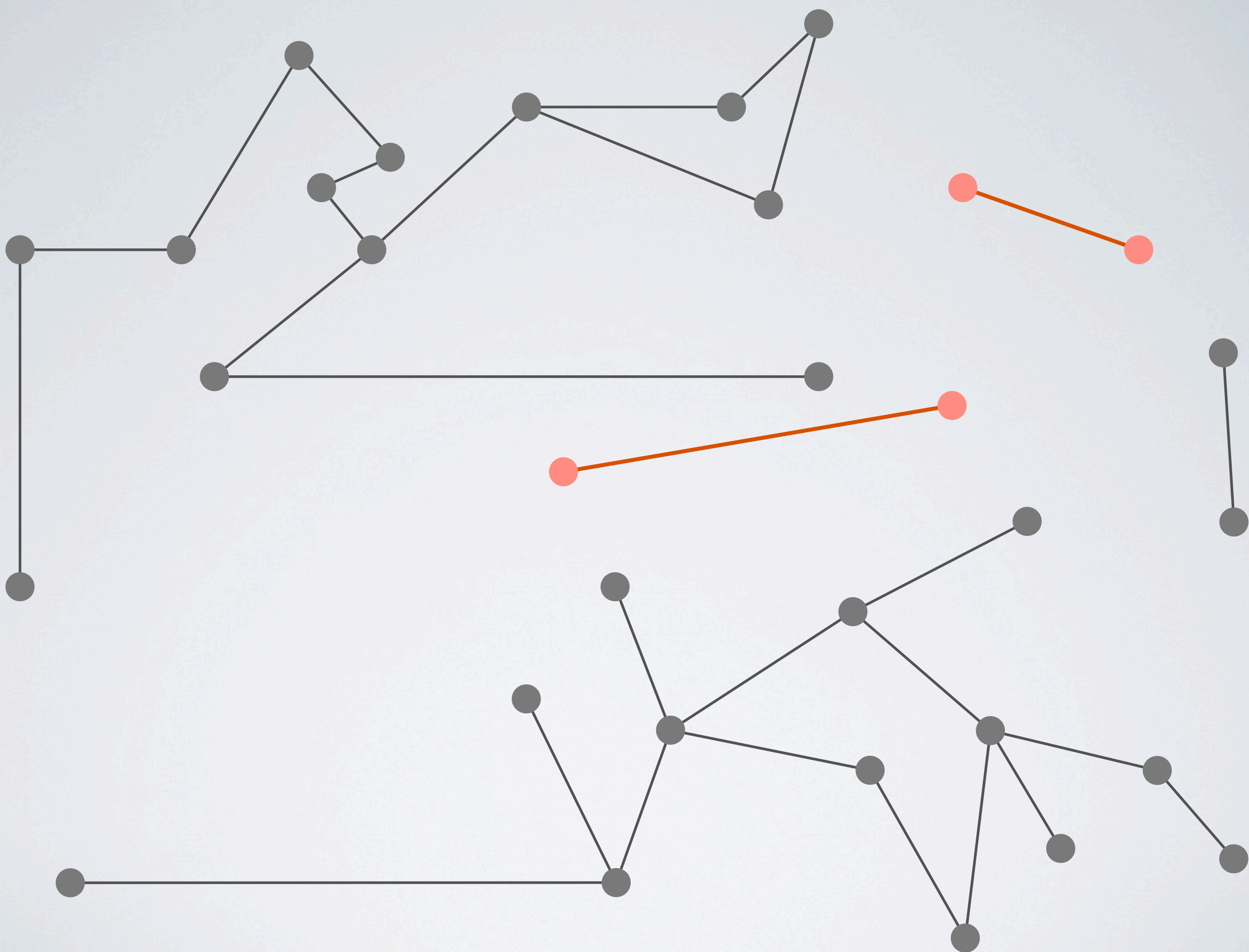
Vertex Cover

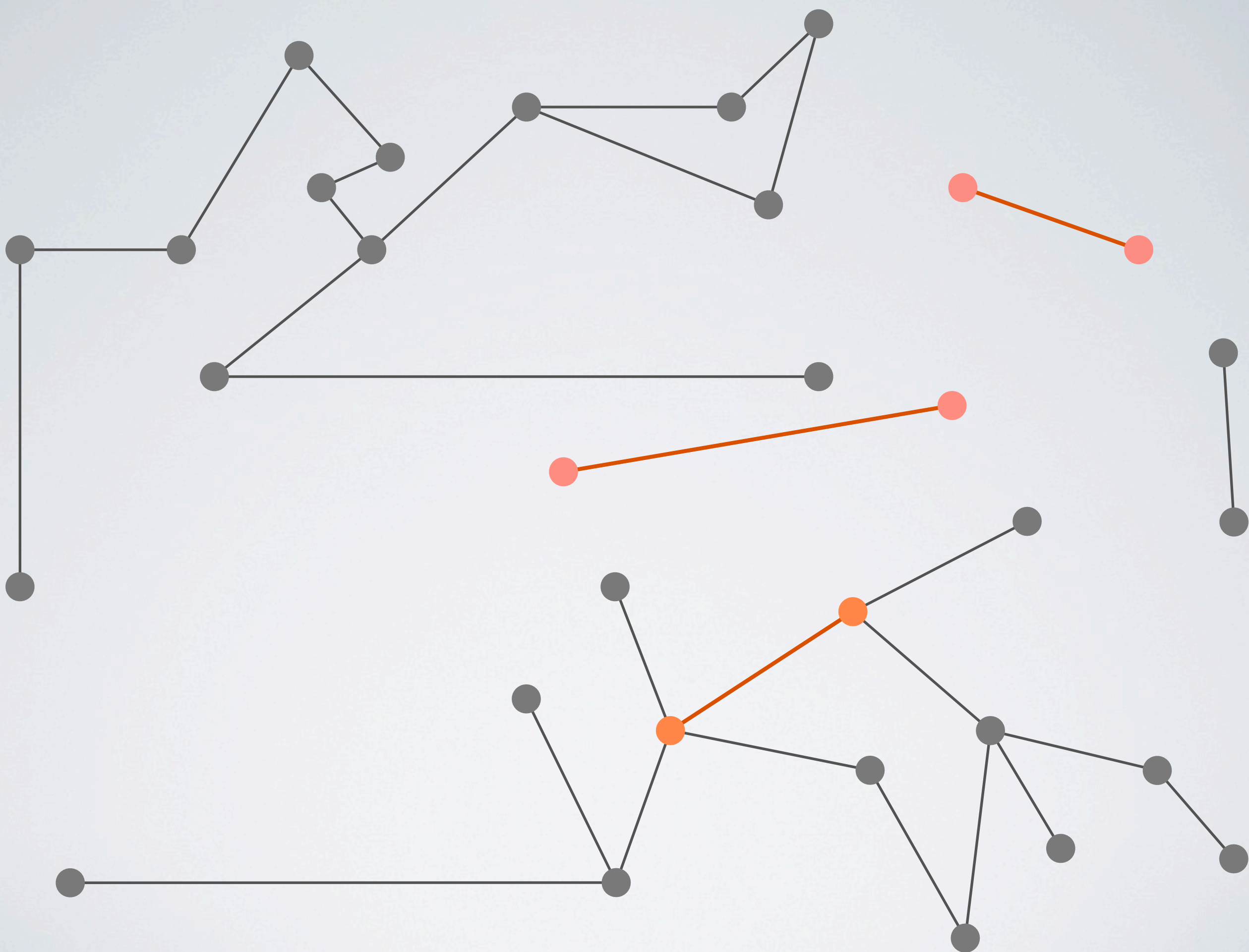


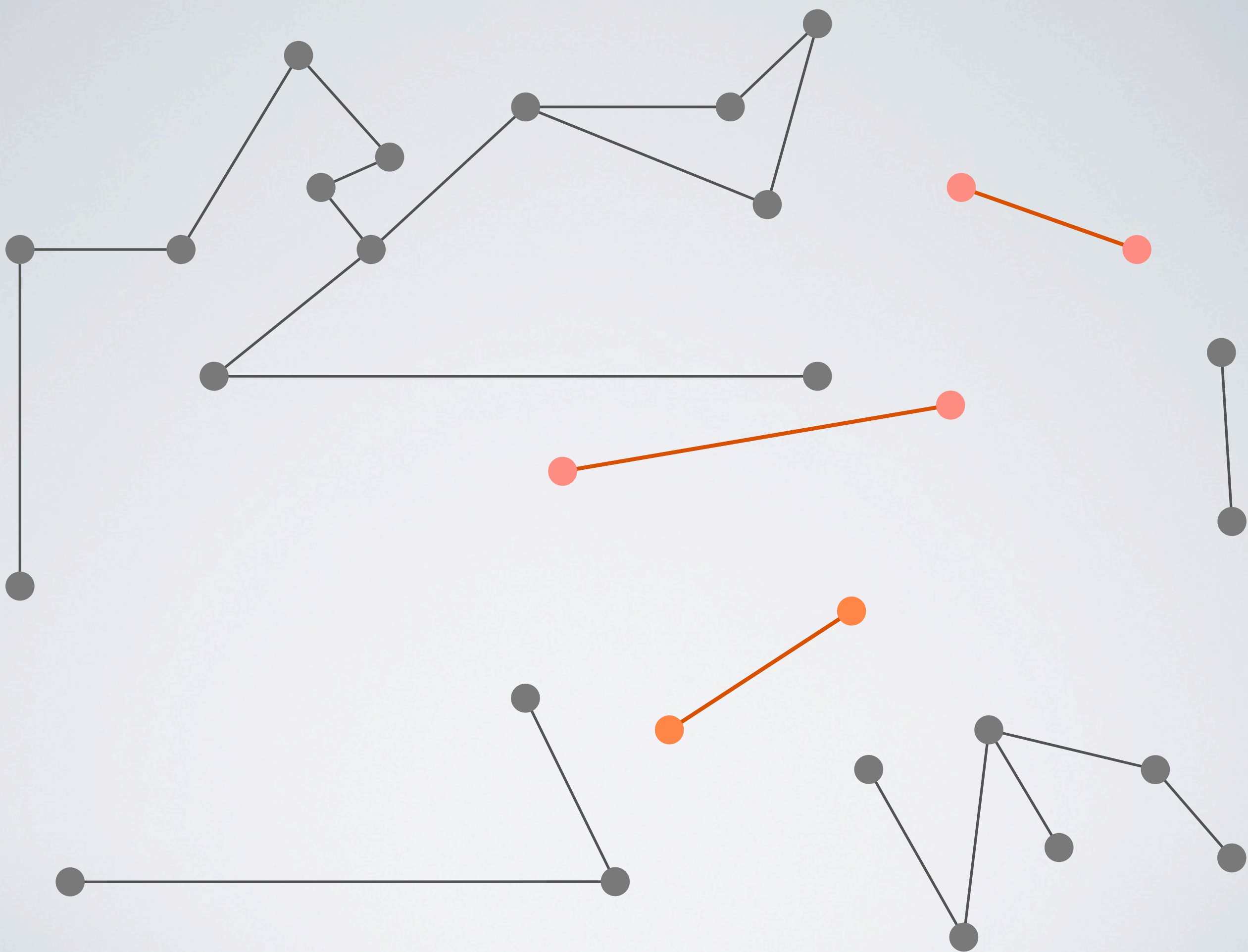


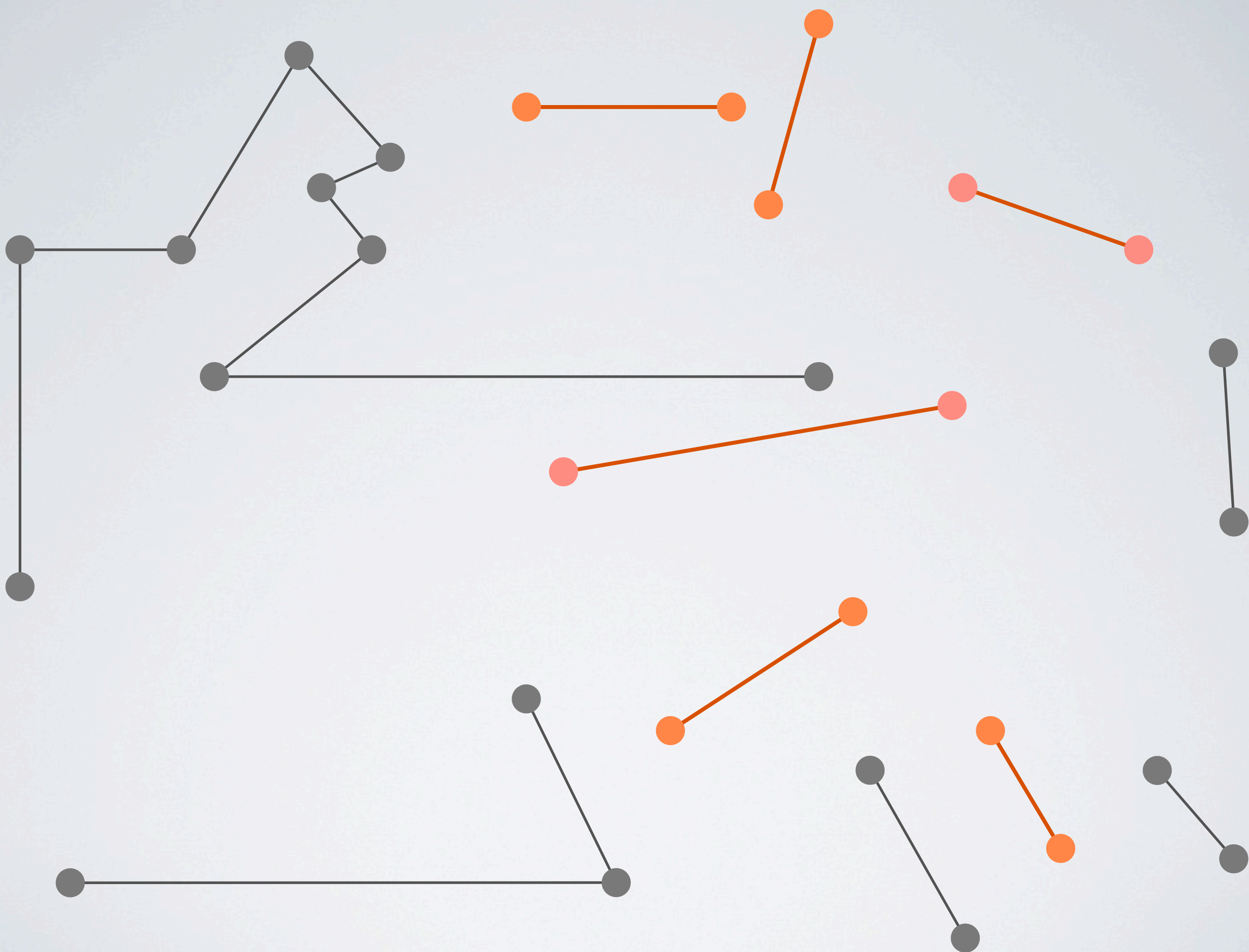


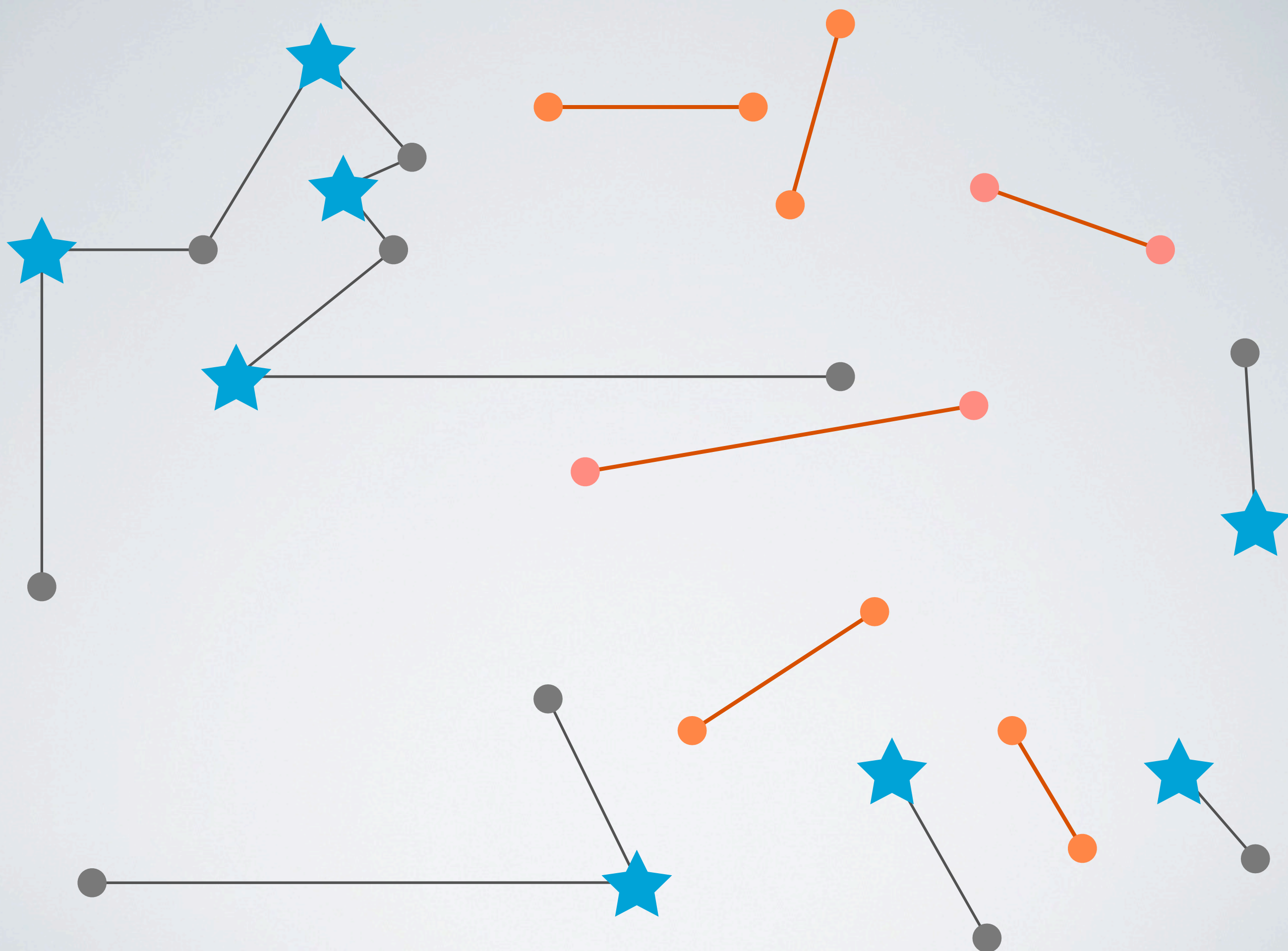






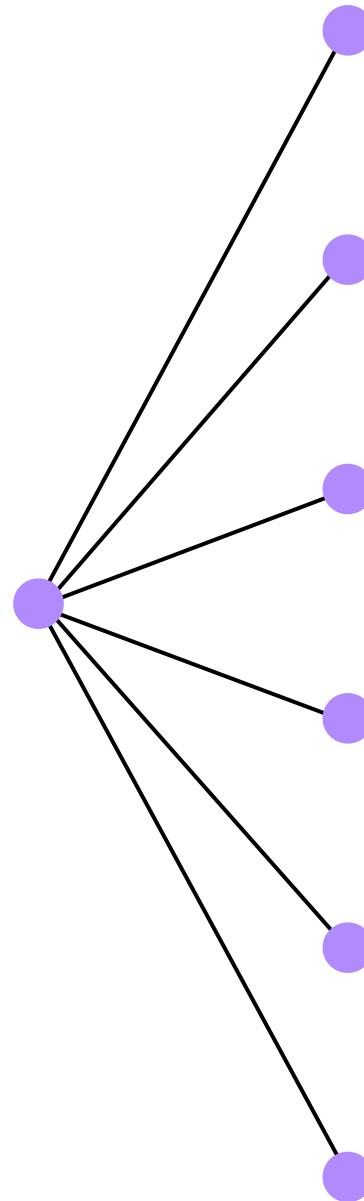






Is there a Vertex Cover with at most k vertices?

A vertex with more than k neighbors.



Throw away all vertices with degree $[k+1]$ or more.

[And decrease the budget appropriately.]



Throw away all vertices with degree $[k+1]$ or more.

[And decrease the budget appropriately.]



After all the high-degree vertices are gone...

...any vertex can cover at most k edges.

...any vertex can cover at most k edges.

Suppose the current budget is $[k-x]$.

...any vertex can cover at most k edges.

Suppose the current budget is $[k-x]$.

If the number of edges in the graph exceeds $k[k-x]$...?

Lots of edges - no small vertex cover possible.

Few edges - brute force becomes feasible.

Lots of edges - no small vertex cover possible.



Few edges - brute force becomes feasible.

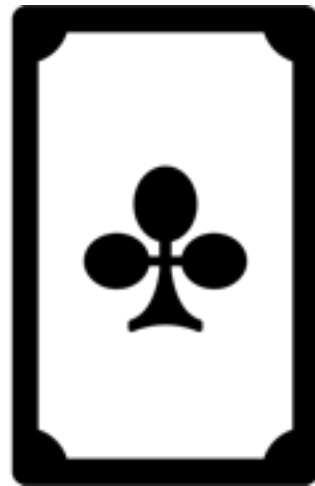


Common Sense



Common Sense

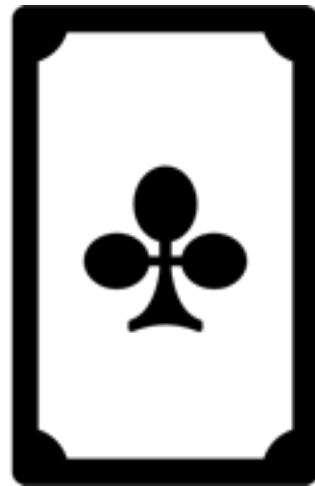
Approximate



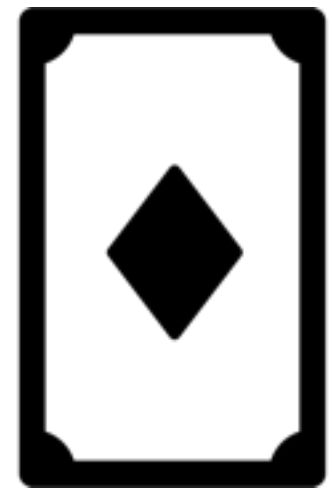
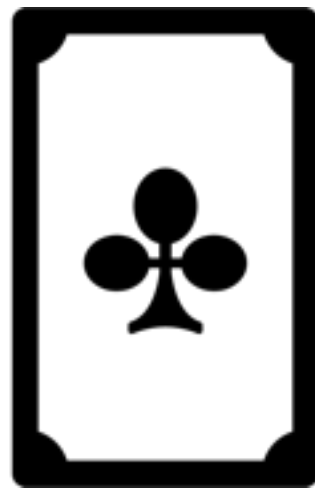
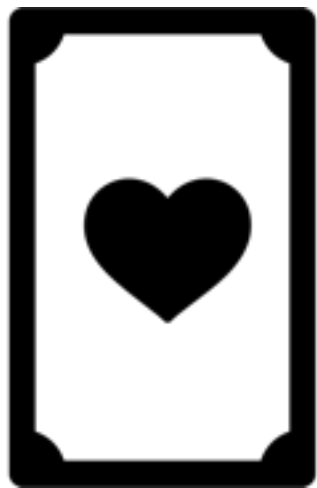
Common Sense

Approximate

Randomize



Common Sense
Approximate
Randomize
Exploit Input Structure



Common Sense
Approximate
Randomize
Exploit Input Structure





Slides and Other Resources

<http://neeldhara.com/summer2013>